Generalized e-Irresolute Mappings in Intuitionistic Fuzzy Topology

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Abstract- The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy ge-irresolute mappings in intuitionistic fuzzy topological space. We investigate some of their properties and obtain several preservation properties and some characterizations concering intuitionistic fuzzy e-irresolute mappings.

Keywords –Intuitionistic fuzzy set, Intuitionistic fuzzy topology, Intuitionistic fuzzy ge-closed sets, Intuitionistic fuzzy ge-open sets, Intuitionistic fuzzy ge-continuity, Intuitionistic fuzzy ge-irresolute.

I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh in his classis paper [18]. Using the concept of fuzzy set Chang [3] introduced the fuzzy topological spaces. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Coker [5] defined the intuitionistic fuzzy topological spaces. This approach provided a wide field for investigation in the area of fuzzy topology and its applications. Thakur and Chaturvedi [14, 15] were the first contributors who considered generalized closed sets and generalized continuity in intuitionistic fuzzy topology. Recently Thakur, Rathor and Thakur [13] introduced the concepts of generalized e-closed sets and generalized e-continuity in intuitionistic fuzzy topology. The present paper studied generalized e-irresolute mappings in intuitionistic fuzzy topology.

II.PRELIMINARIES

This section contains some basic notions and results that are used in sequel. First we present the fundamental definitions obtained by [1, 5].

Definition 2.1 [1] Let X be a nonempty fixed set and I the closed interval [0,1]. An intuitionistic fuzzy set A is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$where the functions $\mu_A: X \to I$ and $\nu_A: X \to I$ denote the degree of membership namely $\mu_A(x)$ and the degree of nonmembership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Obviously, every fuzzy set A on a nonempty set X is an intuitionistic fuzzy set having the form $A = \{< x, \mu_A(x), 1 - \mu_A(x) >: x \in X\}$.
For the basic properties of Intuitionistic fuzzy set the researchers should refer [1].

Definition 2.2 [5] An intuitionistic fuzzy topology in Coker's sense on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfy the following axioms:

(a). $\tilde{0}, \tilde{1} \in \tau$,

(b). $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(c). $\cup G_i \in \tau$ for any arbitrary family $\{G_i: i \in j\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set in X.

Definition 2.3 [5] The Complement A^C of an intuitionistic fuzzy open set A in an intuitionistic fuzzy topological space X, τ is called an intuitionistic fuzzy closed set in X.

Definition 2.4 [17] Let (X, τ) be an Intuitionistic fuzzy topological space and A be any intuitionistic fuzzy set of X. Then the intuitionistic fuzzy e interior of A (In short eint(A)) and intuitionistic fuzzy e closure of A (In short $ecl(A)$) are defined as

$eint(A) = \cup$ { G | G is an Intuitionistic fuzzy e open set in X and $G \subseteq A$},

$ecl(A) = \cap$ { K | K is an Intuitionistic fuzzy e closed set in X and $A \subseteq K$}.

Definition 2.5 An Intuitionistic fuzzy set A in an Intuitionistic fuzzy topological space (X, τ) is called:

(a). Intuitionistic fuzzy ge-closed if $ecl(A) \subseteq O$, whenever $A \subseteq O$ and O is an intuitionistic fuzzy open. [13]

(b). Intuitionistic fuzzy ge-open if $O \subseteq eint(A)$, whenever $O \subseteq A$ and O is an intuitionistic fuzzy closed. [13]

Definition 2.6 [13] A mapping $f: (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic fuzzy ge-continuous if the inverse image of every intuitionistic fuzzy closed set of Y is intuitionistic fuzzy ge-closed in X.

III. Intuitionistic Fuzzy Generalized e-Irresolute Mappings

In this section, we introduce the concept of intuitionistic fuzzy ge-irresolute mappings and study some of their properties in intuitionistic fuzzy topological spaces.

Definition 3.1 *A mapping f from an intuitionistic fuzzy topological space (X, τ) to another intuitionistic fuzzy topological space (Y, σ) is said to be intuitionistic fuzzy ge-irresolute if the inverse image of every intuitionistic fuzzy ge-closed set of Y is intuitionistic fuzzy ge-closed in X.*

Theorem 3.2 *A mapping $f: (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic fuzzy ge-irresolute if and only if the inverse image of every intuitionistic fuzzy ge-open set in Y is intuitionistic fuzzy ge-open in X.*

Proof. It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$, for every intuitionistic fuzzy set U of Y.

Remark 3.3 *Since every intuitionistic fuzzy closed set is intuitionistic fuzzy ge-closed, it is clear that every intuitionistic fuzzy ge-irresolute mapping is intuitionistic fuzzy ge-continuous but the converse may not be true.*

Example 3.4 *Let$X = \{a, b\}$, $Y = \{p, q\}$ and intuitionistic fuzzy sets U and defined as follows:*

$U = \{< a, 0.7, 0.3 >, < b, 0.5, 0.5 >\}$

let $\tau = \{\tilde{0}, U, \tilde{1}\}$ and $\sigma = \{\tilde{0}, \tilde{1}\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f: (X, \tau) \to (Y, \sigma)$ defined by $f(a) = p$ and $f(b) = q$ is intuitionistic fuzzy continuous and hence intuitionistic fuzzy ge-continuous but not intuitionistic fuzzy ge-irresolute.

Example 3.5 *Let$X = \{a, b\}$, $Y = \{p, q\}$ and intuitionistic fuzzy sets V defined as follows:*

$V = \{< a, 0.6, 0.4 >, < b, 0.5, 0.5 >\}$

let $\tau = \{\tilde{0}, \tilde{1}\}$ and $\sigma = \{\tilde{0}, V, \tilde{1}\}$ be intuitionistic fuzzy topologies on X and Y respectively. Then the mapping $f: (X, \tau) \to (Y, \sigma)$ defined by $f(a) = p$ and $f(b) = q$ is intuitionistic fuzzy ge-irresolute but not intuitionistic fuzzy continuous.

Remark 3.6 *Example*(3.4) *and Example* (3.5) *asserts that the concepts of intuitionistic fuzzy ge-irresolute and intuitionistic fuzzy continuous mappings are independent.*

Theorem 3.7 *A mapping $f: (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic fuzzy ge-irresolute if and only if for every intuitionistic fuzzy set A of X.*

(a). $gecl(f(A)) \subseteq f(gecl(A))$.

(b). $gecl(f^{-1}(B)) \subseteq f^{-1}(gecl(B))$.

Proof. Obvious.

Theorem 3.8 *Let$f: (X, \tau) \to (Y, \sigma)$ is bijective intuitionistic fuzzy open and intuitionistic fuzzy ge-continuous then f is intuitionistic fuzzy ge-irresolute.*

Proof. Let A be an intuitionistic fuzzy ge-closed in Y and let $f^{-1}(A) \subseteq G$ where G is intuitionistic fuzzy open in X. Then $A \subseteq f(G)$. Since $f(G)$ is intuitionistic fuzzy open and A is intuitionistic fuzzy ge-closed in Y, $ecl(A) \subseteq f(G)$and $f^{-1}(ecl(A)) \subseteq G$. Since f is intuitionistic fuzzy ge-continuous and $ecl(A)$ is intuitionistic fuzzy e-closed in Y, $ecl(f^{-1}(ecl(A))) \subseteq G$ and also $ecl(f^{-1}(A)) \subseteq G$. Therefore $f^{-1}(A)$ is intuitionistic fuzzy ge-closed in X. Hence f is intuitionistic fuzzy ge-irresolute.

Theorem 3.9 *Let$f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be two intuitionistic fuzzy ge-irresolute mappings, then $gof: (X, \tau) \to (Z, \eta)$ is intuitionistic fuzzy ge-irresolute.*

Proof. Let A be an intuitionistic fuzzy ge-closed in Z, then $g^{-1}(A)$ is an intuitionistic fuzzy ge-closed in Y, because g is an intuitionistic fuzzy ge-irresolute. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy ge-closed in X. Hence gof is intuitionistic fuzzy ge-irresolute.

Theorem 3.10 *Let$f: (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy ge-irresolute and $g: (Y, \sigma) \to (Z, \eta)$ is intuitionistic fuzzy ge-continuous then $gof: (X, \tau) \to (Z, \eta)$ is intuitionistic fuzzy ge-continuous.*

Proof. Let A be an intuitionistic fuzzy closed in Z, then $g^{-1}(A)$ is an intuitionistic fuzzy ge-closed in Y, because g is an intuitionistic fuzzy ge-continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy ge-closed in X. because f is an intuitionistic fuzzy ge-irresolute. Hence gof is intuitionistic fuzzy ge-continuous.

Theorem 3.11 *Let$f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be two intuitionistic fuzzy ge-irresolute mappings, then $gof: (X, \tau) \to (Z, \eta)$ is intuitionistic fuzzy ge-irresolute if B is intuitionistic fuzzy GEO-compact relative to X. then the image $f(B)$ is intuitionistic fuzzy GEO-compact relative to Y.*

Proof. Let $\{A_i: i \in \Lambda\}$ be any collection of intuitionistic fuzzy ge-open set of Y such that $f(B) \subseteq \cup \{A_i: i \in \Lambda\}$. Then $B \subseteq \cup \{f^{-1}(A_i): i \in \Lambda\}$. By using the assumption, there exists a finite subset Λ_o of Λ such that $B \subseteq \cup \{f^{-1}(A_i): i \in \Lambda_o\}$. Therefore $f(B) \subseteq \cup \{A_i: i \in \Lambda_o\}$. Which shows that $f(B)$ is intuitionistic fuzzy GEO-compact relative to Y.

Corollary 3.12 *An intuitionistic fuzzy ge-irresolute image of an intuitionistic fuzzy GEO-compact space is intuitionistic fuzzy GEO-compact.*

Theorem 3.13 *Let$(X \times Y, \tau \times \sigma)$ be the intuitionistic fuzzy product space of non empty intuitionistic fuzzy topological space (x, τ) and (Y, σ). Then the projection mapping $p: X \times Y \to X$ is intuitionistic fuzzy ge-irresolute.*

Proof. Let f be any intuitionistic fuzzy ge-closed set of X. Then $F \times \tilde{1} = (p^{-1}(F))$ is intuitionistic fuzzy ge-closed and hence p is intuitionistic fuzzy ge-irresolute.

Theorem 3.14 *If the product space $(X \times Y, \tau \times \sigma)$ of two non empty intuitionistic fuzzy topological space (x, τ) and (Y, σ) is intuitionistic fuzzy GEO-compact, then each factor space is intuitionistic fuzzy GEO-compact.*

Proof. If $(X \times Y, \tau \times \sigma)$ is intuitionistic fuzzy GEO-compact, then by Corollary (3.12), we obtain that the intuitionistic fuzzy ge-irresolute image of $p(X \times Y) = X$ is intuitionistic fuzzy GEO-compact.

Theorem 3.15 *Let$f: (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy ge- irresolute surjection and (X, τ) is intuitionistic fuzzy GEO-connected, then (Y, σ) is intuitionistic fuzzy GEO-connected.*

Proof. Suppose Y is not intuitionistic fuzzy GEO-connected then there exists a proper intuitionistic fuzzy set G of Y which is both intuitionistic fuzzy ge-open and intuitionistic fuzzy ge-closed, therfore $f^{-1}(G)$ is a proper intuitionistic fuzzy set of X, which is both intuitionistic fuzzy ge-open and intuitionistic fuzzy ge-closed, because f is intuitionistic fuzzy ge-continuous surjection. Therefore X is not intuitionistic fuzzy GEO-connected, which is a contradiction. Hence Y is intuitionistic fuzzy GEO-connected.

Theorem 3.16 *If the product space $(X \times Y, \tau \times \sigma)$ of two non empty intuitionistic fuzzy topological space (x, τ) and (Y, σ) is intuitionistic fuzzy GEO-connected, then each factor intuitionistic fuzzy space is intuitionistic fuzzy GEO-connected.*

Proof. If $(X \times Y, \tau \times \sigma)$ is intuitionistic fuzzy GEO-connected, then the mapping $p: X \times Y \to X$ is intuitionistic fuzzy ge-irresolute, hence by theorem (3.15) the intuitionistic fuzzy ge-irresolute image of $p(X \times Y) = X$ an intuitionistic fuzzy GEO-connected space $X \times Y$ is intuitionistic fuzzy GEO-connected.

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