Retrial Fuzzy Queue Characteristics By Left - Right Representation

N Subashini[1] and N Anusheela[2]

[1] Department of Mathematics, Sri Krishna College of Engineering and Technology, Coimbatore, Tamil Nadu, India.

[2] Department of Mathematics, Government Arts College, Ooty, Tamil Nadu, India

Abstract. In this study, a procedure to find the various performance measures in terms of crisp values for FM/FM/1 queues in which the arrival rate and service rate are triangular fuzzy numbers has been proposed. LR method has the advantage of being short and convenient compared to other methods such as alpha-cuts method. The validity of the model is checked by a numerical example.

1. Introduction

Queuing models have a great extent of applications in service organizations, manufacturing firms, telecommunication networks, computer systems, inventory systems, etc., They are described by a feature such as an arriving customer finding the server busy, waits in a queue until the moment when the server is available to receive him, and leaves the system after service.

Fuzzy Logic was initiated by Zadeh[5]. Fuzzy queueing model was first introduced by Lie and Lee[3]. Further many authors namely Buckely [1], Negi and Lee[4], Chen [2] developed this model.

In this paper, we develop an approach that can provide the system characteristics of fuzzy queues in which the arrival rate, the service rate and the retrial rate are represented by triangular fuzzy numbers.

2. Basic Concepts

2.1 Definition

Let Z denote a universe of discourse. A fuzzy set $\tilde{A}$ in Z is determined by a membership function mapping elements of a domain space or universe of discourse Z to the unit interval [0, 1].

(i.e) $\tilde{A} = \{ \ (x, \ \eta_{\tilde{A}}(x) \ ); \ x \in Z \ \}$

Here $\eta_{\tilde{A}} : Z \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\eta_{\tilde{A}}(x)$ is called the membership value of $x \in Z$ in the fuzzy set $\tilde{A}$. Thus, the function value of $\eta_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$. For each $x \in Z$, $\eta_{\tilde{A}}(x)$=1, x is said mean value, modal value or mode of $\tilde{A}$

2.2 Definition

A fuzzy set $\tilde{A}$ of the universe of discourse Z is called a normal fuzzy set if there exists at least one $x \in Z$ such that $\eta_{\tilde{A}}(x)$ = 1.

2.3 Definition

A fuzzy set $\tilde{A}$ is convex if and only if for any $x_1, x_2 \in Z$, the membership function of $\tilde{A}$ satisfies the condition

$$\eta_{\tilde{A}}(\delta x_1 + (1-\delta)x_2) \geq \min\left(\eta_{\tilde{A}}(x_1), \eta_{\tilde{A}}(x_2)\right), \delta \in [0,1].$$

2.4 Definition

Let $\tilde{A}$ be a fuzzy subset in the universe of discourse Z. The α - *cut* , the support *supp* ($\tilde{A}$), the height *hgt*($\tilde{A}$) and the core *core*($\tilde{A}$) are crisp sets defined respectively as follows.

$$\tilde{A}_\alpha = \{x : \eta_{\tilde{A}}(x) \geq \alpha, x \in Z \text{ and } \alpha \in [0,1]\}$$

$$\sup p(\tilde{A}) = \{x : \eta_{\tilde{A}}(x) > 0, x \in Z \text{ and } \alpha \in [0,1]\}$$

$$hgt(\tilde{A}) = \max\{\eta_{\tilde{A}}(x), x \in Z \text{ and } \alpha \in [0,1]\}$$

$$core(\tilde{A}) = \{x : \eta_{\tilde{A}}(x) = 1, x \in Z \text{ and } \alpha \in [0,1]\}$$

2.5 Definition

A triangular fuzzy number $\tilde{A}(x)$ can be represented by $\tilde{A}(a,b,c;1)$ with membership function $\eta_{\tilde{A}}(x)$ given by

$$\eta_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ \dfrac{x-c}{b-c}, & \mathrm{b} \leq x \leq c \\ 0, & otherwise \end{cases}$$

Such triangular fuzzy number is often noted as $\tilde{A}(a,b,c)$. The L-R representation can be written as $\tilde{A}(a,b,c) = \langle b, b-a, c-b \rangle_{LR}$ for L(x) = R(x) = max(0, 1-x).

2.6 Definition

A fuzzy number $\tilde{M}$ is said to be L-R fuzzy number if and only if there exists three numbers m, $a > 0$, $b > 0$ and two positive, continuous and decreasing functions L and R from real number to [0,1] such that

$(0) = (0) = 1$

$(1) = 0, (x) > 0, \lim_{x\to\infty} (x) = 0$

$(1) = 0, (x) > 0, \lim_{x\to\infty} (x) = 0$

$$\mu_{\tilde{m}}(x) = \begin{cases} L\left(\frac{m-x}{a}\right), & \text{if } x \in [m-a, m] \\ R\left(\frac{x-m}{b}\right), & \text{if } x \in [m, m+b] \end{cases}$$

The L-R representation of the fuzzy number M is $\tilde{M} = \langle m, a, b\rangle_{LR.}$ m is called the mean value or modal value of $\tilde{M}$, a *and* b are called the left spread and right spread of $\tilde{M}$ Conventionally, $\tilde{M} = \langle m, 0, 0\rangle_{LR.}$ is the ordinary real number m, called fuzzy singleton.

$supp(\tilde{M}) =]m - a, m] \cup [m, m + b[=]m - a, m + b[$

2.7 Definition

If there are fuzzy numbers of the $\tilde{M} = \langle m,a,b\rangle_{LR}$ and $\tilde{N} = \langle n,c,d\rangle_{LR}$ same type Then their sum and their difference are also L-R fuzzy numbers of the same type given by

$$\tilde{M} + \tilde{N} = \langle m + n, a + c, b + d\rangle_{LR}$$
$$\tilde{M} - \tilde{N} = \langle m - n, a + d, b + c\rangle_{LR}$$

The product of L-R fuzzy numbers $\tilde{M} = \langle m,a,b\rangle_{LR}$ and $\tilde{N} = \langle n,c,d\rangle_{LR}$ is given by

$\tilde{M}.\tilde{N} = \langle m.n, mc + na - ac, md + nb + bd\rangle_{LR}$

The quotient secant approximation of L-R fuzzy numbers $\tilde{M} = \langle m,a,b\rangle_{LR}$ and $\tilde{N} = \langle n,c,d\rangle_{LR}$ is given by

$$\frac{\tilde{M}}{\tilde{N}} = \frac{\langle m,a,b\rangle_{LR}}{\langle n,c,d\rangle_{LR}} = \left\langle \frac{m}{n}, \frac{md}{n(n+d)} + \frac{a}{n} - \frac{ad}{n(n+d)}, \frac{mc}{n(n-c)} + \frac{b}{n} + \frac{bc}{n(n-c)} \right\rangle_{LR}$$

3. FM/FM/1 - R Queue Model

Consider a queuing system in which the computing network is identifiable as a Markovian single server retrial queue with patient customers and ill-known parameters denoted as FM/FM/1-R. At the end of a high speed manufacturing line in a mobile phone manufacturing unit, there are three quality control inspectors, who do the performance test of the product in EOL (End of Line) test machines. The machine takes 9 seconds to test a product and also the inspectors visually inspect the product under enhanced lighting conditions for aesthetical defects. This is also done sequentially and takes 9 seconds. Upon completion of inspection, the machine sends feedback to a centralized server for the test results and server triggers a common QR code printer with unique serial number for every product. It takes 5 seconds for the server to store the results and generate the unique serial number.

The fuzzified arrival rate $\tilde{\lambda}$, service rate $\tilde{\mu}$ and retrial rate $\tilde{\theta}$ are all triangular fuzzy numbers. Suppose arrival rate, service rate and retrial rate are approximately known and can be represented by convex fuzzy sets.

The system performance measures namely the expected request waiting time in the system, the expected requests number in the system, the expected request waiting time in the buffer and the expected requests number in the buffer are respectively given by

$$E(W) = \frac{\lambda + \theta}{\theta(\mu - \lambda)}$$

$$E(N) = \frac{\lambda(\lambda + \theta)}{\theta(\mu - \lambda)}$$

$$E(W_b) = \frac{\lambda(\mu + \theta)}{\theta\mu(\mu - \lambda)}$$

$$E(N_b) = \frac{\lambda^2(\mu + \theta)}{\theta\mu(\mu - \lambda)}$$

4. Numerical Example

The requests sent to the printer are considered as customers and thus arrival rate is $\tilde{\lambda} = [2, 3, 4]$. The printer is considered as the server and its speed as service rate $\tilde{\mu} = [5, 6, 7]$ and the buffer storage is an orbit and the repeated attempts are retrials rate $\tilde{\theta} = [1, 2, 3]$ are all triangular fuzzy numbers. In their L-R decomposition, the arrival rate, the service rate and the retrial rate are given by $\tilde{\lambda} = [3, 1, 1]$, $\tilde{\mu} = [6, 1, 1]$ and $\tilde{\theta} = [2, 1, 1]$

The expected request waiting time in the system, the expected requests number in the system, the expected request waiting time in the buffer and the expected requests number in the buffer are calculated as

$$E(W) = \frac{\lambda + \theta}{\theta(\mu - \lambda)} = \langle 0.8333,\ 0.6333,\ 6.1666 \rangle$$

$$E(N) = \frac{\lambda(\lambda + \theta)}{\theta(\mu - \lambda)} = \langle 2.5,\ 2.1,\ 25.5 \rangle$$

$$E(W_b) = \frac{\lambda(\mu + \theta)}{\theta\mu(\mu - \lambda)} = \langle 0.6666,\ 0.5633,\ 8 \rangle$$

$$E(N_b) = \frac{\lambda^2(\mu + \theta)}{\theta\mu(\mu - \lambda)} = \langle 2,\ 1.7714,\ 30 \rangle$$

According to the definitions, the modal values of the expected request waiting time in the system, the expected requests number in the system, the expected request waiting time in the buffer and the expected requests number in the buffer are 0.8333, 2.5, 0.6666 and 2 respectively. Their supports are given in the following open intervals.

] 0.8333 - 0.6333, 0.8333 + 6.1666 [=] 0.2, 6.9999 [

] 2.5 – 2.1, 2.5 +25.5 [=] 0.4, 28 [

] 0.6666 - 0.5633, 0.6666 + 8 [=] 0.1033, 8.6666 [

] 2 - 1.7714, 2 + 30 [=] 0.2286, 32 [

5. Results

The modal value of E(W) = 0.8333 gives the most possible value and the support of E(W) =] 0.2, 6.9999 [indicate that the expected request waiting time in the system is approximately between 0.2 and 6.9999.

The modal value of E(N) = 2.5 gives the most possible value and the support of E(N) =] 0.4, 28 [indicate that expected requests number in the system is approximately between 0.4 and 28.

The modal value of $E(W_b)$ = 0.6666 gives the most possible value and the support of $E(W_b)$ =] 0.1033, 8.6666 [indicate that the expected request waiting time in the buffer is approximately between 0.1033 and 8.6666.

The modal value of $E(N_b)$ = 2 gives the most possible value and the support of $E(N_b)$ =] 0.2286, 32 [indicate that the the expected requests number in the buffer is approximately between 0.2286 and 32.

6. Conclusion

The L-R method is used to analyze a fuzzy queue model. The triangular fuzzy number greatly simplifies manipulation. Numerical examples for triangular fuzzy numbers are explained effectively to determine the validity of the suggested model. The method used here is more effective in finding the performance measures of fuzzy queues. The results are given in LR representation. The future work can be done in examining the efficiency of this technique to other queueing models.

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