Analysis of fuzzy queues using Left Right Representation

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Abstract. In this study, a procedure to find the various performance measures in terms of crisp values for FM/FM/1 queues in which the arrival rate and service rate are triangular fuzzy numbers has been proposed. LR method has the advantage of being short and convenient compared to other methods such as alpha-cuts method. The validity of the model is checked by a numerical example.

1. Introduction

Queuing models have a great extent of applications in service organizations, manufacturing firms, telecommunication networks, computer systems, inventory systems, etc. The services are done in any one of the following queue discipline namely first in first out, last in first out, service in random order and selection in priority order.

Fuzzy Logic was initiated by Zadeh[5]. Fuzzy queueing model was first introduced by Lie and Lee[3]. Further many authors namely Buckely [1], Negi and Lee[4], Chen [2] developed this model.

In this paper, we develop an approach that can provide the system characteristics of fuzzy queues in which the arrival rate and the service rate are represented by triangular fuzzy numbers.

2. Basic Concepts

2.1 Definition

Let Z denote a universe of discourse. A fuzzy set $\tilde{A}$ in Z is determined by a membership function mapping elements of a domain space or universe of discourse Z to the unit interval [0, 1].

(i.e) $\tilde{A} = \{ (x, \eta_{\tilde{A}}(x)) ; x \in Z \}$
Here $\eta_A : Z \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\eta_A(x)$ is called the membership value of $x \in Z$ in the fuzzy set $\tilde{A}$. Thus, the function value of $\eta_A(x)$ is termed the grade of membership of $x$ in $\tilde{A}$. For each $x \in Z$, $\eta_A(x)=1$, $x$ is said mean value, modal value or mode of $\tilde{A}$.

2.2 Definition

A fuzzy set $\tilde{A}$ of the universe of discourse $Z$ is called a normal fuzzy set if there exists at least one $x \in Z$ such that $\eta_A(x) = 1$.

2.3 Definition

A fuzzy set $\tilde{A}$ is convex if and only if for any $x_1, x_2 \in Z$, the membership function of $\tilde{A}$ satisfies the condition $\eta_{\tilde{A}}(\delta x_1 + (1-\delta)x_2) \geq \min(\eta_{\tilde{A}}(x_1), \eta_{\tilde{A}}(x_2)), \delta \in [0,1]$.

2.4 Definition

Let $\tilde{A}$ be a fuzzy subset in the universe of discourse $Z$. The $\alpha$-cut, the support $\text{supp}(\tilde{A})$, the height $\text{hgt}(\tilde{A})$ and the core $\text{core}(\tilde{A})$ are crisp sets defined respectively as follows.

- $\tilde{A}_\alpha = \{x : \eta_A(x) \geq \alpha, x \in Z \text{ and } \alpha \in [0,1]\}$
- $\text{supp}(\tilde{A}) = \{x : \eta_A(x) > 0, x \in Z \text{ and } \alpha \in [0,1]\}$
- $\text{hgt}(\tilde{A}) = \max\{\eta_A(x), x \in Z \text{ and } \alpha \in [0,1]\}$
- $\text{core}(\tilde{A}) = \{x : \eta_A(x) = 1, x \in Z \text{ and } \alpha \in [0,1]\}$

2.5 Definition

A triangular fuzzy number $\tilde{A}(x)$ can be represented by $\tilde{A}(a,b,c)$ with membership function $\eta_A(x)$ given by

$$\eta_A(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ \frac{x-c}{b-c}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

Such triangular fuzzy number is often noted as $\tilde{A}(a,b,c)$. The L-R representation can be written as $\tilde{A}(a,b,c) = \langle b, b-a, c-b \rangle_{LR}$ for $L(x) = R(x) = \max(0, 1-x)$. 
2.6 Definition

A fuzzy number $\tilde{M}$ is said to be L-R fuzzy number if and only if there exists three numbers $m, a > 0, b > 0$ and two positive, continuous and decreasing functions $L$ and $R$ from real number to $[0,1]$ such that

$$
(0) = R(0) = 1
$$

$$
(1) = 0, L(x) > 0, \lim_{x \to 1} L(x) = 0
$$

$$
(1) = 0, R(x) > 0, \lim_{x \to 1} R(x) = 0
$$

$$
\mu_m(x) = \begin{cases} 
L \left( \frac{m-x}{a} \right), & \text{if } x \in [m-a,m] \\
R \left( \frac{x-m}{b} \right), & \text{if } x \in [m,m+b]
\end{cases}
$$

The L-R representation of the fuzzy number $M$ is $\tilde{M} = \langle m, a, b \rangle_{LR}$. $m$ is called the mean value or modal value of $\tilde{M}$, and $a$ and $b$ are called the left spread and right spread of $\tilde{M}$.

Conventionally, $\tilde{M} = \langle m, 0,0 \rangle_{LR}$ is the ordinary real number $m$, called fuzzy singleton.

$$
\text{supp}(\tilde{M}) = ]m - a, m[ \cup [m, m + b[ = ]m - a, m + b[
$$

2.7 Definition

If there are fuzzy numbers of the same type $\tilde{M} = \langle m, a, b \rangle_{LR}$ and $\tilde{N} = \langle n, c, d \rangle_{LR}$, then their sum and their difference are also L-R fuzzy numbers of the same type given by

$$
\tilde{M} + \tilde{N} = \langle m + n, a + c, b + d \rangle_{LR}
$$

$$
\tilde{M} - \tilde{N} = \langle m - n, a + d, b + c \rangle_{LR}
$$

The product of L-R fuzzy numbers $\tilde{M} = \langle m, a, b \rangle_{LR}$ and $\tilde{N} = \langle n, c, d \rangle_{LR}$ is given by

$$
\tilde{M} \cdot \tilde{N} = \langle mn, mc + na - ac, md + nb + bd \rangle_{LR}
$$

The quotient secant approximation of L-R fuzzy numbers $\tilde{M} = \langle m, a, b \rangle_{LR}$ and $\tilde{N} = \langle n, c, d \rangle_{LR}$ is given by

$$
\tilde{M} \div \tilde{N} = \langle \frac{m}{n}, \frac{md}{n(n+d)} - \frac{ad}{n(n+d)} + \frac{mc}{n(n-c)} - \frac{bc}{n(n-c)} \rangle_{LR}
$$
3. FM/FM/1 Queue Model

Consider a queuing system in which the customers arrive according to a Poisson process and the service rate follows an exponential distribution. The fuzzified arrival rate $\tilde{\lambda}$ and service rate $\tilde{\mu}$ are all triangular fuzzy numbers. Suppose arrival rate and service rate are approximately known and can be represented by convex fuzzy sets.

The system performance measures namely mean number of customers in the system, the waiting time of customers in the system and the probability that the system is idle are respectively given by

$$L_s = \frac{\tilde{\lambda}}{\tilde{\mu} - \tilde{\lambda}}$$
$$\bar{W}_s = \frac{\tilde{L}_s}{\tilde{\mu}}$$
$$P_s = \frac{\tilde{\lambda}}{\tilde{\mu}}$$

4. Numerical Example

The arrival rate $\tilde{\lambda}$ and service rate $\tilde{\mu}$ are all triangular fuzzy numbers represented by $\tilde{\lambda} = [3,4,7]$, $\tilde{\mu} = [11, 12, 15]$ respectively. In their L-R decomposition, the arrival rates and the service rates are given by $\lambda = [4, 1, 3]$, $\mu = [12, 1, 3]$.

Average number of customers in the system, the waiting time of customers in the system and the probability that the system is idle are calculated as

$$\bar{L}_s = \frac{\tilde{\lambda}}{\tilde{\mu} - \tilde{\lambda}} = \langle 0.5, 0.25, 1.25 \rangle_{LR}$$
$$\bar{W}_s = \frac{\tilde{L}_s}{\tilde{\mu}} = \langle 0.125, 0.0892, 0.4212 \rangle_{LR}$$
$$P_s = \frac{\tilde{\lambda}}{\tilde{\mu}} = \langle 0.333, 0.1333, 0.3030 \rangle_{LR}$$

According to the definitions the modal values of average number of customers in the system, the waiting time of customers in the system and the probability that the system is idle are 0.5, 0.125 and 0.333 respectively. Their supports are given in the following open intervals.

$]0.5 - 0.25, 0.5 + 1.25 [ = ]0.25, 1.75 [$

$]0.125 - 0.0892, 0.125 + 0.4212 [ = ]0.0358, 0.5462 [$

$]0.333 - 0.1333, 0.333 + 0.3030 [ = ]0.1997, 0.6360 [$
5. Results

The modal value of $\bar{L}_x = 0.5$ gives the most possible value and the support of $\bar{L}_x = [0.25, 1.75]$ indicate that the average number of customers in the system is approximately between 0.25 and 1.75.

The modal value of $\bar{W}_x = 0.125$ gives the most possible value and the support of $\bar{W}_x = [0.0358, 0.5462]$ indicate that the waiting time of customers in the system is approximately between 0.0358 and 0.5462.

The modal value of $P = 0.333$ gives the most possible value and the support of $P = [0.1997, 0.6360]$ indicate that the probability that the system is idle is approximately between 0.1997 and 0.6360.

6. Conclusion

The L-R method is used to analyze a fuzzy queue model. The triangular fuzzy number greatly simplifies manipulation. Numerical examples for triangular fuzzy numbers are explained effectively to determine the validity of the suggested model. The method used here is more effective in finding the performance measures of fuzzy queues. The results are given in LR representation. The future work can be done in examining the efficiency of this technique to other queueing models.

References


