A Comparative Study on Icosikaioctagonal Fuzzy Numbers

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ABSTRACT:

In this paper, we proposed a new algorithm to solve icosikaioctagonal fuzzy transportation problem by assuming that a decision maker is uncertain about the specific values of the availability, requirement and transportation costs. We have investigated the results by implementing this method to an unbalanced transportation problem. The problem is to determine the minimum cost from a set of origins to a set of destinations, given constraints on the availability at each origin and the requirement at each destination. We used the methods of Vogel's approximation and Russel's approximation to investigate the transportation minimum cost for the Icosikaioctagonal fuzzy numbers.

Keywords:

Fuzzy numbers, Icosikaioctagonal fuzzy numbers, Fuzzy transportation problem,Crisp values, Ranking function, Vogel's Approximation method and Russel's Approximation method.

Introduction:

The concept of fuzzy sets was first introduced by Zadeh[8] in 1965. Zimmerman [3] formulated the fuzzy linear programming. The transportation problem deals with a special class of linear programming in which the objective is to transport a single commodity from several origins to different destinations so that the total transportation cost is a minimum, explained by Hillier and Liberian [1]. There are several literatures investigated about the fuzzy transportation problem. Nagoorkanni A, Abbas [4] and R. Srinivasan [7] were presented a new method for solving fuzzy transportation problem, P.Pandian and G.Natarajan [5] established a new algorithm for finding fuzzy optimal solution. Ranking fuzzy numbers was initially suggested by Zadeh[8] and also studied theR. Deepa and V Raju [6] solving fuzzy transportation problem using icosikaioctagonal fuzzy numbers.
Inthis paper we examined the fuzzy transportation problem in which the values of availability and requirement cost are represented by Icosikaioctagonal fuzzy numbers. Subsequent to applying the ranking functions, icosikaioctagonal fuzzy numbers are changed over into crisp values after that the optimization of transportation costs can be done by using Vogel's approximation method and Russel approximation method. The comparison study is done for the efficiency of both methods by using the icosikaioctagonal fuzzy numbers.

2. Preliminaries

In this section we characterize some fundamental definitions which will be utilized in this paper.

Definition 2.1 [2]:

In the unit interval [0, 1], each membership function map elements of a given universe set X, which is always a crisp set.

Definition 2.2 [2]:

A Fuzzy set A (fuzzy subset of X) is defined a mapping $A: X \to [0,1]$,where $A(x)$ is the membership degree of x to the fuzzy set A.We denote by $F(x)$,the collection of all fuzzy sets of X.

Definition 2.3 [2]:

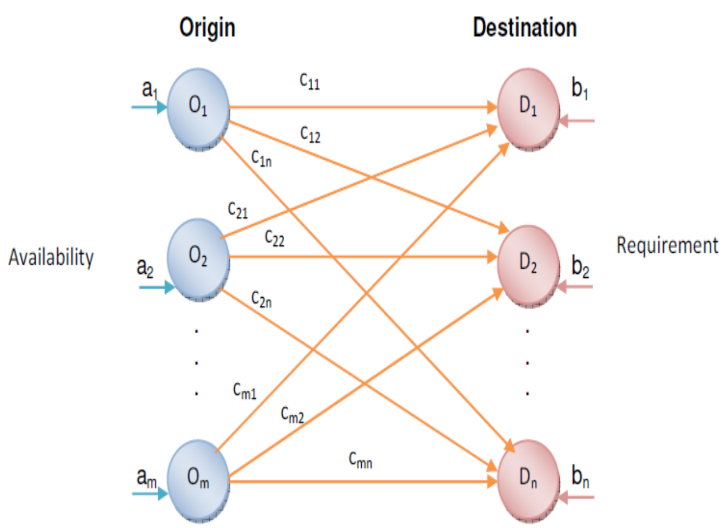
A fuzzy subset of the real line $u: R \rightarrow [0,1]$ is called fuzzy number if it satisfies the following properties.

i) u is normal.
ii) u is fuzzy convex.
iii) u is upper semi continuous on R
iv) u is completely supported.

Definition 2.4 [2]: The Transportation model

The transportation model is represented by the network in figure 1. There are m origins andn destinations, each represented by a node. The arcs represent the routes linking the origins and destinations. Arc*(k,l)* joining the origin$k$ to destination l carries two pieces of information the transportation cost per unit$c_{kl}$and the amount shipped x_{kl}. The amount of availability at origin$k$ is a_k and the amount of destination $l$is b_l. The objective of the model is to minimize the total transportation cost while satisfying all the availability and requirement restrictions.

Figure : 1



Definition 2.5 [2]:

The mathematical formulation of fuzzy Transportation problem

The mathematical formulation of the fuzzy transportation problem whose parameters are Icosikaioctagonal fuzzy numbers under the case that the total availability is equivalent to the total requirement is given by

Minz $= \sum_{k=1}^{m} \sum_{l=1}^{n} c_{kl}\, x_{kl}$

Subject to the constraints

$$\sum_{k=1}^{m} x_{kl} = a_k, \quad k = 1,2,3, \dots, m$$

$$\sum_{l=1}^{n} x_{kl} = b_{l,} \quad l = 1,2,3, \dots, n$$

$$\sum_{k=1}^{m} a_k \;=\sum_{l=1}^{n} b_l \quad , \; k = 1,2,3, \dots \dots m$$
$$l = 1,2,3, \dots \dots n \qquad \text{and } x_{kl} \geq 0$$

Remarks2.1:

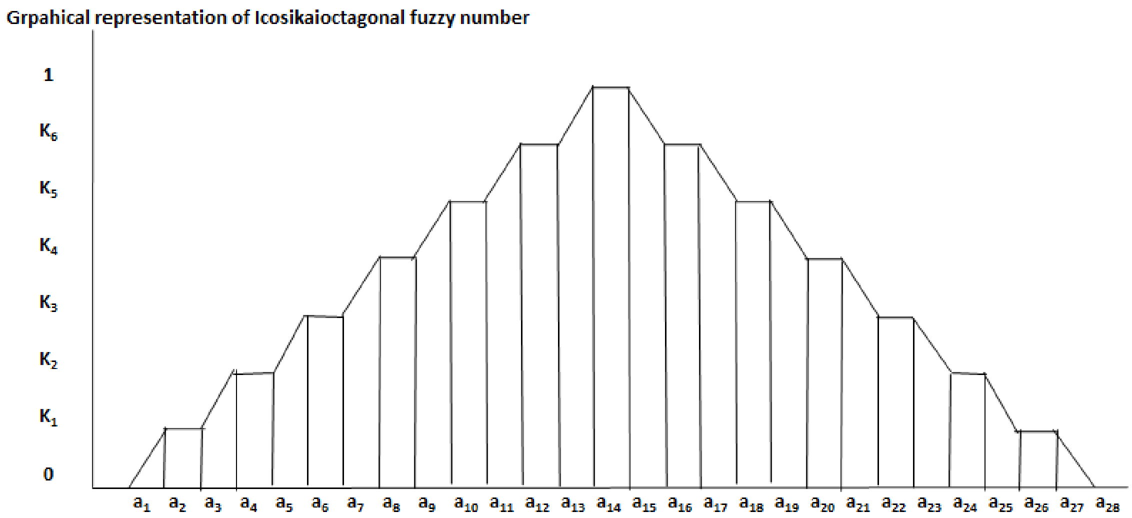
1. The total availabilities must equal to total requirement.
2. The total supply of all the sources is not equal to the total demand of all destinations; the problem is an unbalanced transportation problem.
3. The allocated cells in the transportation table having positive allocation are occupied cells and empty cells are non-occupied cells.
4. A feasible solution that minimizes the transportation cost is an optimal solution.
5. The number of decision variables of the general transportation problem at any stage of feasible solution must be m + n -1.

Definition 2.6 [6]:

ICOSIKAIOCTAGONAL FUZZY NUMBER

A generalized fuzzy number $\overline{M}_{IO} = (g_1 , g_2 , g_3 , g_4 , g_5 , g_6 , g_7 , g_8 , g_9 , g_{10} , g_{11} , g_{12} , g_{13} , g_{14} , g_{15} , g_{16} , g_{17} , g_{18} , g_{19} , g_{20} , g_{21} , g_{22} , g_{23} , g_{24} , g_{25} , g_{26} , g_{27} , g_{28})$ is well known to be icosikaioctagonal fuzzy number its membership function $\lambda_{\overline{M}_{IO}}(T)$ are listed below.

$$\begin{cases}
0 & \dots\dots t \leq g_1 \\
k_1\left(\frac{t-g_1}{g_2-g_1}\right) & \dots\dots g_1 \leq t \leq g_2 \\
k_1 & \dots\dots g_2 \leq t \leq g_3 \\
k_1 + (1-k_1)\left(\frac{t-g_3}{g_4-g_3}\right) & \dots\dots g_3 \leq t \leq g_4 \\
k_2 & \dots\dots g_4 \leq t \leq g_5 \\
k_2 + (1-k_2)\left(\frac{t-g_5}{g_6-g_5}\right) & \dots\dots g_5 \leq t \leq g_6 \\
k_3 & \dots\dots g_6 \leq t \leq g_7 \\
k_3 + (1-k_3)\left(\frac{t-g_7}{g_8-g_7}\right) & \dots\dots g_7 \leq t \leq g_8 \\
k_4 & \dots\dots g_8 \leq t \leq g_9 \\
k_4 + (1-k_4)\left(\frac{t-g_9}{g_{10}-g_9}\right) & \dots\dots g_9 \leq t \leq g_{10} \\
k_5 & \dots\dots g_{10} \leq t \leq g_{11} \\
k_5 + (1-k_5)\left(\frac{t-g_{11}}{g_{12}-g_{11}}\right) & \dots\dots g_{11} \leq t \leq g_{12} \\
k_6 & \dots\dots g_{12} \leq t \leq g_{13} \\
k_6 + (1-k_6)\left(\frac{t-g_{13}}{g_{14}-g_{13}}\right) & \dots\dots g_{13} \leq t \leq g_{14} \\
1 & \dots\dots g_{14} \leq t \leq g_{15} \\
k_6 + (1-k_6)\left(\frac{g_{16}-t}{g_{16}-g_{15}}\right) & \dots\dots g_{15} \leq t \leq g_{16} \\
k_6 & \dots\dots g_{16} \leq t \leq g_{17} \\
k_5 + (1-k_5)\left(\frac{g_{18}-t}{g_{17}-g_{18}}\right) & \dots\dots g_{17} \leq t \leq g_{18} \\
k_5 & \dots\dots g_{18} \leq t \leq g_{19} \\
k_4 + (1-k_4)\left(\frac{g_{20}-t}{g_{19}-g_{20}}\right) & \dots\dots g_{19} \leq t \leq g_{20} \\
k_4 & \dots\dots g_{20} \leq t \leq g_{21} \\
k_3 + (1-k_3)\left(\frac{g_{22}-t}{g_{21}-g_{22}}\right) & \dots\dots g_{21} \leq t \leq g_{22} \\
k_3 & \dots\dots g_{22} \leq t \leq g_{23} \\
k_2 + (1-k_2)\left(\frac{g_{24}-t}{g_{23}-g_{24}}\right) & \dots\dots g_{23} \leq t \leq g_{24} \\
k_2 & \dots\dots g_{24} \leq t \leq g_{25} \\
k_1 + (1-k_1)\left(\frac{g_{26}-t}{g_{25}-g_{26}}\right) & \dots\dots g_{25} \leq t \leq g_{26} \\
k_1 & \dots\dots g_{26} \leq t \leq g_{27} \\
k_1 + (1-k_1)\left(\frac{g_{28}-t}{g_{27}-g_{28}}\right) & \dots\dots g_{27} \leq t \leq g_{28} \\
0 & \dots\dots y \geq g_{28}
\end{cases}$$



Definition 2.7 [6]:

Ranking of Icosikaioctagonal Fuzzy number:

Let $\overline{M}$ be a normal Icosikaioctagonal fuzzy number. The value $H_0^{IO}\left(\overline{M}\right)$ is admitted as the measure of $\overline{M}$ and also it is mapped as follows:

$$H_0^{IO}\left(\overline{M}\right)=\frac{1}{2}\int_0^{k_1}[a_1(b)+a_2(b)\,ds]+\frac{1}{2}\int_{k_1}^{k_2}[e_1(f)+e_2(f)\,ds]+\frac{1}{2}\int_{k_2}^{k_3}[g_1(h)+g_2(h)\,ds]+$$

$$\frac{1}{2}\int_{k_3}^{k_4}[i_1(j)+i_2(j)\,ds]+\frac{1}{2}\int_{k_4}^{k_5}[q_1(r)+q_2(r)\,ds]+\frac{1}{2}\int_{k_5}^{k_6}[o_1(p)+o_2(p)\,ds]+$$

$$\frac{1}{2}\int_{k_6}^{1}[u_1(v)+u_2(v)\,ds]$$

$$H_0^{IO}\left(\overline{M}\right)=\frac{1}{4}\{(g_1+g_2+g_{27}+g_{28})\,k_1+(g_3+g_4+g_{25}+g_{26})\,(k_2-k_1)+(g_5+g_6+g_{23}+g_{24})$$

$$(k_3-k_2)+(g_7+g_8+g_{21}+g_{22})\,(k_4-k_3)\,(g_9+g_{10}+g_{19}+g_{20})\,(k_5-k_4)+$$

$$(g_{11}+g_{12}+g_{17}+g_{18})\,(k_6-k_5)+(g_{13}+g_{14}+g_{15}+g_{16})\,(1-k_6)\}$$

Where $0<k_1<k_2<k_3<k_4<k_5<k_6<1$

Definition 2.8:

Solving the problem of transportation cost optimization using Russel's approximation method can be said to have the same logic or working method using the Vogels approximation method.

The steps of Russell's Approximation Method are described as follows:

1. Arrange the value matrix of transport costs and value of the capacity of each source into columns and row.
2. Find the highest cost value for each row and column.
3. Subtract each cost value on the columns and rows at their highest cost.

4. Selecting the cell that has the greatest negative value from the calculation of step 2 and then allocates goods or products on the cell.
5. Repeat step 2 until step 4 until all products are distributed.
6. If the allocation has been completed, then calculate the distribution cost.

Numerical Example:

Consider availabilities, transportation costs and requirements are Icosikaioctagonal fuzzy transportation of the three origins X_1, X_2, X_3 together with the destinations Y_1, Y_2 , Y_3. The cost values of transporting from origin i to destination j, which are icosikaioctagonal fuzzy numbers are listed in Table 1.

Table 1:

Icosikaioctagonal Fuzzy Transportation table:

Origins \ Destinations	Y_1	Y_2	Y_3	Availability
X_1	-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	-1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26	-23, -22, -20, -18, -16, -14, -12, -10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 29	-11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16
X_2	-12, -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15	-9, -8, -7, -5, -3, -1, 0, 2, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24	-5, -3, -1, 0, 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47	-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18
X_3	-6, -4, -2, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48	-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	-42, -39, -36, -33, -30, -27, -24, -21, -18, -15, -12, -9, -3, 0, 3, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45	-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23
Requirement	-7, -6, -5, -4, -3, -2, -1, 0, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22	-15, -13, -11, -9, -7, -5, -3, -1, 0, 1, 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, 20, 22, 24, 26, 28	-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21	

Solution:

Case I:Vogel's Approximation Method

Icosikaioctagonal fuzzy number $\overline{M}$ is determined as

$$H_0^{IO}\left(\overline{M}\right) = \frac{1}{4}\{(g_1 + g_2 + g_{27} + g_{28})\, k_1 + (g_3 + g_4 + g_{25} + g_{26})\,(k_2 - k_1) + (g_5 + g_6 + g_{23} + g_{24})$$

$$(k_3 - k_2) + (g_7 + g_8 + g_{21} + g_{22})\,(k_4 - k_3)\,(g_9 + g_{10} + g_{19} + g_{20})\,(k_5 - k_4) +$$

$$(g_{11} + g_{12} + g_{17} + g_{18})\,(k_6 - k_5) + (g_{13} + g_{14} + g_{15} + g_{16})\,(1 - k_6)\}$$

Here $k_1 = 0.12$, $k_2 = 0.24$, $k_3 = 0.36$, $k_4 = 0.48$, $k_5 = 0.60$, $k_6 = 0.72$

Now we get the values of $H_0^{IO}\left(\overline{M}\right)$

Destinations / Origins	Y_1	Y_2	Y_3	Availabilities
X_1	4.5	12.5	3	2.5
X_2	1.5	9.42	20.21	4.5
X_3	21	4.5	1.71	7
Requirements	7.5	6.7	12.9	

$$\sum_{k=1}^{m} a_k \neq \sum_{l=1}^{n} b_l$$

The availabilities are less than the requirements, hence the problem is unbalanced transportation problem. Introduced a dummy row and rewrite the matrix in standard format which is given below:

Destinations / Origins	Y_1	Y_2	Y_3	Availabilities
X_1	4.5	12.5	3	2.5
X_2	1.5	9.42	20.21	4.5
X_3	21	4.5	1.71	7
X_4	0	0	0	13.1
Requirements	7.5	6.7	12.9	27.1

$$\sum_{k=1}^{m} a_k = \sum_{l=1}^{n} b_l$$

Now the problem is balanced transportation problem.

By applying Vogel's approximation method

Destinations / Origins	Y_1	Y_2	Y_3
X_1	4.5	12.5	3 [2.5]
X_2	1.5 [4.5]	9.42	20.21
X_3	21	4.5	1.71 [7]
	0 [3]	0 [6.7]	0 [3.4]

The total cost of transportation is

Minimize Z = 3 * 2.5 + 1.5 * 4.5 + 1.71 * 7 + 0 * 3 + 0 * 6.7 + 0 * 3.4

= 26.22

Case II: Russel's Approximation Method

Step :1

Destinations / Origins	Y_1	Y_2	Y_3	Availabilities
X_1	4.5	12.5	3	2.5
X_2	1.5 [4.5]	9.4	20.2	4.5 – 4.5=0
X_3	21	4.5	1.7	7
X_4	0	0	0	13.1
Requirements	7.5-4.5=3	6.7	12.9	

$$\Delta_1 = 4.5 - 33.5 = -29$$

$$\Delta_2 = 12.5 - 25 = -12.5$$

$$\Delta_3 = 3 - 32.7 = -29.7$$

$$\Delta_4 = 1.5 - 41.2 = -39.7$$

$$\Delta_5 = 9.4 - 32.7 = -23.3$$

$$\Delta_6 = 0 - 12.5 = -12.5$$

$$\Delta_7 = 20.2 - 40.2 = -20.2$$

$$\Delta_8 = 21 - 42 = -21$$

$$\Delta_9 = 4.5 - 33.5 = -29$$

$$\Delta_{10} = 1.7 - 41.2 = -39.5$$

$$\Delta_{11} = 0 - 21 = -21$$

$$\Delta_{12} = 0 - 20.2 = -20.2$$

Step:2

Origins \ Destinations	Y_1	Y_2	Y_3	Availabilities
X_1	4.5 [2.5]	12.5	3	2.5 – 2.5 = 0
X_3	21	4.5 [6.7]	1.7	7 – 6.7 = 0.3
X_4	0	0	0	1.31
Requirements	73 – 2.5 = 0.5	6.7 – 6.7 = 0	12.9	

$\Delta_1 = 4.5 - 33.5 = -29$

$\Delta_2 = 12.5 - 25 = -12.5$

$\Delta_3 = 3 - 15.5 = -12.5$

$\Delta_4 = 21 - 42 = -21$

$\Delta_5 = 4.5 - 33.5 = -29$

$\Delta_6 = 1.7 - 24 = -22.3$

$\Delta_7 = 0 - 21 = -21$

$\Delta_8 = 0 - 12.5 = -12.5$

$\Delta_9 = 0 - 3 = -3$

Step 3:

Origins \ Destinations	Y_1	Y_3	Availabilities
X_3	21 [0.3]	1.7	0.3 – 0.3 = 0
X_4	0 [0.2]	0 [12.9]	1.31 – 0.2 = 12.9
Requirements	0.5-0.3 = 0.2 - 0.2 = 0	12.9	

$\Delta_1 = 21 - 42 = -21$

$\Delta_2 = 1.7 - 22.7 = -21$

$\Delta_3 = 0 - 21 = -21$

$\Delta_4 = 0 - 1.7 = -1.7$

The total cost of transportation is

Minimize Z = $4.5 * 1.5 + 4.5 * 2.5 + 4.5 * 6.7 + 21 * 0.3 + 0 * 0.2 + 0 * 12.9$

$= 54.45$

The result of comparison of transportation cost optimization using Vogel's approximation method and Russel's approximation method for Icosikaioctagonal fuzzy number is given in the following table 2:

Table 2:Comparison of VAM and RAM

Method	Optimization Result
VAM	26.22
RAM	54.45

Conclusion:

In this paper the process of determining the fuzzy transportation problem using icosikaioctagonal fuzzy numbers is discussed with appropriate model. The optimal solution acquired through Vogel's approximation method and Russel's approximation method. The proposed ranking function proves that the total transportation cost found by VAM is more optimum when compared with the results obtained from the Russel's approximation method.

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