Rudimentary Operations on Triacontakaidigon Fuzzy Number

T. Malathi[1,*] and P. Senthilkumar[2]

[1] *Research Scholar, Government Arts and Science College, Kangeyam, Tamilnadu, India*

[2] *Assistant Professor, Government Arts and Science College, Kangeyam, Tamilnadu, India*

Abstract

In this paper, a new form of fuzzy number named as "Triacontakaidigon Fuzzy Number" is introduced, as it is not possible to restrict the membership function to any specific form. The α-cut of triacontakaidigon fuzzy number is interpreted, with the use of interval arithmetic of α-cut, basic arithmetic operations are carried out and also illustrated by numerical examples.

Keywords

Fuzzy number, Triacontakaidigon Fuzzy Number, α-cut and Fuzzy operations.

1. Introduction

The seed of fuzzy set concept is introduced by Zadeh[1]. It is to deal with imprecise numerical quantities in a practical way. Fuzzy set theory allows the gradual assessment of the membership of elements in a set which is recounted

[1] Corresponding author* : *malathirscholar@gmail.com*

in the interval[0,1]. Raju and Jayagopal[8] introduced Icosikaioctogonal fuzzy number and also α-cut Icosikaioctogonal fuzzy number was defined and rudimentary operations are carried out using interim arithmetic of α-cut. In this paper, we propose Triacontakaidigon fuzzy number with its membership function. We provide with some of the fundamental arithmetic operations of Triacontakaidigon fuzzy number using arithmetic interval of α-cut and is exemplified by numerical examples. This fuzzy number is very useful in decision making problems.

2. Preliminaries

In this section, we give the preliminaries that are required for this study

Definition 2.1.

A fuzzy set of the real line with $\mu_A(x) : R \longrightarrow [0, 1]$ is called fuzzy number where $\mu_A(x)$is membership function if

(i) A must be normal and convex fuzzy set.

(ii) The support of A must be bounded.

(iii) ${}^{\alpha}A$ must be closed interval for every $\alpha \in [0, 1]$

Definition 2.2.

Let X be a non-empty set. A fuzzy set A of X is defined as

$A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A(x)$ is membership function which maps each element of X to a value between 0 and 1

Definition 2.3.

The support of a fuzzy set A defined on X is a crisp set defined as support (A)=$\{x : \mu_A(x) > 0\}$

2

Definition 2.4.

An α-cut of fuzzy set A is crisp set defined as ${}^{\alpha}[A]=\{x \in X/\mu_A(x) \geq \alpha\}$

Definition 2.5.

A fuzzy set A is convex fuzzy set iff each of it's ${}^{\alpha}A$ is convex set.

Definition 2.6.

A fuzzy number $A = (r_1, r_2, r_3)$, is triangular fuzzy number, if its membership function is given by

$$\mu_A(x) = \begin{cases} \dfrac{x - r_1}{r_2 - r_1}, & for \;\; r_1 \leq x \leq r_2 \\ \dfrac{r_3 - x}{r_3 - r_2}, & for \;\; r_2 \leq x \leq r_3 \\ 0, & for \;\; x > r_3 \end{cases}$$

where $(r_1 \leq r_2 \leq r_3)$

Definition 2.7.

A fuzzy number $A = (r_1, r_2, r_3, r_4)$, is said to be a trapezoidal fuzzy number, if its membership function is given by

$$\mu_A(x) = \begin{cases} 0, & for \;\; x < r_1 \\ \dfrac{x - r_1}{r_2 - r_1}; & for \;\; r_1 \leq x \leq r_2 \\ 1, & for \;\; r_2 \leq x \leq r_3 \\ \dfrac{r_4 - x}{r_4 - r_3}; & for \;\; r_3 \leq x \leq r_4 \\ 0, & for \;\; x > r_4 \end{cases}$$

where $(r_1 \leq r_2 \leq r_3 \leq r_4)$

ISSN No : 1006-7930

3. Triacontakaidigon Fuzzy Number

In this section, a new form of fuzzy number named as Triacontakaidigon Fuzzy Number is introduced. This fuzzy number can be used to solve decision making problems. A fuzzy number $A = l_1, l_2, l_3, ..., l_{32}$ is said to be Triacontakaidigon fuzzy number, where $l_1, l_2, l_3, ..., l_{32}$ are the real numbers which is given below $(0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \leq k_5 \leq k_6 \leq k_7 \leq 1)$

$$\mu_A(x) = \begin{cases} 0, \quad for \quad x < l_1 \\ k_1\left(\dfrac{x-l_1}{l_2-l_1}\right), \quad for \quad l_1 \leq x \leq l_2 \\ k_1, \quad for \quad l_2 \leq x \leq l_3 \\ k_1 + (k_2 - k_1)\left(\dfrac{x-l_3}{l_4-l_3}\right), \quad for \quad l_3 \leq x \leq l_4 \\ k_2, \quad for \quad l_4 \leq x \leq l_5 \\ k_2 + (k_3 - k_2)\left(\dfrac{x-l_5}{l_6-l_5}\right), \quad for \quad l_5 \leq x \leq l_6 \\ k_3, \quad for \quad l_6 \leq x \leq l_7 \\ k_3 + (k_4 - k_3)\left(\dfrac{x-l_7}{l_8-l_7}\right), \quad for \quad l_7 \leq x \leq l_8 \\ k_4, \quad for \quad l_8 \leq x \leq l_9 \\ k_4 + (k_5 - k_4)\left(\dfrac{x-l_9}{l_{10}-l_9}\right), \quad for \quad l_9 \leq x \leq l_{10} \\ k_5, \quad for \quad l_{10} \leq x \leq l_{11} \\ k_5 + (k_6 - k_5)\left(\dfrac{x-l_{11}}{l_{12}-l_{11}}\right), \quad for \quad l_{11} \leq x \leq l_{12} \\ k_6, \quad for \quad l_{12} \leq x \leq l_{13} \\ k_6 + (k_7 - k_6)\left(\dfrac{x-l_{13}}{l_{14}-l_{13}}\right), \quad for \quad l_{13} \leq x \leq l_{14} \\ k_7, \quad for \quad l_{14} \leq x \leq l_{15} \\ k_7 + (1 - k_7)\left(\dfrac{x-l_{15}}{l_{16}-l_{15}}\right), \quad for \quad l_{15} \leq x \leq l_{16} \\ 1, \quad for \quad l_{16} \leq x \leq l_{17} \\ k_7 + (1 - k_7)\left(\dfrac{l_{18}-x}{l_{18}-l_{17}}\right), \quad for \quad l_{17} \leq x \leq l_{18} \\ k_7, \quad for \quad l_{18} \leq x \leq l_{19} \end{cases}$$

4

$$\mu_A(x) = \begin{cases} k_6 + (k_7 - k_6)\left(\dfrac{l_{20} - x}{l_{20} - l_{19}}\right), & for \;\; l_{19} \leq x \leq l_{20} \\ k_6, \;\; for \;\; l_{20} \leq x \leq l_{21} \\ k_5 + (k_6 - k_5)\left(\dfrac{l_{22} - x}{l_{22} - l_{21}}\right), & for \;\; l_{21} \leq x \leq l_{22} \\ k_5, \;\; for \;\; l_{22} \leq x \leq l_{23} \\ k_4 + (k_5 - k_4)\left(\dfrac{l_{24} - x}{l_{24} - l_{23}}\right), & for \;\; l_{23} \leq x \leq l_{24} \\ k_4, \;\; for \;\; l_{24} \leq x \leq l_{25} \\ k_3 + (k_4 - k_3)\left(\dfrac{l_{26} - x}{l_{26} - l_{25}}\right), & for \;\; l_{25} \leq x \leq l_{26} \\ k_3, \;\; for \;\; l_{26} \leq x \leq l_{27} \\ k_2 + (k_3 - k_2)\left(\dfrac{l_{28} - x}{l_{28} - l_{27}}\right), & for \;\; l_{27} \leq x \leq l_{28} \\ k_2, \;\; for \;\; l_{28} \leq x \leq l_{27} \\ k_1 + (k_2 - k_1)\left(\dfrac{l_{30} - x}{l_{30} - l_{29}}\right), & for \;\; l_{29} \leq x \leq l_{30} \\ k_1, \;\; for \;\; r_{30} \leq x \leq r_{31} \\ k_1\left(\dfrac{l_{32} - x}{l_{32} - l_{31}}\right), \;\; for \;\; l_{31} \leq x \leq l_{32} \\ 0, \;\; for \; x > l_{32} \end{cases}$$

4. α-cut

Definition 4.1.

For $\alpha \in [0,1], the$ α-cut of Triacontakaidigon fuzzy number $A = l_1, l_2, l_3, ..., l_{32}$ is defined as,

$$[A]_\alpha = \begin{cases} l_1 + \left(\frac{\alpha}{k_1}\right)(l_2 - l_1), l_{32} - \left(\frac{\alpha}{k_1}\right)(l_{32} - l_{31})] & for \quad \alpha \in [0, k_1] \\ l_3 + \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(l_4 - l_3), l_{30} - \left(\frac{\alpha - k_1}{k_2 - k_1}\right)(l_{30} - l_{29}) & for \quad \alpha \in [k_1, k_2] \\ l_5 + \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(l_6 - l_5), l_{28} - \left(\frac{\alpha - k_2}{k_3 - k_2}\right)(l_{28} - l_{27}) & for \quad \alpha \in [k_2, k_3] \\ l_7 + \left(\frac{\alpha - k_3}{k_4 - k_3}\right)(l_8 - l_7), l_{26} - \left(\frac{\alpha - k_3}{k_3 - k_2}\right)(l_{26} - l_{25}) & for \quad \alpha \in [k_3, k_4] \\ l_9 + \left(\frac{\alpha - k_4}{k_5 - k_4}\right)(l_{10} - l_9), l_{24} - \left(\frac{\alpha - k_4}{k_5 - k_4}\right)(l_{24} - l_{23}) & for \quad \alpha \in [k_4, k_5] \\ l_{11} + \left(\frac{\alpha - k_5}{k_6 - k_5}\right)(l_{12} - l_{11}), l_{22} - \left(\frac{\alpha - k_5}{k_6 - k_5}\right)(l_{22} - l_{21}) & for \quad \alpha \in [k_5, k_6] \\ l_{13} + \left(\frac{\alpha - k_6}{k_7 - k_6}\right)(l_{14} - l_{13}), l_{20} - \left(\frac{\alpha - k_6}{k_7 - k_6}\right)(l_{20} - l_{19}) & for \quad \alpha \in [k_6, k_7] \\ l_{15} + \left(\frac{\alpha - k_7}{1 - k_7}\right)(l_{16} - l_{15}), l_{18} - \left(\frac{\alpha - k_7}{1 - k_7}\right)(a_{18} - l_{17}) & for \quad \alpha \in [k_7, 1] \end{cases}$$

4.2 Arithmetic operations on Triacontakaidigon Fuzzy Numbers

If $A = (l_1, l_2, l_3, ..., l_{32})$ and $B = (m_1, m_2, m_3, ..., m_{32})$ then,

- Addition is defined as,

$A + B = (l_1 + m_1, l_2 + m_2, ..., l_{32} + m_{32})$.

- Subtraction is defined as,

$A - B = (l_1 - m_1, l_2 - m_2, ..., l_{32} - m_{32})$.

- Multiplication is defined as,

$A * B = (l_1 * m_1, l_2 * m_2, ..., l_{32} * m_{32})$.

- Scalar multiplication for a positive integer k is defined as,

$k * A = (kl_1, kl_2, kl_3, ..., kl_{32})$.

5. Operations on Triacontakaidigon fuzzy number using α-cut

The α-cut of Triacontakaidigon fuzzy number$[A] = (l_1, l_2, l_3, ..., l_{32})$

for all $\in [0, 1]$, $When$ $k_1 = \frac{1}{8}, k_2 = \frac{2}{8}, k_3 = \frac{3}{8}, k_4 = \frac{4}{8}, k_5 = \frac{5}{8}, k_6 = \frac{6}{8}, k_7 = \frac{7}{8}$ is given by

$$[A]_\alpha = \begin{cases} [l_1 + 8\alpha(l_2 - l_1), l_{32} - 8\alpha(l_{32} - l_{31})] & for \quad \alpha \in [0, \frac{1}{8}] \\ [l_3 + (8\alpha - 1)(l_4 - l_3), l_{30} - (8\alpha - 1)(l_{30} - l_{29})] & for \quad \alpha \in [\frac{1}{8}, \frac{2}{8}] \\ [l_5 + (8\alpha - 2)(l_6 - l_5), l_{28} - (8\alpha - 2)(l_{28} - l_{27})] & for \quad \alpha \in [\frac{2}{8}, \frac{3}{8}] \\ [l_7 + (8\alpha - 3)(l_8 - l_7), l_{26} - (8\alpha - 3)(l_{26} - l_{25})] & for \quad \alpha \in [\frac{3}{8}, \frac{4}{8}] \\ [l_9 + (8\alpha - 4)(l_{10} - l_9), l_{24} - (8\alpha - 4)(l_{24} - l_{23})] & for \quad \alpha \in [\frac{4}{8}, \frac{5}{8}] \\ [l_{11} + (8\alpha - 5)(l_{12} - l_{11}), l_{22} - (8\alpha - 5)(l_{22} - l_{21})] & for \quad \alpha \in [\frac{5}{8}, \frac{6}{8}] \\ [l_{13} + (8\alpha - 6)(l_{14} - l_{13}), l_{20} - (8\alpha - 6)l_{20} - l_{19})] & for \quad \alpha \in [\frac{6}{8}, \frac{7}{8}] \\ [l_{15} + (8\alpha - 7)(l_{16} - l_{15}), l_{18} - (8\alpha - 7)(l_{18} - l_{17})] & for \quad \alpha \in [\frac{7}{8}, 1] \end{cases}$$

5.1 Addition of two Triacontakaidigon Fuzzy Numbers

If $A = (l_1, l_2, l_3, ..., l_{32})$ and $B = (m_1, m_2, m_3, ..., m_{32})$ be two Triacontakaidigon fuzzy numbers. Let us add the α-cuts of $[A_{Tkdi}]_\alpha$ and $[B_{Tkdi}]_\alpha$

7

of A and B respectively, using interval as defined below,

$$[A]_\alpha+[B]_\alpha=\begin{cases}[l_1+8\alpha(l_2-l_1),l_{32}-8\alpha(l_{32}-l_{31})] \\ \quad +[m_1+8\alpha(m_2-m_1),m_{32}-8\alpha(m_{32}-m_{31})] & for \quad \alpha\in[0,\frac{1}{8}] \\ [l_3+(8\alpha-1)(l_4-l_3),l_{30}-(8\alpha-1)(l_{30}-l_{29})] \\ \quad +[m_3+(8\alpha-1)(m_4-m_3),m_{30}-(8\alpha-1)(m_{30}-m_{29})] & for \quad \alpha\in[\frac{1}{8},\frac{2}{8}] \\ [l_5+(8\alpha-2)(l_6-l_5),l_{28}-(8\alpha-2)(l_{28}-l_{27})] \\ \quad +[m_5+(8\alpha-2)(m_6-b_5),m_{28}-(8\alpha-2)(m_{28}-m_{27})] & for \quad \alpha\in[\frac{2}{8},\frac{3}{8}] \\ [l_7+(8\alpha-3)(l_8-l_7),l_{26}-(8\alpha-3)(l_{26}-l_{25})] \\ \quad +[m_7+(8\alpha-3)(m_8-m_7),m_{26}-(8\alpha-3)(m_{26}-m_{25})] & for \quad \alpha\in[\frac{3}{8},\frac{4}{8}] \\ [l_9+(8\alpha-4)(l_{10}-l_9),l_{24}-(8\alpha-4)(l_{24}-l_{23})] \\ \quad +[m_9+(8\alpha-4)(m_{10}-m_9),m_{24}-(8\alpha-4)(m_{24}-m_{23})] & for \quad \alpha\in[\frac{4}{8},\frac{5}{8}] \\ [l_{11}+(8\alpha-5)(l_{12}-l_{11}),l_{22}-(8\alpha-5)(l_{22}-l_{21})] \\ \quad +[m_{11}+(8\alpha-5)(m_{12}-m_{11}),m_{22}-(8\alpha-5)(m_{22}-m_{21})] & for \quad \alpha\in[\frac{5}{8},\frac{6}{8}] \\ l_{13}+(8\alpha-6)(l_{14}-l_{13}),l_{20}-(8\alpha-6)(l_{20}-l_{19})] \\ \quad +m_{13}+(8\alpha-6)(m_{14}-m_{13}),m_{20}-8\alpha-6)(m_{20}-m_{19})] & for \quad \alpha\in[\frac{6}{8},\frac{7}{8}] \\ [l_{15}+(8\alpha-7)(l_{16}-l_{15}),l_{18}-(8\alpha-7)(l_{18}-l_{17})] \\ \quad +[m_{15}+(8\alpha-7)(m_{16}-m_{15}),m_{18}-(8\alpha-7)(m_{18}-m_{17})] & for \quad \alpha\in[\frac{7}{8},1]\end{cases}$$

5.2 Example

Let A=[1,2,3,4,5,......,32] , B=[2,4,6,8,10,.........,64].

For $\alpha\in[0,\frac{1}{8}]$, we have $[A]_\alpha$=$[1+8\alpha,32-8\alpha]$ and $[B]_\alpha$=$[2+16\alpha,64-16\alpha]$

Thus $[A]_\alpha+[B]_\alpha=[3+24\alpha,96-24\alpha]$.

when α=0,$[A]_\alpha+[B]_\alpha$=[3,96] and $\alpha=\dfrac{1}{8}$, $[\mathrm{A}]_\alpha+[B]_\alpha$=[6,93]

For $\alpha\in[\frac{1}{8},\frac{2}{8}]$, we have $[A]_\alpha$=$[2+8\alpha,31-8\alpha]$ and $[B]_\alpha$=$[4+16\alpha,62-16\alpha]$

Thus $[A]_\alpha+[B]_\alpha=[6+24\alpha,93-24\alpha]$.

when α=$\frac{1}{8}$,$[\mathrm{A}]_\alpha + [B]_\alpha$=[9,90] and α=$\frac{2}{8}$,$[\mathrm{A}]_\alpha + [B]_\alpha$=[12,87]

For $\alpha \in [\frac{2}{8}, \frac{3}{8}]$, we have $[A]_\alpha$=$[3+8\alpha, 30-8\alpha]$ and $[B]_\alpha$=$[6+16\alpha, 60-16\alpha]$

Thus $[A]_\alpha + [B]_\alpha = [9 + 24\alpha, 90 - 24\alpha]$.

when α=$\frac{2}{8}$,$[\mathrm{A}]_\alpha + [B]_\alpha$=[15,84] and α=$\frac{3}{8}$, $[\mathrm{A}]_\alpha + [B]_\alpha$=[18,81]

For $\alpha \in [\frac{3}{8}, \frac{4}{8}]$, we have $[A]_\alpha$=$[4+8\alpha, 29-8\alpha]$ and $[B]_\alpha$=$[8+16\alpha, 58-16\alpha]$

Thus $[A]_\alpha + [B]_\alpha = [12 + 24\alpha, 87 - 24\alpha]$.

when α=$\frac{3}{8}$,$[\mathrm{A}]_\alpha + [B]_\alpha$=[21,78] and α=$\frac{4}{8}$, $[\mathrm{A}]_\alpha + [B]_\alpha$=[24,75]

For $\alpha \in [\frac{4}{8}, \frac{5}{8}]$, we have $[A]_\alpha$=$[5+8\alpha, 28-8\alpha]$ and $[B]_\alpha$=$[10+16\alpha, 56-16\alpha]$

Thus $[A]_\alpha + [B]_\alpha = [15 + 24\alpha, 84 - 24\alpha]$.

when α=$\frac{4}{8}$,$[\mathrm{A}]_\alpha + [B]_\alpha$=[27,72] and α=$\frac{5}{8}$, $[\mathrm{A}]_\alpha + [B]_\alpha$ =[30,69]

For $\alpha \in [\frac{5}{8}, \frac{6}{8}]$, we have $[A]_\alpha$=$[6+8\alpha, 27-8\alpha]$ and $[B]_\alpha$=$[12+16\alpha, 54-16\alpha]$

Thus $[A]_\alpha + [B]_\alpha = [18 + 24\alpha, 81 - 24\alpha]$.

when α=$\frac{5}{8}$,$[\mathrm{A}]_\alpha + [B]_\alpha$=[33,66] and α=$\frac{6}{8}$,$[\mathrm{A}]_\alpha + [B]_\alpha$=[36,63]

For $\alpha \in [\frac{6}{8}, \frac{7}{8}]$ we have $[A]_\alpha$=$[7+8\alpha, 26-8\alpha]$ and $[B]_\alpha$=$[14+16\alpha, 52-16\alpha]$

Thus $[A]_\alpha + [B]_\alpha = [21 + 24\alpha, 78 - 24\alpha]$

when α =$\frac{1}{8}$,$[\mathrm{A}]_\alpha + [B]_\alpha$=[39,60] and α=$\frac{2}{8}$, $[\mathrm{A}]_\alpha + [B]_\alpha$=[42,57]

For $\alpha \in [\frac{7}{8}, 1]$, we have $[A]_\alpha$=$[8+8\alpha, 25-8\alpha]$ and $[B]_\alpha$=$[16+16\alpha, 50-16\alpha]$

Thus $[A]_\alpha + [B]_\alpha = [23 + 24\alpha, 75 - 24\alpha]$.

when α =$\frac{1}{8}$,$[\mathrm{A}]_\alpha + [B]_\alpha$=[45,54] and α=$\frac{2}{8}$, $[\mathrm{A}]_\alpha + [B]_\alpha$=[48,51]

5.3 Subtraction of two Triacontakaidigon Fuzzy Numbers:

If $A = (l_1, l_2, l_3, ..., l_{32})$ and $B = (m_1, m_2, m_3, ..., m_{32})$ be two Triacontakaidigon fuzzy numbers. Let us subtract the α-cuts of $[A_{Tkdi}]_\alpha$ and $[B_{Tkdi}]_\alpha$ of A and B respectively, using interval arithmetic as defined below.

$$[A]_\alpha - [B]_\alpha = \begin{cases} [l_1 + 8\alpha(l_2 - l_1), l_{32} - 8\alpha(l_{32} - l_{31})] \\ \quad -[m_1 + 8\alpha(m_2 - m_1), m_{32} - 8\alpha(m_{32} - m_{31})] & for \quad \alpha \in [0, \frac{1}{8}] \\ [l_3 + (8\alpha - 1)(l_4 - l_3), l_{30} - (8\alpha - 1)(l_{30} - l_{29})] \\ \quad -[m_3 + (8\alpha - 1)(m_4 - m_3), m_{30} - (8\alpha - 1)(m_{30} - m_{29})] & for \quad \alpha \in [\frac{1}{8}, \frac{2}{8}] \\ [l_5 + (8\alpha - 2)(l_6 - a_5), l_{28} - (8\alpha - 2)(l_{28} - l_{27})] \\ \quad -[m_5 + (8\alpha - 2)(m_6 - m_5), m_{28} - (8\alpha - 2)(m_{28} - m_{27})] & for \quad \alpha \in [\frac{2}{8}, \frac{3}{8}] \\ [l_7 + (8\alpha - 3)(l_8 - l_7), l_{26} - (8\alpha - 3)(l_{26} - l_{25})] \\ \quad -[m_7 + (8\alpha - 3)(m_8 - m_7), m_{26} - (8\alpha - 3)(m_{26} - m_{25})] & for \quad \alpha \in [\frac{3}{8}, \frac{4}{8}] \\ [l_9 + (8\alpha - 4)(l_{10} - l_9), l_{24} - (8\alpha - 4)(l_{24} - l_{23})] \\ \quad -[m_9 + (8\alpha - 4)(m_{10} - m_9), m_{24} - (8\alpha - 4)(m_{24} - m_{23})] & for \quad \alpha \in [\frac{4}{8}, \frac{5}{8}] \\ [l_{11} + (8\alpha - 5)(l_{12} - l_{11}), l_{22} - (8\alpha - 5)(l_{22} - l_{21})] \\ \quad -[m_{11} + (8\alpha - 5)(m_{12} - m_{11}), m_{22} - (8\alpha - 5)(m_{22} - m_{21})] & for \quad \alpha \in [\frac{5}{8}, \frac{6}{8}] \end{cases}$$

$$[A]_\alpha - [B]_\alpha = \begin{cases} [l_{13} + (8\alpha - 6)(l_{14} - l_{13}), l_{20} - (8\alpha - 6)(l_{20} - l_{19})] \\ \quad -[m_{13} + (8\alpha - 6)(m_{14} - m_{13}), m_{20} - (8\alpha - 6)(m_{20} - m_{19})] & for \quad \alpha \in [\frac{6}{8}, \frac{7}{8}] \\ [l_{15} + (8\alpha - 7)(l_{16} - l_{15}), l_{18} - (8\alpha - 7)(l_{18} - l_{17})] \\ \quad -[m_{15} + (8\alpha - 7)(m_{16} - m_{15}), m_{18} - (8\alpha - 7)(m_{18} - m_{17})] \\ & for \quad \alpha \in [\frac{7}{8}, 1] \end{cases}$$

5.4 Example

Let A=[2,4,6,8,10,.........64] *and* B=[0,1,2,3,4,.........31]

For $\alpha \in [0, \frac{1}{8}], we have$ $[\text{A}]_\alpha$=$[2 + 16\alpha, 64 - 16\alpha]$ and $[B]_\alpha$=$[8\alpha, 31 - 8\alpha]$

Thus $[A]_\alpha - [B]_\alpha = [2 + 8\alpha, 33 - 8\alpha]$.

when α=0, $[\text{A}]_\alpha - [B]_\alpha$=[2,33] and α=$\frac{1}{8}$, $[\text{A}]_\alpha - [B]_\alpha$=[3,32]

For $\alpha \in [\frac{1}{8}, \frac{2}{8}]$, we have $[A]_\alpha$=$[4+16\alpha, 62-16\alpha]$ and $[B]_\alpha$= $[1+8\alpha, 30-8\alpha]$

Thus $[A]_\alpha - [B]_\alpha = [3 + 8\alpha, 32 - 8\alpha]$.

when α=$\frac{1}{8}$,$[\text{A}]_\alpha - [B]_\alpha$=[4,31] and α=$\frac{2}{8}$, $[\text{A}]_\alpha - [B]_\alpha$=[5,30]

For $\alpha \in [\frac{2}{8}, \frac{3}{8}]$, we have $[A]_\alpha$=$[6+16\alpha, 60-16\alpha]$ and $[B]_\alpha$=$[2+8\alpha, 29-8\alpha]$

Thus $[A]_\alpha - [B]_\alpha = [4 + 8\alpha, 31 - 8\alpha]$.

and α=$\frac{3}{8}$, $[\text{A}]_\alpha - [B]_\alpha$=[7,28]

For $\alpha \in [\frac{3}{8}, \frac{4}{8}]$, we have $[A]_\alpha$=$[8+16\alpha, 58-16\alpha]$ and $[B]_\alpha$=$[3+8\alpha, 28-8\alpha]$

Thus $[A]_\alpha - [B]_\alpha = [5 + 8\alpha, 30 - 8\alpha]$.

when α=$\frac{3}{8}$, $[\text{A}]_\alpha - [B]_\alpha$=[8,27] and α=$\frac{4}{8}$, $[\text{A}]_\alpha - [B]_\alpha$=[9,26]

For $\alpha \in [\frac{4}{8}, \frac{5}{8}]$, we have $[A]_\alpha$=$[10+16\alpha, 56-16\alpha]$ and $[B]_\alpha$=$[4+8\alpha, 27-8\alpha]$

Thus $[A]_\alpha - [B]_\alpha = [6+8\alpha, 29-8\alpha]$.

when $\alpha=\dfrac{4}{8}$, $[\mathrm{A}]_\alpha - [B]_\alpha$=[10,25] and $\alpha=\dfrac{5}{8}$, $[\mathrm{A}]_\alpha - [B]_\alpha$=[11,24]

For $\alpha \in [\frac{5}{8}, \frac{6}{8}]$, we have $[A]_\alpha$=$[12+16\alpha, 54-16\alpha]$ and $[B]_\alpha$=$[5+8\alpha, 26-8\alpha]$

Thus $[A]_\alpha - [B]_\alpha = [7+8\alpha, 28-8\alpha]$.

when $\alpha=\dfrac{5}{8}$,$[\mathrm{A}]_\alpha - [B]_\alpha$=[12,23] and $\alpha=\dfrac{6}{8}$, $[\mathrm{A}]_\alpha - [B]_\alpha$=[13,22]

For $\alpha \in [\frac{6}{8}, \frac{7}{8}]$, we have $[A]_\alpha$=$[14+16\alpha, 52-16\alpha]$

and $[B]_\alpha$=$[6+8\alpha, 25-16\alpha]$

Thus $[A]_\alpha - [B]_\alpha$=$[8+8\alpha, 27-8\alpha]$.

when $\alpha=\dfrac{6}{8}$,$[\mathrm{A}]_\alpha - [B]_\alpha$=[14,21] and $\alpha=\dfrac{7}{8}$, $[\mathrm{A}]_\alpha - [B]_\alpha$=[15,20]

For $\alpha \in [\frac{7}{8}, 1]$, we have $[A]_\alpha$=$[16+16\alpha, 50-16\alpha]$ and $[B]_\alpha$=$[7+8\alpha, 24-8\alpha]$

Thus $[A]_\alpha - [B]_\alpha = [9+8\alpha, 26-8\alpha]$.

when $\alpha=\dfrac{7}{8}$,$[\mathrm{A}]_\alpha - [B]_\alpha$=[16,19] and α=1, $[A]_\alpha - [B]_\alpha$=[17,18]

5.5 Multiplication of two Triacontakaidigon Fuzzy Numbers

If A=$(l_1, l_2, l_3, ..., l_{32})$ and B=$(m_1, m_2, m_3, ..., m_{32})$ be two Triacontakaidigon fuzzy numbers. Let us multiply the α-cuts of $[\mathrm{A}_{Tkdi}]_\alpha$ and $[B_{Tkdi}]_\alpha$ of A and B respectively, using interval arithmetic as defined below.

$$[\mathrm{A}]_\alpha * [B]_\alpha = \begin{cases} [l_1+8\alpha(l_2-l_1), l_{32}-8\alpha(l_{32}-l_{31})] \\ \quad *[m_1+8\alpha(m_2-m_1), m_{32}-8\alpha(m_{32}-m_{31})] & for \;\; \alpha \in [0,\frac{1}{8}] \\ [l_3+(8\alpha-1)(l_4-l_3), l_{30}-(8\alpha-1)(l_{30}-l_{29})] \\ \quad *[m_3+(8\alpha-1)(m_4-m_3), m_{30}-(8\alpha-1)(m_{30}-m_{29})] & for \;\; \alpha \in [\frac{1}{8},\frac{2}{8}] \\ [l_5+(8\alpha-2)(l_6-l_5), l_{28}-(8\alpha-2)(l_{28}-l_{27})] \\ \quad *[m_5+(8\alpha-2)(m_6-m_5), m_{28}-(8\alpha-2)(m_{28}-m_{27})] & for \;\; \alpha \in [\frac{2}{8},\frac{3}{8}] \\ [l_7+(8\alpha-3)(l_8-l_7), l_{26}-(8\alpha-3)(l_{26}-l_{25})] \\ \quad *[m_7+(8\alpha-3)(m_8-m_7), m_{26}-(8\alpha-3)(m_{26}-m_{25})] & for \;\; \alpha \in [\frac{3}{8},\frac{4}{8}] \\ [l_9+(8\alpha-4)(l_{10}-l_9), l_{24}-(8\alpha-4)(l_{24}-l_{23})] \\ \quad *[m_9+(8\alpha-4)(m_{10}-m_9), m_{24}-(8\alpha-4)(m_{24}-m_{23})] & for \;\; \alpha \in [\frac{4}{8},\frac{5}{8}] \\ [l_{11}+(8\alpha-5)(l_{12}-l_{11}), l_{22}-(8\alpha-5)(l_{22}-l_{21})] \\ \quad *[m_{11}+(8\alpha-5)(m_{12}-m_{11}), m_{22}-(8\alpha-5)(m_{22}-m_{21})] & for \;\; \alpha \in [\frac{5}{8},\frac{6}{8}] \\ [l_{13}+(8\alpha-6)(l_{14}-l_{13}), l_{20}-(8\alpha-6)(l_{20}-l_{19})] \\ \quad *[m_{13}+(8\alpha-6)(m_{14}-m_{13}), m_{20}-(8\alpha-6)(m_{20}-m_{19})] & for \;\; \alpha \in [\frac{6}{8},\frac{7}{8}] \\ [l_{15}+(8\alpha-7)(l_{16}-l_{15}), l_{18}-(8\alpha-7)(l_{18}-l_{17})] \\ \quad *[m_{15}+(8\alpha-7)(m_{16}-m_{15}), m_{18}-(8\alpha-7)(m_{18}-m_{17})] & for \;\; \alpha \in [\frac{7}{8},1] \end{cases}$$

5.6 Example

Let A=[2,4,6,8,10,........,64] and B=[0,1,2,3,4,........,31]

For $\alpha \in [0, \frac{1}{8}]$, we have $[A]_\alpha$=$[1+8\alpha, 32-8\alpha]$ and $[B]_\alpha$=$[0+8\alpha, 31-8\alpha]$

Thus $[A]_\alpha * [B]_\alpha = [8\alpha + 64\alpha^2, 992 - 504\alpha + 64\alpha^2]$.

When α=0, $[A]_\alpha * [B]_\alpha$=[0,992] and α=$\frac{1}{8}$ $[\text{A}]_\alpha * [B]_\alpha = [9, 930]$

For $\alpha \in [\frac{1}{8}, \frac{2}{8}]$, we have $[A]_\alpha$=$[2 + 8\alpha, 31 - 8\alpha]$ and $[B]_\alpha$=$[1{+}8\alpha, 30 - 8\alpha]$

Thus $[A]_\alpha * [B]_\alpha = [2 + 24\alpha + 64\alpha^2, 930 - 488\alpha + 64\alpha^2]$.

When α =$\frac{5}{8}$, $[\text{A}]_\alpha * [B]_\alpha$=[6,443] and α =$\frac{6}{8}$, $[\text{A}]_\alpha * [B]_\alpha$ =[9,810]

For $\alpha \in [\frac{2}{8}, \frac{3}{8}]$, we have $[A]_\alpha = [3{+}8\alpha, 30{-}8\alpha]$ and $[B]_\alpha$= $[2{+}8\alpha, 29{-}8\alpha]$,

Thus $[A]_\alpha * [B]_\alpha = [6 + 40\alpha + 64\alpha^2, 870 - 472\alpha + 64\alpha^2]$.

When α=$\frac{5}{8}$, $[\text{A}]_\alpha * [B]_\alpha$=[20,756] and α=$\frac{6}{8}$, $[\text{A}]_\alpha * [B]_\alpha$=[30,702]

For $\alpha \in [\frac{3}{8}, \frac{4}{8}]$, we have $[A]_\alpha$=$[4{+}8\alpha, 29{-}8\alpha]$ and $[B]_\alpha$ =$[3{+}8\alpha, 28{-}8\alpha]$

Thus $[A]_\alpha * [B]_\alpha = [12 + 56\alpha + 64\alpha^2, 812 - 456\alpha + 64\alpha^2]$.

When α=$\frac{5}{8}$,$[\text{A}]_\alpha * [B]_\alpha$ =[42,650] and α=$\frac{6}{8}$, $[\text{A}]_\alpha * [B]_\alpha$ =[56,600]

For $\alpha \in [\frac{4}{8}, \frac{5}{8}]$, we have $[A]_\alpha$=$[5 + 8\alpha, 28 - 8\alpha]$ and $[B]_\alpha$=$[4{+}8\alpha, 27 - 8\alpha]$

Thus $[A]_\alpha * [B]_\alpha = [20 + 72\alpha + 64\alpha^2, 756 - 440\alpha + 64\alpha^2]$.

When α=$\frac{5}{8}$,$[\text{A}]_\alpha * [B]_\alpha$=[72,552] and α=$\frac{6}{8}$, $[\text{A}]_\alpha * [B]_\alpha$=[90,506]

For $\alpha \in [\frac{5}{8}, \frac{6}{8}]$, we have $[A]_\alpha$=$[6{+}8\alpha, 27{-}8\alpha]$ and $[B]_\alpha$=$[5{+}8\alpha, 26{-}8\alpha]$

Thus $[A]_\alpha - [B]_\alpha = [30 + 88\alpha + 64\alpha^2, 702 - 424\alpha + 64\alpha^2]$.

when α=$\frac{5}{8}$, $[\text{A}]_\alpha * [B]_\alpha$=[10,462] and α=$\frac{6}{8}$, $[\text{A}]_\alpha * [B]_\alpha$=[132,420]

For $\alpha \in [\frac{6}{8}, \frac{7}{8}]$, we have $[A]_\alpha$=$[7{+}8\alpha, 25{-}8\alpha]$ and $[B]_\alpha$=$[6{+}8\alpha, 52{-}16\alpha]$

Thus $[A]_\alpha * [B]_\alpha = [42 + 104\alpha + 64\alpha^2 650 - 408\alpha - 64\alpha^2]$.

14

When $\alpha = \dfrac{6}{8}$, $[A]_\alpha * [B]_\alpha$ =[156,380] and $\alpha = \dfrac{7}{8}$, $[A]_\alpha * [B]_\alpha$ =[182,342]

For $\alpha \in [\frac{7}{8}, 1]$, we have $[A]_\alpha = [8+8\alpha, 7+8\alpha]$ and $[B]_\alpha$ =[25-8α, $24-8\alpha$]

Thus $[A]_\alpha * [B]_\alpha = [56 + 120\alpha + 64\alpha^2, 600 - 392\alpha + 64\alpha^2]$.

When $\alpha = \dfrac{7}{8}$, $[A]_\alpha * [B]_\alpha + 64\alpha^2$=[210,306] and α=1, $[A]_\alpha * [B]_\alpha$ =[240,615]

6. Conclusion

In this paper, a new form of fuzzy number named as the Triacontakaidigon fuzzy number was introduced. The arithmetic operations are performed with an arithmetic interval of α-cuts and are illustrated with numerical examples. Triacontakaidigon fuzzy number can be applied to that problem, which has thirty two points in representation. In future, it may be applied in operations research problems.

References

[1] Abhinav Bansal., “Some nonlinear arithmetic operations on triangular fuzzy numbers(m,α,β)”, Advances in fuzzy mathematics, Page No.147-156, (2010)

[2] Abhinav Bansal., “Trapezoidal fuzzy numbers(a,b,c,d):Arithmetic Behavior”, international journal of physical and mathematical sciences”, Vol. 2, no. 1,

[3] Dubois .D., and Prade .H., “Operations on fuzzy numbers, International Journal of Systems Science”, Vol.9, no.6.,pp.613-626, (1978).

[4] Raju V and Jayagopal. R., “A Rudimentary Operations on Octagonal

Fuzzy Numbers", International Journal of Research in Advent Technology, Vol. 6, no. 6, 1320-1323, (2018).

[5] Rezvani. S., "Multiplication Operation on Trapezoidal Fuzzy numbers", Journal of Physical Sciences,Vol no-15,17-26, (2011)

[6] Raju. V and Jayagopal. V., "An Approach on Icosikaioctagonal Fuzzy number-Traditional Operations on Icosikaioctagonal fuzzy number", A Journal of composition theory, Vol.XII, Issue X, Page No: 727-734, (2019).

[7] Zadeh. L.A., "Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems", Vol. 1, no. 1, 3-28, (1978).