

Using New Integral Transform “GALM Transform” to Solving Differential Equations

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Abstract

In this work, new integral transform is proposed, and this transform is applied to attain the exact solutions for the linear ordinary differential equations with constant coefficients. This integral transform which we called GALM transform was also applied in nuclear physics and some medical applications.

The basic integral transforms like the Fourier transform and Laplace transform have been around for quite some time, new integral transforms are continually being developed to address specific problems or provide alternative perspectives on existing ones, new Integral transforms can be used in many fields, such as physics (including nuclear physics), engineering (where they can be used to solve some electrical circuit problems), medicine, pharmacokinetics, biotechnology, and finding answers to problems related to heart disease and cancer [Kurt 2013]. Elzaki, Aboodh, Mohanad, Kamal, Mahgoub and SEE transforms are good examples of new transforms for addressing differential equations, also GALM transform and a portion of its pivotal results are used helpfully to address differential equations.

This new transform is gotten and obtained from indispensable Laplace transform and after demonstration of some other integral transforms . GALM transform was introduced by Gharib M. Gharib, Ahmad alsawaer, Loiy kharabsheh and Maha Alsaoudi to utilize differential equations in the time area.

Keywords: GALM transform, Ordinary differential equations, Partial differential equations, Integral transform, Integral equations.

1 Introduction

This new transform is gotten and obtained from indispensable Laplace transform and after demonstration of some other integral transforms . GALM transform was introduced by Gharib M. Gharib, Ahmad alsawaer, Loiy kharabsheh and Maha Alsaoudi to utilize differential equations in the time area, [Gharib et al. 2021,Gharib et al. 2023A,J. Jawdat et al. 2023,Gharib et al. 2023B, Gharib et al. 2023C, Gharib et al. 2008].

While the basic integral transforms like the Fourier transform and Laplace transform have been around for quite some time, new integral transforms are continually being developed to address specific problems or provide alternative perspectives on existing ones, new Integral transforms can be used in many fields, such as physics (including nuclear physics[Kuffi et al. 2020]), engineering (where they can be used to solve some electrical circuit problems), medicine, pharmacokinetics, biotechnology, and finding answers to problems related to heart disease and cancer [Kurt 2013]. Elzaki, Aboodh, Mohanad, Kamal, Mahgoub and SEE transforms are good examples of new transforms for addressing differential equations, also GALM transform and a portion of its pivotal results are used helpfully to address differential equations[Mansour et al. 2021A, Aboodh et al. 2013, J. Jawdat et al. (2023), Maktoof et al. 2021,Mansour et al. 2021B, Mahdi et al. 2022, Kuffi et al. 2020, Kuffi et al. 2021, Elzaki et al. 2011, Kamal et al. 2016, Mahgoub et al. 2016, Kuffi et al. 2023A].

The definition of the GALM transform of time mapping $f(t)$ represented by the operator $G\{f(t)\}$ or simply $T(s)$ is:

$$G\{f(t)\} = T(s) = S^n \int_{t=0}^{\infty} a^{-t} f(st)dt. \quad (1)$$

where $a > 1$, $t \in [0, \infty)$ and $s \in \mathbb{Z}$

The parameter (s) is independent of time (t) and may take on values that are either real or complex .

In this paper we will determine the GALM integral transform's concept, essential characteristics, solving different problems in physics, nuclear physics, engineering, or any life application by convert the ordinary differential equation into an easy and simple algebraic equation, then by taking the inverse of this transform to obtain the exact solution. [Mohammed et al. 2023A, Mohand et al. 2017, Mansour et al. 2023, Zamil et al. 2023].

Through our study and our knowledge of many integral transforms with one parameter or two parameters or more, there is no preference between one transform and another, but there is an important matter, which is that each transform has a use for some applications through after converting differential equations, integral equation or systems into algebraic equations. Conversion is made easier by simplifying that application or example[Mahdi et al. 2023A, Mahdi

et al. 2023B, Mahdi et al. 2023C, Moazzam et al. 2023A, Kuffi et al. 2023B, Moazzam et al. 2023B, Mahgoub et al. 2017].

2 Methodology

2.1 GALM Transform of common Functions

It is going to be assumed that the existence of equation (1). The existence sufficient conditions of the GALM integral transformation is the function $f(t)$ being of exponential nature and piece wise continuous function for $0 < t < \infty$ ($t \geq 0$). The existence of the GALM transformation is contingent upon certain conditions. In the absence of these conditions, the transformation may or may not be present[Mohammed et al. 2023B, Kuffi et al. 2022A].

This section concerns the evaluation of the GALM integral transform for fundamental functions.

1) If $f(t) = k$, where k is a constant function, then by applying definition of GALM transform we get :

$$\begin{aligned} G\{k\} &= s^n \int_0^{\infty} k a^{-t} dt = s^n \lim_{A \rightarrow \infty} \left[\int_0^A k a^{-t} dt \right] \\ &= s^n \lim_{A \rightarrow \infty} \left[\frac{ka^{-t}}{\ln(a)} \right]_A^0 = \frac{k s^n}{\ln(a)} \end{aligned}$$

2) If $f(t) = t$ then:

$$G\{t\} = s^n \int_0^{\infty} s t a^{-t} dt = s^{n+1} \lim_{A \rightarrow \infty} \left[\int_0^A t a^{-t} dt \right]$$

integration by parts ,we get

$$\begin{aligned} &= s^{n+1} \lim_{A \rightarrow \infty} \left[\frac{t a^{-t}}{\ln(a)} + \frac{a^{-t}}{(\ln(a))^2} \right]_A^0 \\ &= \frac{s^{n+1}}{(\ln(a))^2} \end{aligned}$$

Also:

$$(i) G\{t^2\} = \frac{2s^{n+2}}{(\ln(a))^3}.$$

$$(ii) G\{t^3\} = \frac{6s^{n+3}}{(\ln(a))^4}.$$

(iii) In general , if $m \in \mathbb{Z}$ is a positive number, then :

$$G\{t^m\} = \frac{m! s^{n+m}}{(\ln(a))^{m+1}}$$

3) Let $f(t) = e^{bt}$, where b is a constant, then:

$$\begin{aligned} G\{e^{bt}\} &= s^n \int_0^\infty e^{bst} a^{-t} dt = s^n \lim_{A \rightarrow \infty} \left[\int_0^A \left(\frac{a}{e^{bs}} \right)^{-t} dt \right] \\ &= s^n \lim_{A \rightarrow \infty} \left[\left(\frac{a}{e^{bs}} \right)^{-t} \cdot \frac{1}{\ln(a) - bs} \right]_A^0 \\ G\{e^{bt}\} &= \frac{s^n}{\ln(a) - bs} \end{aligned}$$

4) Let $f(t) = \sin(bt)$, where b is a constant number, then:

$$\begin{aligned} G\{\sin(bt)\} &= s^n \int_0^\infty \sin(bst) a^{-t} dt \\ &= s^n \lim_{A \rightarrow \infty} \left[\int_0^A \sin(bst) a^{-t} dt \right] = s^n \lim_{A \rightarrow \infty} \left[\left[\frac{a^{-t} \sin(bst)}{\ln(a)} \right]_A^0 + \frac{bs}{\ln(a)} \int_0^A a^{-t} \cos(bst) dt \right] \end{aligned}$$

After simple computations, we get:

$$G\{\sin(bt)\} = \frac{b s^{n+1}}{(\ln(a))^2 + b^2 s^2}$$

5) Let $f(t) = \cos(bt)$, then:

$$G\{\cos(bt)\} = s^n \int_0^\infty \cos(bst) a^{-t} dt = s^n \lim_{A \rightarrow \infty} \left[\int_0^A \cos(bst) a^{-t} dt \right]$$

After simple computations, we get:

$$G\{\cos(bt)\} = \frac{s^n \ln(a)}{(\ln(a))^2 + b^2 s^2}$$

6) Let $f(t) = \sinh(bt)$, then:

$$G\{\sinh(bt)\} = s^n \int_0^\infty \sinh(bst) a^{-t} dt = s^n \lim_{A \rightarrow \infty} \left[\int_0^A \sinh(bst) a^{-t} dt \right]$$

After simple computations, we get:

$$G\{\sinh(bt)\} = \frac{b s^{n+1}}{(\ln(a))^2 - b^2 s^2}$$

7) Let $f(t) = \cosh(bt)$, then:

$$G\{\cosh(bt)\} = s^n \int_0^\infty \cosh(bst) a^{-t} dt = s^n \lim_{A \rightarrow \infty} \left[\int_0^A \cosh(bst) a^{-t} dt \right]$$

After simple computations, we get:

$$G\{\cosh(bt)\} = \frac{s^n \ln(a)}{(\ln(a))^2 - b^2 s^2}$$

8) Property of shifting in GALM integral transform

If GALM integral transform of the mapping $f(t)$ is $T(S)$, then GALM transform of mapping $\{e^{bt} f(t)\}$ is given by

$$\frac{\ln(a)}{\ln(a) - bs} \cdot T\left(s \cdot \frac{\ln(a)}{\ln(a) - bs}\right)$$

Proof

$$= S^n \int_{t=0}^\infty f(st) e^{-(\ln(a)-bs)t} dt$$

Now, let $x \ln(a) = (\ln(a) - bs)t$, then

$$t = \frac{x \ln(a)}{\ln(a) - bs} \quad \text{and} \quad dt = \frac{\ln(a)}{\ln(a) - bs} dx$$

$$\begin{aligned} G\{e^{bt} f(t)\} &= S^n \frac{\ln(a)}{\ln(a) - bs} \int_{t=0}^\infty f\left(s \frac{x \ln(a)}{\ln(a) - bs}\right) a^{-x} dx \\ &= \frac{\ln(a)}{\ln(a) - bs} \cdot T\left(s \cdot \frac{\ln(a)}{\ln(a) - bs}\right) \end{aligned}$$

Theorem(2.1)

$$(i) \quad G\{f'(t)\} = \frac{T(s) \ln(a)}{s} - s^{n-1} f(0)$$

$$(ii) \quad G\{f''(t)\} = \frac{T(s) (\ln(a))^2}{s^2} - s^{n-2} \ln(a) f(0) - s^{n-1} f'(0)$$

$$(iii) \quad G\{f'''(t)\} = \frac{T(s) (\ln(a))^3}{s^3} - s^{n-3} (\ln(a))^2 f(0) - s^{n-2} \ln(a) f'(0) - s^{n-1} f''(0)$$

$$(iv) G\{f^{(m)}(t)\} = \frac{T(s) (\ln(a))^m}{s^m} - \sum_{k=0}^{m-1} s^{n-m+k} (\ln(a))^{m-k-1} f^{(k)}(0)$$

Proofs

(i) By using the condition (1) we get:

$$G\{f'(t)\} = s^n \int_0^\infty f'(st) a^{-t} dt = s^n \lim_{A \rightarrow \infty} \left[\int_0^A f'(st) a^{-t} dt \right]$$

Integrating by parts, we get:

$$G\{f'(t)\} = \frac{T(s) \ln(a)}{s} - s^{n-1} f(0)$$

(ii) Let $f'(t) = g(t)$

Also, by the definition, we have:

$$\begin{aligned} G\{f''(t)\} &= G\{g'(t)\} = \frac{G\{g(t)\} \ln(a)}{s} - s^{n-1} g(0) \\ &= \frac{\ln(a)}{s} \left[\frac{T(s) \ln(a)}{s} - s^{n-1} f(0) \right] - s^{n-1} f'(0) \end{aligned}$$

Then we get

$$G\{f''(t)\} = \frac{T(s) (\ln(a))^2}{s^2} - s^{n-2} \ln(a) f(0) - s^{n-1} f'(0)$$

(iii) Can be proved In the same way.

(iv) Can be proved by Mathematical Induction.

2.2 The Inverse of GALM Integral Transform

In this section, we demonstrate the inverse of GALM integral transform of some common functions: [Mohammed et al. 2023C, Moazzam et al. 2023C].

$$(1) \mathbf{G}^{-1} \left\{ \frac{k s^n}{\ln(a)} \right\} = \mathbf{k}, \text{ where } k \text{ is a constant number.}$$

$$(2) \mathbf{G}^{-1} \left\{ \frac{m! s^{n+m}}{(\ln(a))^{m+1}} \right\} = \mathbf{t}^m, \text{ where } n > 0 \text{ integer number.}$$

$$(3) \mathbf{G}^{-1} \left\{ \frac{s^n}{\ln(a) - bs} \right\} = \mathbf{e}^{bt}, \text{ where } b \text{ is a constant number.}$$

$$(4) \mathbf{G}^{-1} \left\{ \frac{b s^{n+1}}{(\ln(a))^2 + b^2 s^2} \right\} = \mathbf{sin}(bt) .$$

$$(5) \mathbf{G}^{-1} \left\{ \frac{s^n \ln(a)}{(\ln(a))^2 + b^2 s^2} \right\} = \mathbf{cos}(bt) .$$

$$(6) \mathbf{G}^{-1} \left\{ \frac{b s^{n+1}}{(\ln(a))^2 - b^2 s^2} \right\} = \mathbf{sinh}(bt) .$$

$$(7) \mathbf{G}^{-1} \left\{ \frac{s^n \ln(a)}{(\ln(a))^2 - b^2 s^2} \right\} = \mathbf{cosh}(bt) .$$

2.3 Applications of GALM Integral Transform In Ordinary Differential Equations (ODEs)

This section demonstrates GALM transform capability and usefulness to handle and utilize ordinary differential equations when it is applied to attain the exact solutions for such equations by tackling several problems .[Moazzam et al. 2023B]

Example 1

Consider the differential equation:

$$y'' + y = 0 , y'(0) = y(0) = 1$$

Solution: apply GALM integral transform on both sides, we get:

$$\begin{aligned} G\{y''\} + G\{y\} &= G\{0\} \\ \frac{G\{y\}(\ln(a))^2}{s^2} - s^{n-2} \ln(a) y(0) - s^{n-1} y'(0) + G\{y\} &= 0 \end{aligned}$$

So

$$G\{y\} = [s^{n-2} \ln(a) + s^{n-1}] \cdot \frac{s^2}{(\ln(a))^2 + s^2} = \frac{s^n \ln(a)}{(\ln(a))^2 + s^2} + \frac{s^{n+1}}{(\ln(a))^2 + s^2}$$

using inverse of GALM transform on both sides, we get:

$$y = G^{-1}(G\{y\}) = G^{-1} \left[\frac{s^n \ln(a)}{(\ln(a))^2 + s^2} \right] + G^{-1} \left[\frac{s^{n+1}}{(\ln(a))^2 + s^2} \right]$$

The exact solution:

$$y(t) = \mathbf{cos}(t) + \mathbf{sin}(t)$$

Example 2

Find the solution of differential equation:

$$y'' + y = t , y(0) = 0 , y'(0) = 2$$

Solution: using GALM definition :

$$\frac{G\{y\}(\ln(a))^2}{s^2} - s^{n-2} \ln(a) y(0) - s^{n-1} y'(0) + G\{y\} = \frac{s^{n+1}}{(\ln(a))^2}$$

So

$$G\{y\} = \left(2s^{n-1} + \frac{s^{n+1}}{(\ln(a))^2} \right) \cdot \frac{s^2}{(\ln(a))^2 + s^2}$$

Then

$$G\{y\} = \frac{2s^{n+1}}{(\ln(a))^2 + s^2} + \left[\frac{s^n}{(\ln(a))^2} \cdot \frac{s^3}{[(\ln(a))^2 + s^2]} \right]$$

$$G\{y\} = \left\{ \frac{2s^{n+1}}{(\ln(a))^2 + s^2} \right\} + \left\{ \frac{s^n}{(\ln(a))^2} \left[s - \frac{s(\ln(a))^2}{(\ln(a))^2 + s^2} \right] \right\}$$

Obtain the inverse GALM transform of this equation as:

$$y = G^{-1}(G\{y\}) = G^{-1} \left\{ \frac{2s^{n+1}}{(\ln(a))^2 + s^2} \right\} + G^{-1} \left\{ \frac{s^{n+1}}{(\ln(a))^2} \right\} - G^{-1} \left\{ \frac{2s^{n+1}}{(\ln(a))^2 + s^2} \right\}$$

$$y(t) = t + \sin(t)$$

Example 3

Consider the 2nd request differential equation:

$$y'' + 3y' + 2y = 0, y(0) = 3, y'(0) = 0$$

Solution:

$$\frac{G\{y\}(\ln(a))^2}{s^2} - s^{n-2} \ln(a) y(0) - s^{n-1} y'(0) + 3 \frac{G\{y\} \ln(a)}{s} - 3 s^{n-1} y(0) + 2G\{y\} = 0$$

$$G\{y\} \left[\frac{(\ln(a))^2}{s^2} + \frac{3 \ln(a)}{s} + 2 \right] = 3 s^{n-2} \ln(a) + 9 s^{n-1}$$

$$G\{y\} = \frac{3 s^{n+1} \ln(a) + 9 s^{n+2}}{s[(\ln(a))^2 + 3 s \ln(a) + 2s^2]}$$

$$G\{y\} = s^n \left[\frac{3 \ln(a) + 9s}{(2s + \ln(a))(s + \ln(a))} \right]$$

Now

$$\frac{3 \ln(a) + 9s}{(2s + \ln(a))(s + \ln(a))} = \frac{A}{(s + \ln(a))} + \frac{B}{(2s + \ln(a))}$$

After simplifying , we get: A = 6 and B = -3.

So,

$$G\{y\} = \frac{6 s^n}{(s + \ln(a))} - \frac{3 s^n}{(\ln(a) + 2s)}$$

The exact solution is:

$$y(t) = 6 e^{-t} - 3 e^{-2t}$$

Example 4

Consider the second-order linear non homogeneous differential equation:

$$y'' + 9y = \cos(2t) \quad , y(0) = 1 \quad , y'(0) = \frac{12}{5}$$

Solution:

$$\frac{G\{y\} (\ln(a))^2}{s^2} - s^{n-2} \ln(a) y(0) - s^{n-1} y'(0) + 9 G\{y\} = \frac{s^n \ln(a)}{(\ln(a))^2 + 4s^2}$$

$$G\{y\} = \left(\frac{s^n \ln(a)}{(\ln(a))^2 + 4s^2} + s^{n-2} \ln(a) + \frac{12}{5} \cdot s^{n-1} \right) \cdot \frac{s^2}{(\ln(a))^2 + 9s^2}$$

$$G\{y\} = s^n \ln(a) \cdot \frac{s^2}{((\ln(a))^2 + 4s^2)((\ln(a))^2 + 9s^2)} + \frac{s^n \ln(a)}{(\ln(a))^2 + 9s^2} + \frac{12}{5} \cdot \frac{s^{n+1}}{(\ln(a))^2 + 9s^2}$$

But

$$\frac{s^2}{((\ln(a))^2 + 4s^2)((\ln(a))^2 + 9s^2)} = \frac{A}{(\ln(a))^2 + 4s^2} + \frac{B}{(\ln(a))^2 + 9s^2}$$

After simplifying , we get: A = $\frac{1}{5}$, B = $-\frac{1}{5}$

Taking inverse GALM transform, we get:

$$y = \frac{1}{5} \cos(2t) + \frac{4}{5} \cos(3t) + \frac{4}{5} \sin(3t)$$

3 Applications

3.1 GALM Integral Transform in "Nuclear Physics"

This application is grounded in the fundamental principles of nuclear physics. consider the first-order ordinary differential equation [Kuffi et al. 2022B].

$$\frac{dN}{dt} = -\lambda N(t).$$

The essential relation that characterizes the process of radioactive decay is presented in the equation above. The function $N(t)$ represents the quantity of atoms that have not sustained decay within a given time period t , while the constant λ denotes the constant rate of decay. The GALM integral transform denoted by $G\{\cdot\}$ can be utilized to obtain:

$$G\{N'(t)\} + \lambda G\{N(t)\} = 0,$$

There,

$$\frac{T(S) \ln(a)}{s} - s^{n-1} N(0) + \lambda T(S) = 0.$$

Here $G\{N(t)\} = T(S)$ and $N(0) = N_0$.

So,

$$\frac{T(S) \ln(a)}{s} - s^{n-1} N_0 + \lambda T(s) = 0,$$

Then:

$$\left(\frac{\ln(a)}{s} + \lambda\right) T(s) = s^{n-1} N_0, \text{ so } T(s) = N_0 \cdot \frac{s^n}{1 + s\lambda}.$$

Now, applying inverse GALM integral transform on both sides, we have:

$$N(t) = N_0 e^{-\lambda t}.$$

This is the proper form of radioactive decay.

3.2 Application of "Pharmacokinetics"

In Pharmacokinetics we have used ordinary differential equations to examine the rate of decrease in concentration (C) of a medicines as a function of time (t) which is indispensable factor to determine amount of dose, frequency of dose, etc [Mansour et al. 2021A].

Consider the problem of Pharmacokinetics

$$\frac{d}{dt} C(t) + \lambda C(t) = \frac{\gamma}{VOL.}, \quad \text{where } t > 0 \}, \quad \text{with } C(0) = 0$$

The function C(t) represents the concentration of the drug in the blood stream at a specific time t. (concentration time function)

The symbol (λ) denotes an elimination rate constant

The term "Vol." refers to the volume of distribution of the drug

Gamma (γ) refers to the infusion proportion measured in milligrams per minute (mg/min).

GALM transform can be applied to both sides of equation gives :

$$G\{C'(t)\} + \lambda G\{C(t)\} = \frac{\gamma}{VOL.} G\{1\}$$

There,

$$\frac{T(S) \ln(a)}{s} - s^{n-1} C(0) + \lambda T(S) = \frac{\gamma}{VOL.} \cdot \frac{s^n}{\ln(a)}$$

So,

$$\begin{aligned} T(s) &= \frac{\gamma}{VOL.} \cdot \frac{s^n}{\ln(a)} \cdot \frac{s}{\ln(a) + s\lambda} = \frac{\gamma}{VOL.} \cdot \frac{s^n}{\ln(a)} \left[\frac{1}{\lambda} - \frac{\ln(a)}{\lambda(\ln(a) + s\lambda)} \right] \\ &= \frac{\gamma}{\lambda VOL.} \left[\frac{s^n}{\ln(a)} - \frac{s^n}{\ln(a) + s\lambda} \right] \end{aligned}$$

Take inverse of GALM transform , we have:

$$C(t) = \frac{\gamma}{\lambda VOL.} (1 - e^{-\lambda t})$$

3.3 Blood Glucose Concentration

through continual vascular glucose injection, the concentration of glucose in the blood B(t) is overtaking the base line value at the start of the infusion . The mapping B(t) is represented by the following equation .

$$B'(t) + k B(t) = \frac{\delta}{\gamma}$$

Where $t \in (0, \infty)$ and $B(0) = 0$

k - constant Velocity of elimination .

δ - the average of infusion.

γ - volume occupied by distributing glucose.

The GALM integral transform can applied to determine the concentricity of glucose in the bloodstream.

applying GALM integral transform to above equation ,we get :

$$G\{B'(t)\} + k G\{B(t)\} = \frac{\delta}{\gamma} G\{1\}$$

By utilizing the initial value problem (I,V,P) and the integral

Transform outlined in section 2.1 .

$$\frac{T(S) \ln(a)}{s} - s^{n-1} B(0) + k T(S) = \frac{\delta}{\gamma} \cdot \frac{s^n}{\ln(a)}$$

Then ,

$$\begin{aligned} T(s) &= \frac{\delta}{\gamma} \cdot \frac{s^n}{\ln(a)} \cdot \frac{s}{\ln(a) + sk} = \frac{\delta}{\gamma} \cdot \frac{s^n}{\ln(a)} \left[\frac{1}{k} - \frac{\ln(a)}{k(\ln(a) + sk)} \right] \\ &= \frac{\delta}{\gamma k} \left[\frac{s^n}{\ln(a)} - \frac{s^n}{\ln(a) + sk} \right] \end{aligned}$$

After performing basic calculations along with taking inverse GALM integral transform to the given expression, the resultant value obtained represents the concentration of glucose in the blood:

$$B(t) = \frac{\delta}{\gamma} (1 - e^{-\lambda t})$$

4 Conclusions

New integral transform was introduced through this paper . We found that conversion of linear ordinary differential equations with constant coefficients and higher orders turns into simple algebraic equations that are simpler and easier than other transforms. In this paper important medical applications were presented, as well as the application of nuclear physics.

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