Effects of Hall current and Viscous dissipation on MHD Free Convective Casson fluid over a semi-infinite aligned vertical plate

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Abstract- The Scope of the present study explores the significance of a steady magnetohydrodynamics free convective Casson fluid past an semi-infinite aligned vertical plate. A transverse uniform magnetic field is considered. Hall current and Viscous dissipation are taken into account and in presence of thermal radiation with heat source and chemical reaction are analyzed. The mathematical modeling equations are transformed into ordinary differential equations and then solved analytically by using perturbation technique. The flow field on various aspects velocity, temperature and concentration are discussed and illustrated graphically. Velocity profile decreases when increasing magnetic parameter while the reverse trend persists for Casson fluid. The Skin friction, Nusselt number and Sherwood number are calculated and formulated in a table.

Keywords – Magnetohydrodynamics, Casson fluid, Thermal radiation, Heat source, Chemical reaction.

I. INTRODUCTION

The Emerging study of non-Newtonian fluids has got more interest among researchers in the last few decades, because of its immeasurable applications in science, engineering, technological, industries especially in extract of crude oil from petroleum. One cannot describe all the properties of non-Newtonian fluids, in a single constitutive equation. Casson fluid is also one of them and it is most popular among researchers due to its immense applications such as wire drawing, food processing, paper production, chemical and cosmetic industries, drilling, blood flow in narrow arteries, metallurgy, water pipe channels, pharmaceutical, thermal insulation and many others. In 1959, Casson [1] was investigated the Casson fluid model to found the behavior of flow of pigment oil suspensions. Examples of Casson fluids are tomato sauce, blood, honey, ketchup, toothpaste, shampoo, jelly, paints, concentrated fruit juice etc.

The unsteady two-dimensional flow of a non-Newtonian fluid over a stretching surface was discussed by Mukhopadhyay et al. [2]. Poply et al. [3] was investigated the aligned MHD flow in a Casson nanofluid by using RungeKuttaFehlberg method.

Many investigators have been included the MHD effects, along with heat and mass transfer, thermal radiation, chemical reaction and porous medium. It has widely used in various aspects like geophysics, astrophysical, nuclear fusion, heat pipes, filtration process etc. Patel et al. [4] analyzed MHD Casson fluid flow embedded in porous medium with radiation and chemical reaction by using laplace transform technique. Chemical reaction effect on MHD flow of Casson fluid with porous stretching sheet was inspected by Hari Krishna et al. [5]. Sandhya et al. [6] analyzed Radiation and chemical reaction effects on MHD Casson flow past a moving porous plate. Sulochana et al. [7] discussed the effect of frictional heating on Casson nanofluid over an inclined porous plate. MHD flow of a Casson fluid over an exponentially inclined permeable surface with thermal radiation and chemical reaction by Bala Anki Reddy[8].Panigrahi et al. [9] explained the heat and mass transfer of MHD Casson nanofluid flow with Newtonian heating and chemical reaction.

In 1879, Edwin Hall was first introduced the concept of Hall effects. Due to applied Ohm's law, some of the authors have not considered the Hall term. As a result, current which is perpendicular to electrical and magnetic field states Hall current effect. Its applications are used in nuclear fusion, MHD generators, refrigeration coils, electric transformers etc. Vijayaragavan et al. [10-14] studied Joule heating and thermal radiation effects, aligned magnetic field, Hall current effect, mixed convective, dual solutions of heat and mass transfer with Dufour effect and chemical reaction. The impact of Hall current on the entropy generation of radiative MHD Casson fluid by Opanuga et al. [15].RamRedddy et al. [16] elaborated Soret, Joule heating and Hall effects in a Casson fluid saturated porous medium in a vertical channel. Nandkeolyar et al. [17] studied numerical treatment of unsteady three-dimensional hydromagnetic flow of a Casson fluid with Hall effects by using Spectral quasilinearization method (SQLM). The influence of Hall and ion slip current on steady MHD Casson fluid past an infinite vertical porous plate in presence of Soret and chemical reaction by Rajakumar et al. [18]. Sulochana et al. [19] discussed unsteady MHD Casson fluid flow through vertical plate in presence of Hall current. Abd El-Aziz et al. [20] described the effect of Hall current on an unsteady free convection slip flow of a Casson fluid past an infinite vertical permeable plate in presence of suction. Sridhar et al. [21] illustrated numerical approach of MHD Casson fluid over an exponentially permeable stretching sheet with chemical reaction and Hall effect by using Keller-box method. Sandeep et al. [22] described heat and mass transfer in MHD Casson fluid over an exponentially stretching surface. Heat and mass transfer in MHD Casson fluid flow past an oscillating vertical plate with ramped wall temperature by Patel et al. [23]. Das et al. [24] examined unsteady MHD radiative chemically reactive Casson fluid by using fourth-order Runge-Kutta technique. Rudraswamy et al. [25] analyzed cross diffusion effect on MHD mixed convection flow of Casson fluid over a vertical plate. Sandeep et al. [26] explored the effects of induced magnetic field and homogeneous – heterogeneous reactions on stagnation flow of Casson fluid. Ali et al. [27] extended heat and mass transfer phenomenon for Casson fluid over shrinking wall subject to Lorentz force and heat source/sink. Raju et al. [28] discussed the viscous and joule's dissipation on Casson fluid with inclined magnetic field and multiple slips. Impact of chemical reaction on MHD natural convective flow through stretching sheet in presence of heat source/sink and viscous dissipation by Reddy et al. [29]. Das et al. [30] discussed Newtonian heating effect on unsteady hydromagnetic Casson fluid flow past a flat plate by using laplace technique. Magnetohydrodynamic axisymmetric flow of Casson fluid with thermal conductivity solved by homotopy analysis method (HAM) by Naz et al. [31].Scrutinization of thermal radiation, viscous dissipation and joule heating effects on marangoni convective two-phase flow of Casson fluid by Mahanthesh et al. [32].Jithender Reddy et al. [33] analyzed the influence of viscous dissipation on unsteady MHD natural convective fluid over an oscillating vertical plate via finite element method (FEM). Dero et al. [34] described the effects of viscous dissipation and chemical reaction on Casson nanofluid by using the shooting method with Maple implementation. Balamurugan et al. [35] studied the effect of viscous dissipation Ohmic heating and Hall current on MHD flow past an inclined porous plate in presence of Dufour effect, heat source and chemical reaction.

Motivated by all these aforementioned investigations , we are interested to inspect the effects of Hall current and Viscous dissipation on MHD free convective flow of Casson fluid past through a semi-infinite aligned vertical plate in presence of thermal radiation with heat source/sink and chemical reaction. The mathematical modeling equations of the non-linear partial differential equations are transformed into ordinary differential equations and then solved analytically by using perturbation technique. The effect of various pertinent parameters on velocity, temperature and concentration are illustrated. Also the local skin friction, Nusselt number, Shrewood number are calculated and given in the tabular form.

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II.MATHEMATICAL FORMULATION

Let us consider a steady two-dimensional MHD free convective flow, which is of viscous, incompressible Casson fluid past a semi-infinite aligned vertical plate (as shown in Fig 1.) having boundary layers and subjected to thermal and concentration buoyancy forces. Fixing the plate to be along x^* axis and y^* axis which is perpendicular to it. The wall is maintained at constant temperature T_w and concentration C_w, higher than the ambient temperature T_∞^* and concentration C_∞^* respectively. A transverse uniform magnetic field $B_\circ$ is applied, which is strong enough to produce the hall current in the flow field. Here we considering a two-dimensional , so the physical variable are independent of x^*, because of the length of the plate is large enough . The effects of hall current, viscous dissipation, thermal radiation, heat source/sink and chemical reaction are considered.

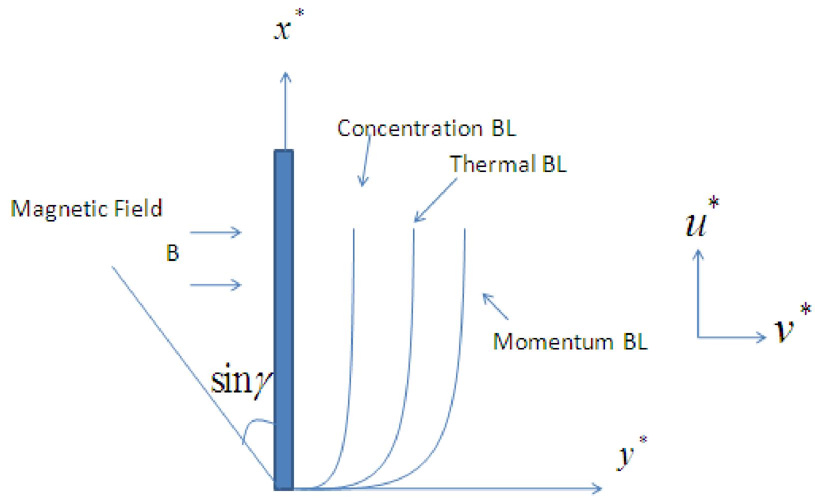


Figure 1. Physical Sketch of the problem

The Rheological equation for an isotropic and incompressible flow of Casson fluid is given by

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + P_y/\sqrt{2\pi}\right)e_{ij}, & \pi \succ \pi_C \\ 2\left(\mu_B + P_y/\sqrt{2\pi}\right)e_{ij}, & \pi \prec \pi_C \end{cases}$$

Where $\pi = e_{ij}e_{ij}$ and e_{ij} are the (i, j)th component of the deformation rate, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, P_y is the yield stress and π_C is the critical value of this product based on the non-Newtonian model. Under these assumptions , the governing equations are as follows:

Continuity Equation:

$$\frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

$$\left(v^* = -v_\circ,\ a\ constant\right)$$

Momentum Equation:

$$\rho v^* \frac{\partial u^*}{\partial y^*} = \mu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_\circ^2 \sin^2\gamma}{\left(1+m^2\right)}u^* + \rho g \beta_T\left(T^* - T_\infty^*\right) + \rho g \beta_C\left(C^* - C_\infty^*\right) \quad (2)$$

Energy Equation:

$$v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{Q_\circ}{\rho C_p}\left(T^* - T_\infty^*\right) - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*} + \frac{\mu}{\rho C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{\sigma B_\circ^2}{\rho C_p}u^{*2} \quad (3)$$

 ISSN No : 1006-7930

Concentration Equation:

$$v^* \frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - R^*\left(C^* - C^*_\infty\right) \quad (4)$$

With following boundary conditions

$$u=0, T^* = T_w, C^* = C_w, \; at \; y=0 \quad (5)$$

$$u \to 0, T^* \to T_\infty, C^* \to C_\infty, at \; y \to \infty \quad (6)$$

Where x^*, y^* denote the directions of dimensional distances along and perpendicular to the plate . u^*, v^* denote the dimensional velocities along x^*, y^* directions respectively. g be the gravitational acceleration, T^* is the dimensional temperature of the fluid near the plate, T^*_∞ is the free stream dimensional temperature, C^* is the dimensional concentration, C^*_∞ is free stream concentration, β is the Casson parameter, $v = \mu / \rho$ is the kinematic viscosity, k is the thermal conductivity of the fluid , ρ is the density of the fluid, m is the hall current parameter , σ is the electrical conductivity, C_p is the specific heat capacity at constant pressure, β_T and β_C are thermal and concentration expansion coefficient , $Q_\circ$ is the heat absorption coefficient, D is mass diffusivity, R is the chemical reaction, F is radiation parameter, q_r^* is the radiative heat flux.

Assume a fluid for an optically thin limit,

The radiative heat flux is given by

$$\frac{\partial q_r^*}{\partial y^*} = 4\left(T^* - T^*_\infty\right) I' \quad (7)$$

Where $I' = \int_0^\infty K_{\lambda w} \left(\frac{\partial e_{b\lambda}}{\partial T}\right)_w d\lambda,$ $K_{\lambda w}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is Planck's function.

Introducing the non -dimensional quantities

$$\left.\begin{aligned}
&y = \frac{y^* v_\circ}{v}, \quad u = \frac{u^*}{v_\circ}, \quad \theta = \frac{T^* - T^*_\infty}{T^*_w - T^*_\infty}, \quad \phi = \frac{C^* - C^*_\infty}{C^*_w - C^*_\infty}, \quad M = \frac{\sigma B_\circ^2 v}{\rho v_\circ^2} \\
&Gr = \frac{\rho g \beta_T v^2 \left(T^*_w - T^*_\infty\right)}{v_\circ^3 \mu}, \quad Gm = \frac{\rho g \beta_C v^2 \left(C^*_w - C^*_\infty\right)}{v_\circ^3 \mu}, \quad \Pr = \frac{\mu C_p}{k} \\
&S = \frac{Q_\circ v}{\rho C_p v_\circ^2}, \quad Ec = \frac{v_\circ^2}{C_p\left(T^*_w - T^*_\infty\right)}, \quad Sc = \frac{v}{D}, \quad F = \frac{4 v I'}{\rho C_p v_\circ^2}, \quad R = \frac{R^* v}{v_\circ^2}
\end{aligned}\right\} \quad (8)$$

Making use of Eqns (7) & (8), the governing eqns (2)-(4) are reduced into non-dimensional form as follows

$$\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - P_1 u + Gr\theta + Gr\phi = 0 \quad (9)$$

$$\frac{\partial^2\theta}{\partial y^2}+\Pr\frac{\partial\theta}{\partial y}+\Pr\left(S-F\right)\theta+\Pr Ec\left(\frac{\partial u}{\partial y}\right)^2+\Pr M\ Ec\,u^2=0$$

(10)

$$\frac{\partial^2\phi}{\partial y^2}+Sc\frac{\partial\phi}{\partial y}-Sc\,R\phi=0$$ (11)

Where $M_1=\dfrac{M}{1+m^2}$, $P_1=M_1\sin^2\gamma$

With the following boundary conditions

$u=0,\ \theta=1,\ \phi=1,\ at\ \ y=0$ (12)

$u\to 0,\ \theta\to 0,\ \phi\to 0,\ at\ \ y\to\infty$ (13)

III. METHOD OF SOLUTION

The set of partial differential equations (9)-(11) that cannot be solved in closed form. So these equations are transformed into a set of ordinary differential equations and solved analytically. Let us take

$u(y)=u_\circ(y)+Ecu_1(y)+o(Ec^2)$ (14)

$\theta(y)=\theta_\circ(y)+Ec\theta_1(y)+o(Ec^2)$ (15)

$\phi(y)=\phi_\circ(y)+\mathrm{Ec}\,\phi_1(y)+o\left(Ec^2\right)$ (16)

Using these equations (14)-(16) into (9)-(11), we can reduce the set of partial differential equations into ordinary differential equations, and then equating the coefficient of zero order and first order of Ec, we get the set of ordinary differential equations

$$\left(1+\frac{1}{\beta}\right)u_\circ''+u_\circ'-P_1u_\circ=-Gr\theta_\circ-Gm\phi_\circ$$

(17)

$\theta_\circ''+\Pr\theta_\circ'+\Pr\left(S-F\right)\theta_\circ=0$ (18)

$\phi_\circ''+Sc\phi_\circ'-ScR\phi_\circ=0$ (19)

Equating the coefficient of first order of Ec, we get

$$\left(1+\frac{1}{\beta}\right)u_1''+u_1'-P_1u_1=-Gr\theta_1-Gm\phi_1$$

(20)

$\theta_1''+\Pr\theta_1'+\Pr\left(S-F\right)\theta_1=-\Pr u_\circ'^2-\Pr Mu_\circ^2$ (21)

$\phi_1''+Sc\phi_1'-ScR\phi_1=0$ (22)

The corresponding boundary conditions are

$u_\circ=0,u_1=0,\theta_\circ=1,\theta_1=0,\phi_\circ=1,\phi_1=0,\ at\ \ y=0$ (23)

$u_\circ=0,\mathrm{u}_1=0,\theta_\circ=0,\theta_1=0,\phi_\circ=0,\phi_1=0,\ at\ \ y\to\infty$ (24)

Solving the above differential equations with following boundary conditions, we get the solution as follows

$$u_\circ=\left(A_3+A_4\right)e^{-A_2y}-A_3e^{-m_2y}-A_4e^{-m_1y}$$

(25)

$\theta_\circ=e^{-m_2y}$ (26)

$$\phi_\circ = e^{-m_1 y} \quad (27)$$

$$u_1 = A_{16}e^{-A_2 y} - A_{17}e^{-m_2 y} + A_{18}e^{-2A_2 y} + A_{19}e^{-2m_2 y} + A_{20}e^{-2m_1 y} \quad (28)$$

$$\theta_1 = A_{15}e^{-m_2 y} - A_5 e^{-2A_2 y} - A_6 e^{-2m_2 y} - A_7 e^{-2m_1 y} \quad (29)$$

$$\phi_1 = 0 \quad (30)$$

Substituting the above solutions (25)-(30) in (14)-(16) , we get the final solution of velocity, temperature and concentration in the boundary layer as follows

$$u(y) = (A_3 + A_4)e^{-A_2 y} - A_3 e^{-m_2 y} - A_4 e^{-m_1 y} + Ec\left(A_{16}e^{-A_2 y} - A_{17}e^{-m_2 y} + A_{18}e^{-2A_2 y} + A_{19}e^{-2m_2 y} + A_{20}e^{-2m_1 y}\right) \quad (31)$$

$$\theta(y) = e^{-m_2 y} + Ec\left(A_{15}e^{-m_2 y} - A_5 e^{-2A_2 y} - A_6 e^{-2m_2 y} - A_7 e^{-2m_1 y}\right) \quad (32)$$

$$\phi(y) = e^{-m_1 y} \quad (33)$$

3.1. Skin Friction –

In the non-dimensional form the skin friction for the velocity u is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= \left[-A_2(A_3 + A_4) + m_2 A_3 + m_1 A_4\right] + Ec\left[-A_2 A_{16} + m_2 A_{17} - 2A_2 A_{18} - 2m_2 A_{19} - 2m_1 A_{20}\right] \quad (34)$$

3.2. Nusselt Number –

The rate of heat transfer which is in non-dimensional form Nusselt number is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$= m_2 + Ec\left[m_2 A_{15} - 2A_2 A_5 - 2m_2 A_6 - 2m_1 A_7\right] \quad (35)$$

3.3. Sherwood Number –

The rate of mass transfer coefficient which is in non-dimensional form Sherwood number is given by

$$Sh = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$

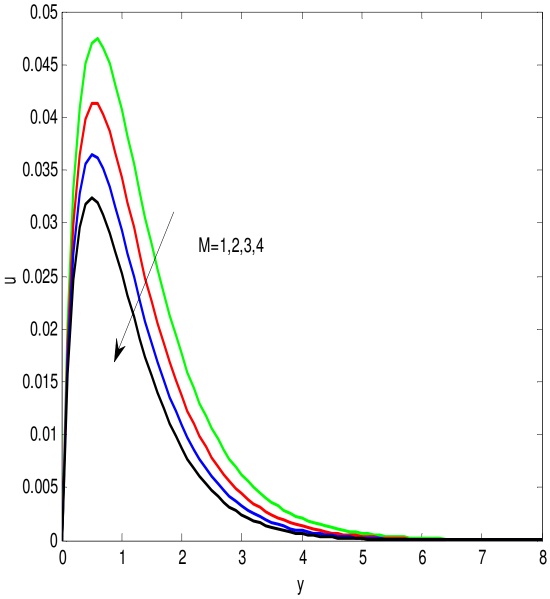
$$= m_1 \quad (36)$$

IV. RESULTS AND DISCUSSION

The system of non-linear ordinary differential equations (9)-(11) having boundary conditions are solved analytically. Results are expressed in terms of graphs for the pertinent parameters. Magnetic parameter M, Grashof number Gr, modified Grashof number Gm, Casson parameter β, Hall Current parameter m, Heat Source parameter S, Radiation parameter F, Prandtl number $\Pr$, Schmidt number Sc, Chemical Reaction parameter R, on velocity , temperature and concentration field. Also Skin Friction τ, Nusselt number Nu and Sherwood number Sh have been computed numerically and numerical values are formulated in the tabular form. The influence of $\Pr = 7.0$, $Sc = 2.0$, $Gr = 2$, $Gm = 2$, $R = 0.1$, $M = 1$, $S = 1$, $F = 1$, $\beta = 0.5$, $m = 3$, $\gamma = \pi/4$, $Ec = 0.01$ fixing these values as constant unless they are mentioned.

Figure 2.depicts that the velocity profile for different values of Magnetic parameter M. In figure. 2 the velocity profile decreases with increasing value of magnetic parameter because of the fact that an increasing value of M enhances the Lorentz force that acts against the flow. Various values of Grashof number Gr and modified Grashof number Gm are represented in figures 3-4. In these figures the maximum peak value is obtained because of the absence of buoyancy forces, it improves the boundary layer thickness and the velocity increases with increasing values of Grashof number and modified Grashof number. Velocity profile for different value of Casson parameter β is shown in figure 5. It is noted that velocity increases with the raising values of Casson fluid parameter due to the effect of the cooling plate.The strength of the magnetic field B_0 of an electrically conducting fluid produces Hall current. Figure 6represents the impact of hall current parameter m on velocity profile for different value of m. The velocity profile increases with increasing value of hall current parameter. But the opposite trend is seen in figure 7. In figure 7 displays the velocity profile for different value of radiation parameter F. Raising value in radiation parameter shows that the velocity profile decreases. Figures 8-9 shows that the velocity profile and temperature profile both increases with increasing value of Heat source parameter S. Figure 10 depicts that the temperature decreases with increasing value of Prandtl number $\Pr$. Because of the fact that the thermal conductivity of fluid decreases with increasing value of Prandtl number $\Pr$ and noted that decreases the thermal boundary layer thickness. Reverse trend is seen in figure 11. In Figure 11 represents that the temperature profile for different values of Radiation parameter F. It is noticed that the temperature profile decreases as increasing in Radiation parameter F. Figures 12-13 represent the concentration profiles for different values of Schmidt number Sc and Chemical reaction R. It is observed that the concentration profile for different values of Schmidt number and chemcial reaction. It is noticed that the concentration profile decreases with increasing values of Schmidt number Sc and Chemical reaction parameter R.

Fig 2 Velocity Profile for different values of Fig 4 Velocity Profile for different values of modified
Magnetic parameter M Grashof number Gm



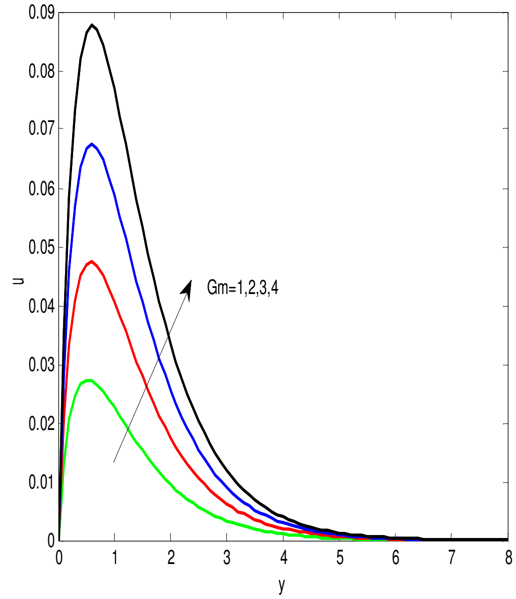
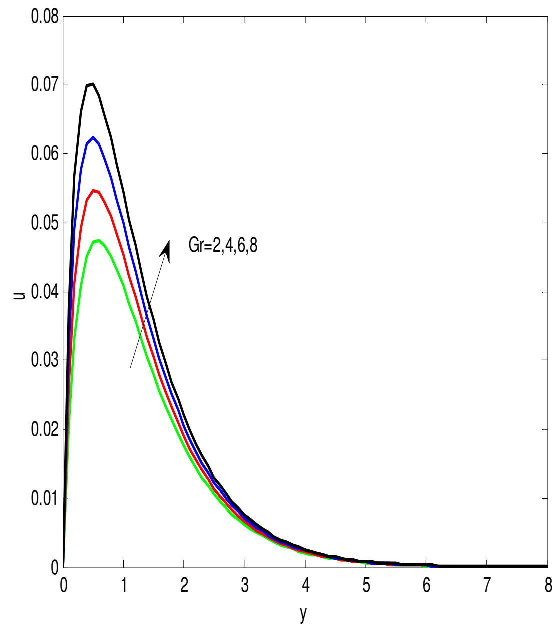


Fig 3.Velocity Profile for different values ofFig 5. Velocity Profile for different values of Casson parameter β
Grashof number Gr



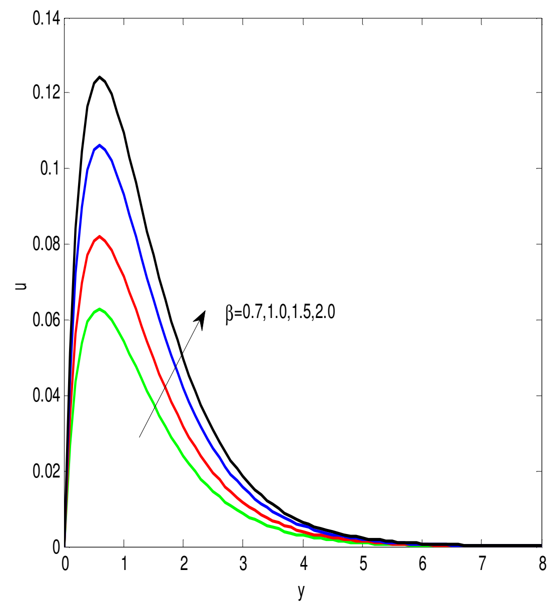
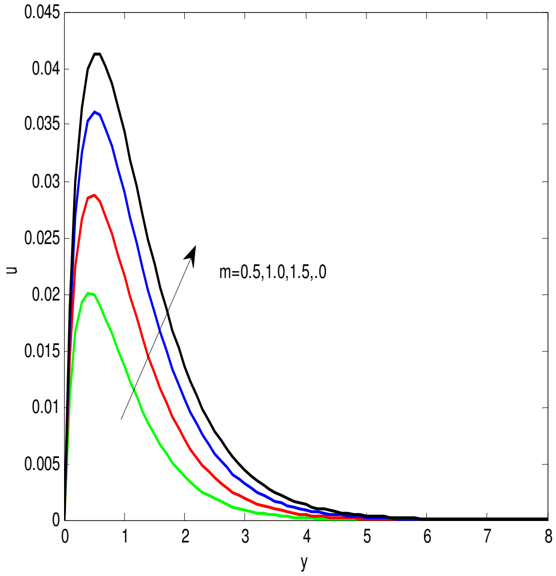


Fig 6.Velocity Profile for different values of Hall current parameter m Fig 7. Velocity Profile for different values of

heat source parameter S



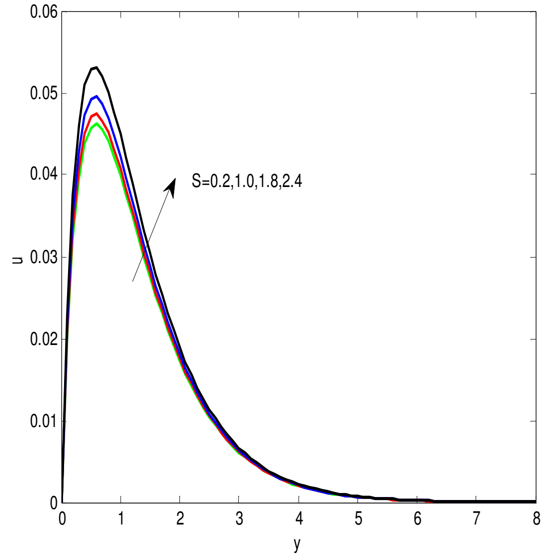
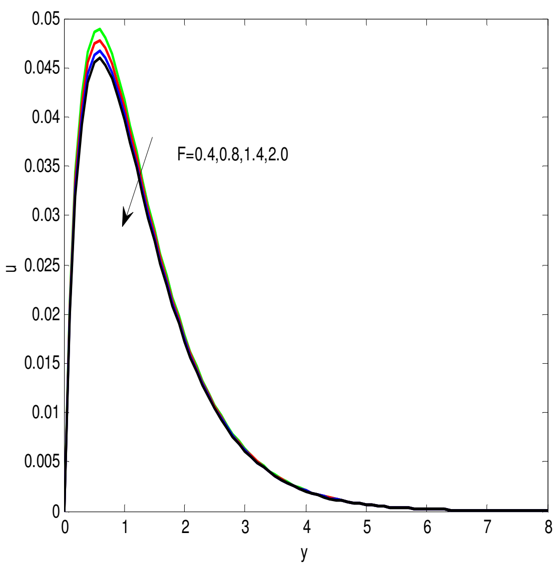


Fig 7.Velocity Profile for different values of Radiation parameter F

Fig 9. Temperature Profile for different values of Heat

Source parameter S



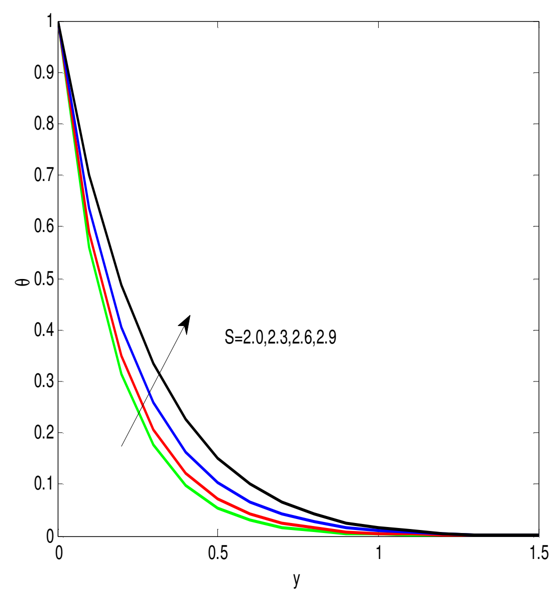
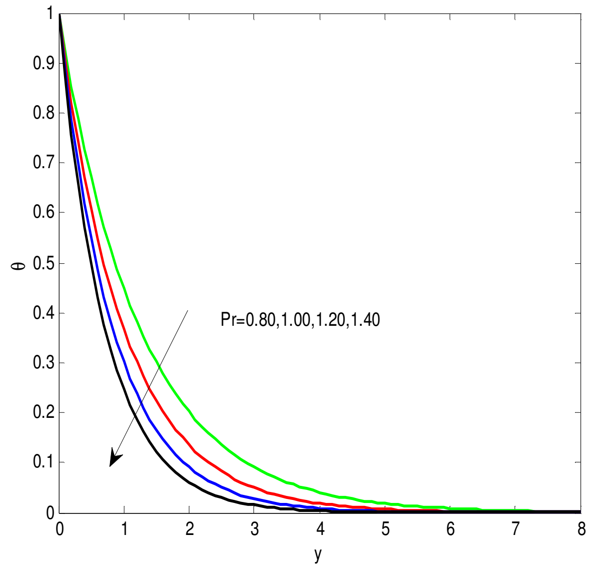


Fig 10. Temperature Profile for different values of Prandtl number $\Pr$



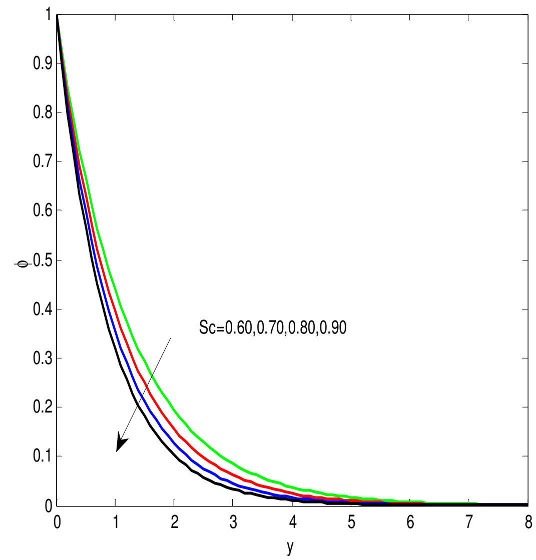


Fig 11. Temperature Profile for different values of Radiation parameter F

Fig 13. Concentration Profile for different values of Chemical reaction R

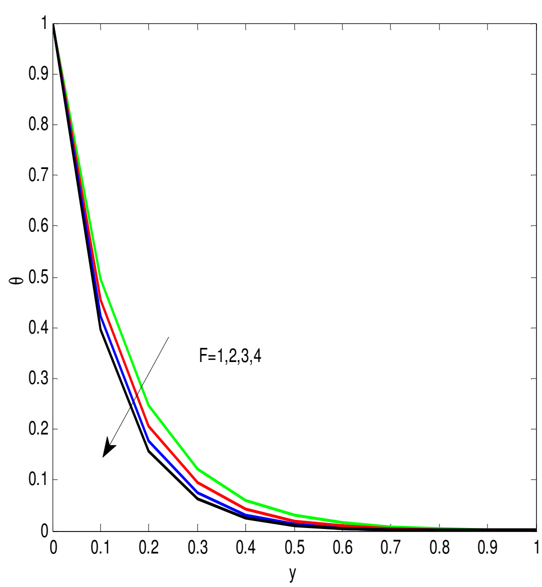


Fig 12. Concentration Profile for different values of Schmidt number Sc

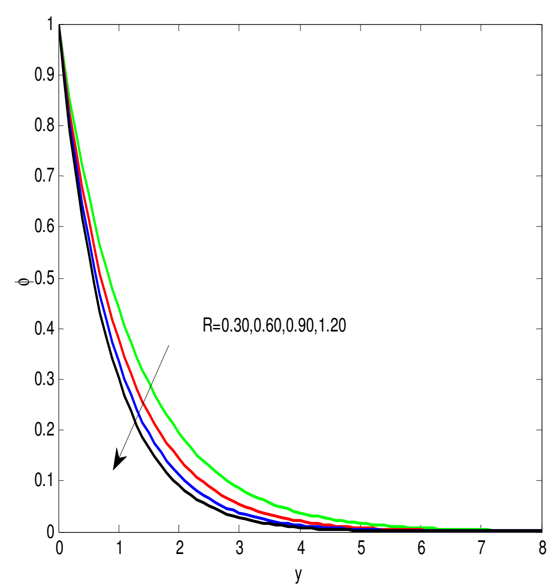


Table 1. Numerical values for Skin Friction (τ)

Gm	Gr	M	β	m	F	S	τ
1	2	1	0.5	3	1	1	0.1712
2	2	1	0.5	3	1	1	0.2585
3	2	1	0.5	3	1	1	0.3458
4	2	1	0.5	3	1	1	0.4332
2	**2**	1	0.5	3	1	1	0.2585
2	**4**	1	0.5	3	1	1	0.3424
2	**6**	1	0.5	3	1	1	0.4262
2	**8**	1	0.5	3	1	1	0.5101
2	2	**1**	0.5	3	1	1	0.2585
2	2	**2**	0.5	3	1	1	0.2379
2	2	**3**	0.5	3	1	1	0.2199
2	2	**4**	0.5	3	1	1	0.2036
2	2	1	**0.7**	3	1	1	0.3367
2	2	1	**1.0**	3	1	1	0.4325
2	2	1	**1.5**	3	1	1	0.5524
2	2	1	**2.0**	3	1	1	0.6403
2	2	1	0.5	**0.5**	1	1	0.1492
2	2	1	0.5	**1.0**	1	1	0.1887
2	2	1	0.5	**1.5**	1	1	0.2186
2	2	1	0.5	**2.0**	1	1	0.2379
2	2	1	0.5	3	**0.2**	1	0.2659
2	2	1	0.5	3	**0.8**	1	0.2607
2	2	1	0.5	3	**1.4**	1	0.2548
2	2	1	0.5	3	**2.0**	1	0.2503
2	2	1	0.5	3	1	**0.2**	0.2516
2	2	1	0.5	3	1	**1.0**	0.2585
2	2	1	0.5	3	1	**1.8**	0.2691
2	2	1	0.5	3	1	**2.4**	0.2841

Table 2.Numerical Values for Nusselt Number (Nu)Table 3. Numerical Values forSherwood Number (Sh)

S	Pr	F	Nu
2.0	7.0	1	5.7913
2.3	7.0	1	5.2748
2.6	7.0	1	4.5247
2.9	7.0	1	3.5000
1	**0.80**	1	0.8000
1	**1.00**	1	1
1	**1.20**	1	1.2000
1	**1.40**	1	1.4000
1	7.0	**1**	7
1	7.0	**2**	7.8875
1	7.0	**3**	8.6235
1	7.0	**4**	9.2663

Sc	R	Sh
0.60	0.30	0.8196
0.70	0.30	0.9266
0.80	0.30	1.0325
0.90	0.30	1.1374
0.60	**0.30**	0.8196
0.60	**0.60**	0.9708
0.60	**0.90**	1.0937
0.60	**1.20**	1.2000

V.CONCLUSION

In this article, we have investigated the effect of hall current and viscous dissipation of a steady magnetohydrodynamics free convective Casson fluid past an semi-infinite aligned vertical plate in presence of thermal radiation with heat source and chemical reaction. The non-linear partial differential equations (9)-(11) are transformed into ordinary differential equations and then solved analytically using perturbation technique. The effect of various pertinent parameters on velocity, temperature and concentration profiles are discussed and presented graphically. Also we formulated the values of Skin friction, Nusselt number and Sherwood number in tabular form. We concluded that

1) An increasing value of Grashof number, modified Grashof number, Casson fluid parameter, Hall current parameter enhances the velocity profile while reverse trend is seen for increasing value of Magnetic parameter.
2) The fluid temperature decreases while raising value of Prandtl number.
3) Both the velocity and temperature profiles increases while increasing value of heat source parameter but the reverse trend is seen for raising value of radiation parameter.
4) The concentration profile decreases when the raising value of both Schmidt number and chemical reaction
5) The local skin friction increases with increasing values of Grashof number, modified Grashof number, Casson fluid, Hall current and while decreases with increasing value of Magnetic and Radiation parameter.
6) The Nusselt number increases with increasing value of heat source parameter and while decreases with raising value of Prandtl number and Radiation parameter.
7) Both Schmidt number and Chemical reaction increases the Sherwood number.

Nomenclature –

x^*, y^* : Dimensional distance along and perpendicular to the plate
u^*, v^* : Dimensional velocities
g : Gravitational Acceleration
T^* : Dimensional Temperature
T_∞^* : Free stream dimensional Temperature
C^* : Dimensional Concentration
C_∞^* : Free stream dimensional Concentration
β : Casson parameter
$v = \mu / \rho$: Kinematic viscosity
Gr : Grashof number
Gm : Modified Grashof number
q_r^* : Radiative Heat flux
k : Thermal conductivity of the fluid
ρ : Density of the fluid
σ : Electrical Conductivity
C_p : Specific heat capacity at constant pressure
β_T : Thermal expansion coefficient
β_C : Concentration expansion coefficient
$Q_\circ$: Heat absorption coefficient
F : Radiation parameter
D : Mass Diffusivity
R : Chemical reaction

Appendix –

$$m_1 = \frac{Sc + \sqrt{Sc^2 + 4RSc}}{2}, m_2 = \frac{\Pr + \sqrt{\Pr^2 - 4\Pr(S-F)}}{2}, \quad A_2 = \frac{1 + \sqrt{1 + 4P_1\left(1 + \frac{1}{\beta}\right)}}{2}$$

$$A_3 = \frac{Gr}{\left(1 + \frac{1}{\beta}\right)m_2^2 - m_2 - P_1} \quad A_4 = \frac{Gm}{\left(1 + \frac{1}{\beta}\right)m_1^2 - m_1 - P_1} \quad A_5 = \frac{\Pr(A_3 + A_4)^2 (M + A_2^2)}{4A_2^2 - 2\Pr A_2 + \Pr(S-F)}$$

$$A_6 = \frac{\Pr A_3^2 (M + m_2^2)}{4m_2^2 - 2\Pr m_2 + \Pr(S-F)}, A_{15} = A_5 + A_6 + A_7, \quad A_{16} = A_{17} - A_{18} - A_{19} - A_{20}$$

$$A_7 = \frac{\Pr A_4^2 \left(M + m_1^2\right)}{4m_1^2 - 2\Pr m_1 + \Pr\left(S - F\right)}, A_{17} = \frac{GrA_{15}}{\left(1+\frac{1}{\beta}\right)m_2^2 - m_2 - P_1}, A_{18} = \frac{GrA_5}{4\left(1+\frac{1}{\beta}\right)A_2^2 - 2A_2 - P_1}$$

$$A_{19} = \frac{GrA_6}{4\left(1+\frac{1}{\beta}\right)m_2^2 - 2m_2 - P_1}, A_{20} = \frac{GrA_7}{4\left(1+\frac{1}{\beta}\right)m_1^2 - 2m_1 - P_1}$$

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