A Statistical Analysis of Partially Balanced Incomplete Block Designs using Fuzzy Approach

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Abstract

The Partially Balanced Incomplete Block Designs (PBIBD) are play an important role in incomplete block designs, especially in agricultural field. The analysis of PBIBD m associative classes has received less attention than the balanced case. Usually, PBIBD is used with two-association classes is PBIBD(2) defined on any study of designs, At the time practical situations not required precise observations due to the complication and uncertainty involved such problems, it is sometimes unrealistic or insensible to require precise judgments. In such situations PBIBD(2) for fuzzy observations is inevitable. In this paper, a statistical analysis of PBIBD(2) with intra-block analysis using α-cut interval method of trapezoidal fuzzy numbers through a numerical illustration.

Keywords: Fuzzy Partially Balanced Incomplete Block Designs, Decision Rule, Trapezoidal Fuzzy Numbers, α-Cut Method.

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I. Introduction

Incomplete block designs are commonly used when prevention is necessary when the number of treatments is large. The origins of incomplete block designs introduced the concept of Balanced Incomplete Block Designs (BIBD) and their analysis using both intra-inter block analysis. BIBD is not available for all treatment combinations. Such situations the PBIBD is most inevitable. PBIBD(2) is the difference between the effects of a pair of treatments is assessed with the same accuracy for all pairs in a set. The data provided in this study are vague and therefore we need an extended version of the PBIBD(2) intra-block analysis to investigate these vague observations. In 1965 Lotfi A. Zadeh [20] introduced a fuzzy set theory. This theory provides appealing coordination bonds for integrating membership values that represent uncertain information. A fuzzy number is a generalization of a normal, real number, which does not point to a single value, but rather to a set of connected possible values,where each possible value has a membership function of its own weight between 0 and 1. Triangular Fuzzy Numbers (TFNs) are uniquely appropriated by triplet with membership functions which can be more sophisticatedly and efficiently method. TZFNs are located on the left and right can be adjusted as desired by choosing the compact resistance values of a variable. TZFNs have lots of uses in the efficient adjustment of imprecise information. Many authors have discussed this method. Some suitable references are Chanas [4] has proposed the interval approximation of L-R fuzzy number for two conditions width and Hamming distance respectively. Montenegro et al. [10] has provided one-way ANOVA test for two different approaches to the small and large population model and their approaches are based on measuring the differences between fuzzy numbers. Wu [18] has introduced a technique of imposing fuzzy data upon the traditional ANOVA model using h-level sets. Wu [19] has proposed a fuzzy confidence interval with the help of a central limit theorem to create unknown fuzzy parameters used for the fuzzy random variable. Behera and Singh [3] have investigated impact of continuous use of fertilizer on fractions of manganese in soil and stated that it directly influenced randomized block designs. Nakama et al. [11] have developed Minkowski support function is used to derive a metric for intervals and convert them into Hilbert-space value functions through two-way ANOVA interval-valued data. The testing the null hypothesis a bootstrap scheme is developed and the p-values of the observed test statistics are estimated. Gonzalez-Rodriguez et al. [7] have proposed a one-way ANOVA experimental approach to fuzzy observations and their hilbert space considered functional data of fuzzy observations to be functional data. Gnanapriya et al. [6] have proposed identifying decision level through balanced incomplete block design using trapezoidal membership functions based on α-cut interval method. Acha [1] has proposed two different methods of vector space and classical ANOVA used in data available from the Nigerian stock market. Asady [2] has described trapezoidal fuzzy numbers as the best way to precisely measure the distance between fuzzy numbers. Jiryei et al [8] have discussed triangular and exponential fuzzy contexts for estimating fuzzy parameters random variables are basically analyzed by ANOVA. Nourbakhsh et al. [12] has proposed one- way ANOVA extended triangular fuzzy numbers using two real-life data are shown in this method. Sharma et al. [17] have proposed a complete web solution is most useful for researchers and students by creating client-server architecture in partially balanced incomplete block designs. Parthiban and Gajivaradhan [13] have proposed a new technique for testing hypotheses under one-factor ANOVA model. Of these, the results are determined by the α-cut interval model using fuzzy consumer examples and pursued by the author [14] they proposed two-way ANOVA process under different types of trapezoidal fuzzy models using different models with trapezoidal fuzzy numbers and also the author [15] continued they discussed with numerical examples using three-factor ANOVA model based on decision-making problems under trapezoidal fuzzy environments. In this paper, a statistical analysis of PBIBD(2) with intra-block analysis for trapezoidal fuzzy numbers through α-cut interval through an example.

II. Preliminaries

2.1 Definition of BIBD

A set of v treatments in b blocks each with containing $(k < v)$ treatments is said to be a balanced incomplete block design if it satisfies the following parametric relations: (i) $vr = bk = N$, (ii) $\lambda(v-1) = r(k-1)$ and (iii) $b \geq v$. The quantities v, b, r, k and λ are usually called the parameters. Where, v = treatments, b = blocks, k = size of the block, r = replication, λ = all pair of treatments appear together in blocks.

2.2 Definition of PBIBD

A set of v treatments distributed in blocks each containing distinct treatments is said to form a PBIBD design.

(i) Every treatment occurs in r blocks.

(ii) Two treatments occur together in $\lambda_1, \lambda_2, ..., \lambda_m$ blocks.

(iii) Given a treatment Θ each of n_i treatments occurs with Θ in λ_i blocks $(i = 1, 2, \ldots, m)$ so that $\sum_i n_i = v - 1$ and $\sum_i n_i \lambda_i = r(k-1)$.

2.3 Definition of TZFNs

The trapezoidal fuzzy numbers are identifying by four parameters, $\tilde{A} = a, b, c, d \in \mathbb{R}$ as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x < a \\ \dfrac{x-a}{b-a} & ; a \le x \le b \\ 1 & ; b \le x \le c \\ \dfrac{d-x}{d-c} & ; c \le x \le d \\ 0 & ; x > d \end{cases}$$

If we denote α cut interval for fuzzy number $\tilde{A}$ as $\tilde{A}_\alpha$, the getting for an interval is defined as $\tilde{A}_\alpha = [a + (b-a)\alpha, d - (d-c)\alpha]; \alpha \in [0,1]$. When $b = c$; the trapezoidal fuzzy number coincides with triangular one. TZFNs defined by lower limit a , upper limit d , a lower support limit b and the upper support limit c. The following aspects of the membership functions are defined. They are: (i) Core of membership function is $\mu_{\tilde{A}}(x) > 0$. (ii) Support of membership function is $\mu_{\tilde{A}}(x) = 1$. (iii) Boundaries usually denoted by the interval $0 < \mu_{\tilde{A}}(x) > 1$. Figure 2.3.1 shows the details.

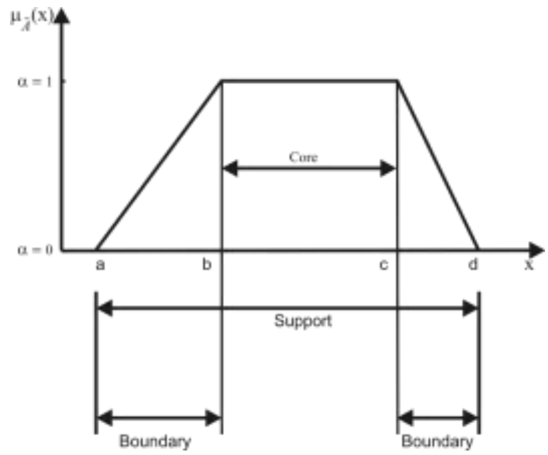

Figure 2.3.1 Trapezoidal Fuzzy Numbers

III. A Statistical Analysis of Partially Balanced Incomplete Block Designs with Two Associate Classes using Trapezoidal Fuzzy Observations

In this section, to test the hypotheses of PBIBD(2) with intra block analysis using α -cut interval method of trapezoidal fuzzy numbers is proposed. Using this condition, convert the crisp PBIBD(2) intra block analysis to fuzzy PBIBD(2) intra block analysis. Fetching the fuzzy PBIBD(2), let us consider lower and upper levels of PBIBD's. In this planned to approach, a crisp PBIBD(2) intra block models are chosen in terms of lower and upper level. To conclude, to analyzed lower-level and upper-level model using crisp PBIBD(2) intra block procedure. The generalized linear model of fuzzy PBIBD is $\tilde{y}_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$. Where

$\tilde{y}_{ij}$ is the observation in the j^{th} block receiving the i^{th} treatments. τ_i is the fixed effect due to the i^{th} treatment, β_j is the fixed effect due to the j^{th} block and ε_{ij} is the random error effect which follows $N(0,\sigma_e^2)$. A fuzzy PBIBD(2) linear model can be divided into two models. Namely, the lower-level and upper-level models are respectively.

3.1 Fuzzy PBIBD(2) lower-level model

The lower-level model is $a_{ij}+(b_{ij}-a_{ij})\alpha$; $i=1,2,...,v$; $j=1,2,...,b$, the α -cut interval method can be represented as in the table 1.

Table 1: The Layout of Lower-Level Model for α -Cut Interval Method of PBIBD(2)

Treatments i	**Blocks** j					
	1	2	…	j	…	b
1	$a_{11}+(b_{11}-a_{11})\alpha$	-	…	$a_{1j}+(b_{1j}-a_{1j})\alpha$	…	$a_{1b}+(b_{1b}-a_{1b})\alpha$
2	$a_{21}+(b_{21}-a_{21})\alpha$	$a_{22}+(b_{22}-a_{22})\alpha$	…	$a_{2j}+(b_{2j}-a_{2j})\alpha$	…	-
⋮	⋮	⋮	…	⋮	⋮	⋮
i	$a_{i1}+(b_{i1}-a_{i1})\alpha$	$a_{i2}+(b_{i2}-a_{i2})\alpha$	…	-	…	$a_{ib}+(b_{ib}-a_{ib})\alpha$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
v	-	$a_{v2}+(b_{v2}-a_{v2})\alpha$	…	$a_{vj}+(b_{vj}-a_{vj})\alpha$	…	$a_{vb}+(b_{vb}-a_{vb})\alpha$

All the sum of squares is calculated using the classical PBIBD(2) with intra-block analysis are given below

The Total Sum of Squares (TSS) is

$$TSS^L=\sum_i\sum_j\left[a_{ij}+\left(b_{ij}-a_{ij}\right)\alpha\right]^2-\frac{(G)^2}{N}$$

The Correction Factor (CF) is $CF^L=\frac{(G)^2}{N}$

The Grand Total (GT) is $G^L=\sum_i\sum_j\left[a_{ij}+\left(b_{ij}-a_{ij}\right)\alpha\right]$

The Blocks (unadjusted) Sum of Squares (BSS) is

$$BSS^L=\frac{1}{k}\sum_j\left[a_{ij}+\left(b_{ij}-a_{ij}\right)\alpha\right]^2-\frac{(G)^2}{N}$$

The Treatments (adjusted) Sum of Squares (Ad.TrtSS) is

$$Ad.TrtSS^L=\sum_i\tilde{t}_iQ_i$$

Where, $Q_i^L=\left[\left(\sum_j\left[a_{ij}+\left(b_{ij}-a_{ij}\right)\alpha\right]\right)-\frac{1}{k}\sum_j n_{ij}\left(\sum_i\left[a_{ij}+\left(b_{ij}-a_{ij}\right)\alpha\right]\right)\right]$

$$\tilde{t}_i^L=\frac{k\left\{\tilde{B}_2\left[\left(\sum_j\left[a_{ij}+\left(b_{ij}-a_{ij}\right)\alpha\right]\right)-\frac{1}{k}\sum_j n_{ij}\left(\sum_i\left[a_{ij}+\left(b_{ij}-a_{ij}\right)\alpha\right]\right)\right]-\tilde{B}_1\tilde{S}_2\left[\left(\sum_j\left[a_{ij}+\left(b_{ij}-a_{ij}\right)\alpha\right]\right)-\frac{1}{k}\sum_j n_{ij}\left(\sum_i\left[a_{ij}+\left(b_{ij}-a_{ij}\right)\alpha\right]\right)\right]\right\}}{\tilde{A}_1\tilde{B}_2-\tilde{A}_2\tilde{B}_1}$$

Where,
$$\tilde{A}_1=r(k-1)\ ;\ \tilde{A}_2=(\lambda_1-\lambda_2)\,p_{12}^1$$
$$\tilde{B}_1=(\lambda_1-\lambda_2)\ ;\ \tilde{B}_2=r(k-1)+\lambda_1+(\lambda_1-\lambda_2)(p_{22}^2-p_{22}^1)$$

The parameters of the first and second associate matrices is given by

$$\tilde{p}_{ij}^{1}=\begin{pmatrix} p_{11}^{1} & p_{12}^{1} \\ p_{21}^{1} & p_{22}^{1} \end{pmatrix};\ \tilde{p}_{ij}^{2}=\begin{pmatrix} p_{11}^{2} & p_{12}^{2} \\ p_{21}^{2} & p_{22}^{2} \end{pmatrix}$$

Error Sum of Squares (ESS) is obtained by subtraction

$$ESS^{L}=TSS^{L}-BSS^{L}-Ad.TrtSS^{L}$$

3.1.1 Test of hypotheses for two sets of blocks and treatments

$$\tilde{H}_{0}^{L}:b_{1}^{L}=b_{2}^{L}=\cdots=b_{b}^{L}\ Vs\ \tilde{H}_{1}^{L}:b_{1}^{L}\neq b_{2}^{L}\neq\cdots\neq b_{b}^{L}$$

$$\tilde{H}_{0}^{L}:t_{1}^{L}=t_{2}^{L}=\cdots=t_{v}^{L}\ Vs\ \tilde{H}_{1}^{L}:t_{1}^{L}\neq t_{2}^{L}\neq\cdots\neq t_{v}^{L}$$

All these values are referred in the ANOVA table and the inference is drawn

Table 2: ANOVA Table for Fuzzy PBIBD(2) of Lower-Level Model

SV	df	SS	MSS	$\tilde{F}$ - Ratio
Blocks (Unadjusted)	$(b-1)$	BSS^{L}	MSS_{B}^{L}	$\tilde{F}_{B}=\frac{MSS_{B}^{L}}{MSS_{E}^{L}}$
Treatments (Adjusted)	$(v-1)$	$Ad.TrtSS^{L}$	$MSS_{Ad.Trt}^{L}$	$\tilde{F}_{Ad.Trt}=\frac{MSS_{Ad.Trt}^{L}}{MSS_{E}^{L}}$
Total Due to Remainder	$(N-v-b+1)$	ESS^{L}	MSS_{E}^{L}	-
Total	$(N-1)$	TSS^{L}	-	-

3.1.2 Decision Rules of Lower-Level Model

(i) If $\tilde{F}_{B}^{L}<F_{t}$ at level of significance r with $[(b-1),(N-v-b+1)]$ degrees of freedom then the null hypothesis $\tilde{H}_{0}^{L}$ is accepted in the α-cut interval method sense, if not the alternative hypothesis $\tilde{H}_{1}^{L}$ is accepted.

(ii) If $\tilde{F}_{Ad.Trt}^{L}<F_{t}$ at level of significance r with $[(v-1),(N-v-b+1)]$ degrees of freedom then the null hypothesis $\tilde{H}_{0}^{L}$ is accepted in the α-cut interval method sense, if not the alternative hypothesis $\tilde{H}_{1}^{L}$ is accepted.

Note: Here, the notation for level of significance is to be r as an alternative to α , so as to avoid getting confused with α-cut value found in trapezoidal fuzzy numbers.

3.2 Fuzzy PBIBD(2) of Upper-Level Model

Similarly the fuzzy PBIBD(2) for upper-level model SS's are given below

The correction factor (CF) is $CF^{U}=\frac{(G)^{2}}{N}$

The Grand Total (GT) is $G^{U}=\sum_{i}\sum_{j}\left[d_{ij}-\left(d_{ij}-c_{ij}\right)\alpha\right]$

The Total Sum of Squares (TSS) is

$$TSS^{U}=\sum_{i}\sum_{j}\left[d_{ij}-\left(d_{ij}-c_{ij}\right)\alpha\right]^{2}-\frac{(G)^{2}}{N}$$

The Blocks (unadjusted) Sum of Squares (BSS) is

$$BSS^U = \frac{1}{k}\sum_j\left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]^2 - \frac{(G)^2}{N}$$

The Treatments (Adjusted) Sum of Squares (Ad.TrtSS) is

$$Ad.TrtSS^U = \sum_i \tilde{t}_i Q_i$$

Where, $Q_i^U = \left[\left(\sum_j\left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]\right) - \frac{1}{k}\sum_j n_{ij}\left(\sum_i\left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]\right)\right]$

$$\tilde{t}_i^U = \frac{k\left\{\tilde{B}_2\left[\left(\sum_j\left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]\right) - \frac{1}{k}\sum_j n_{ij}\left(\sum_i\left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]\right)\right] - \tilde{B}_1\tilde{S}_2\left[\left(\sum_j\left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]\right) - \frac{1}{k}\sum_j n_{ij}\left(\sum_i\left[d_{ij} - \left(d_{ij} - c_{ij}\right)\alpha\right]\right)\right]\right\}}{\tilde{A}_1\tilde{B}_2 - \tilde{A}_2\tilde{B}_1}$$

Where, $\tilde{A}_1 = r(k-1)$; $\tilde{A}_2 = (\lambda_1 - \lambda_2)p_{12}^1$

$\tilde{B}_1 = (\lambda_1 - \lambda_2)$; $\tilde{B}_2 = r(k-1) + \lambda_1 + (\lambda_1 - \lambda_2)(p_{22}^2 - p_{22}^1)$

The parameters of the first and second associate matrices is given by

$$\tilde{p}_{ij}^1 = \begin{pmatrix} p_{11}^1 & p_{12}^1 \\ p_{21}^1 & p_{22}^1 \end{pmatrix} ; \tilde{p}_{ij}^2 = \begin{pmatrix} p_{11}^2 & p_{12}^2 \\ p_{21}^2 & p_{22}^2 \end{pmatrix}$$

The Error sum of squares is obtained by subtraction

$$ESS^U = TSS^U - BSS^U - Ad.TrtSS^U$$

3.2.1 Test of hypotheses for two sets of blocks and treatments

$$\tilde{H}_0^U : b_1^U = b_2^U = \cdots = b_b^U \text{ Vs } \tilde{H}_1^U : b_1^U \neq b_2^U \neq \cdots \neq b_b^U$$

$$\tilde{H}_0^U : t_1^U = t_2^U = \cdots = t_v^U \text{ Vs } \tilde{H}_1^U : t_1^U \neq t_2^U \neq \cdots \neq t_v^U$$

All these values are referred in the ANOVA table and the inference is drawn

Table 4: ANOVA Table for Upper-Level Fuzzy PBIBD(2) with Intra-Block Analysis

SV	df	SS	MSS	$\tilde{F}$ - Ratio
Blocks (Unadjusted)	$(b-1)$	BSS^U	MSS_B^U	$\tilde{F}_B = \frac{MSS_B^U}{MSS_E^U}$
Treatments (Adjusted)	$(v-1)$	$Ad.TrtSS^U$	$MSS_{Ad.Trt}^U$	$\tilde{F}_{Ad.Trt} = \frac{MSS_{Ad.Trt}^U}{MSS_E^U}$
Total Due to Remainder	$(N-v-b+1)$	ESS^U	MSS_E^U	-
Total	$(N-1)$	TSS^U	-	-

3.2.2 Decision Rules of Upper - Level Model

(i) If $\tilde{F}_B^U < F_t$ at level of significance r with $\left[(b-1),(N-v-b+1)\right]$ degrees of freedom then the null hypothesis $\tilde{H}_0^U$ is accepted in the α -cut interval method sense, if not the alternative hypothesis $\tilde{H}_1^U$ is accepted.

(ii) If $\tilde{F}_{Ad.Trt}^U < F_t$ at level of significance r with $\left[(v-1),(N-v-b+1)\right]$ degrees of freedom then the null hypothesis $\tilde{H}_0^U$ is accepted in the α -cut interval method sense, if not the alternative hypothesis $\tilde{H}_1^U$ is accepted.

IV. Application

The yields were collected through primary data in the ragi yield in the Salem District (East / West) regions of Tamilnadu. Nine varieties of ragi crops with nine different types of fertilizers are examined in a PBIBD(2) with intra block analysis. The exact observations in the PBIBD(2) evaluations examined were approximately. Therefore, fuzzy observations are needed to quantify the blurring nature of this yield. For the purpose of the present data are in terms of TZFN in kilograms per hectare.

Table 5: Table for Yield of Ragi Crops

Fertilizers	Varieties				
	Karusuruttai	ML365	CO15	GPU67	CO14
N	-	-	[39,41,44,45]	-	-
P	-	[34,36,39,41]	-	-	-
K	[31,32,35,37]	-	-	-	-
Ca	[31,34,37,39]	[39,41,43,44]	-	-	[48,49,51,52]
Mg	-	-	[30,33,36,37]	-	[37,39,41,42]
Zn	-	-	-	-	[49,52,55,57]
Fe	-	[38,42,44,45]	[49,52,55,57]	[58,59,60,62]	-
B	[35,36,40,41]	-	-	[40,43,46,47]	-
Cl	-	-	-	[29,31,33,35]	-

Conti…

Fertilizers	Varieties			
	Paiyur1	Paiyur2	CO13	CORa14
N	-	[45,46,50,52]	-	[59,61,63,64]
P	-	-	[54,55,59,62]	[65,66,68,70]
K	[48,52,55,57]	-	-	[57,58,60,62]
Ca	-	-	-	-
Mg	[38,41,42,44]	-	-	-
Zn	-	[57,58,60,62]	[59,61,63,64]	-
Fe	-	-	-	-
B	-	[47,49,52,53]	-	-
Cl	[45,46,50,52]	-	[49,51,53,54]	-

Test whether there is any significant difference between varieties and fertilizers of ragi crops yield.

Table 6: Table for Lower and Upper Levels using α -Cut Interval Method with TZFNs

Fertilizers	Varieties				
	Karusuruttai	ML365	CO15	GPU67	CO14
N	-	-	[39+ α , 45- α]	-	-
P	-	[34+2 α , 41-2 α]	-	-	-
K	[31+ α , 37-2 α]	-	-	-	-
Ca	[31+3 α , 39-2 α]	[39,+2 α , 44- α]	-	-	[48+ α , 52- α]
Mg	-	-	[30+3 α , 37- α]	-	[37+2 α , 42- α]
Zn	-	-	-	-	[49+3 α , 57-2 α]
Fe	-	[38+4 α , 45- α]	[49+3 α , 57-2 α]	[58+ α , 62-2 α]	-
B	[35+ α , 41- α]	-	-	[40+3 α , 47- α]	-
Cl	-	-	-	[29+2 α , 35-2 α]	-

Conti…

Fertilizers	Varieties			
	Paiyur1	Paiyur2	CO13	CORa14
N	-	[45+ α , 52-2 α]	-	[59+2 α , 64- α]
P	-	-	[54+ α , 62-3 α]	[65+ α , 70-2 α]
K	[48+4 α , 57-2 α]	-	-	[57+ α , 62-2 α]
Ca	-	-	-	-
Mg	[38+3 α , 44-2 α]	-	-	-
Zn	-	[57+ α , 62-2 α]	[59+2 α , 64- α]	-
Fe	-	-	-	-
B	-	[47+2 α , 53- α]	-	-
Cl	[45+ α , 52-2 α]	-	[49+2 α , 54- α]	-

Table 7: Lower-Level Model for α - Cut Interval Method using TZFNs

Fertilizers	Varieties				
	Karusuruttai	ML365	CO15	GPU67	CO14
N	-	-	[39+α]	-	-
P	-	[34+2α]	-	-	-
K	[31+α]	-	-	-	-
Ca	[31+3α]	[39+2α]	-	-	[48+α]
Mg	-	-	[30+3α]	-	[37+2α]
Zn	-	-	-	-	[49+3α]
Fe	-	[38+4α]	[49+3α]	[58+α]	-
B	[35+α]	-	-	[40+3α]	-
Cl	-	-	-	[29+2α]	-

Conti…

Fertilizers	Varieties			
	Paiyur1	Paiyur2	CO13	CORa14
N	-	[45+α]	-	[59+2α]
P	-	-	[54+α]	[65+α]
K	[48+4α]	-	-	[57+α]
Ca	-	-	-	-
Mg	[38+3α]	-	-	-
Zn	-	[57+α]	[59+2α]	-
Fe	-	-	-	-
B	-	[47+2α]	-	-
Cl	[45+α]	-	[49+2α]	-

(i) $\tilde{H}_0^L$:There is no significant difference between varieties of yield in ragi crops.

(ii) $\tilde{H}_0^L$: There is no significant difference between fertilizers of yield in ragi crops.

Correction Factor (CF) is $CF^L = \frac{2916\alpha^2 + 130680\alpha + 1464100}{27}$

Total Sum of Squares (TSS) is

$$TSS^L = \frac{648\alpha^2 - 7344\alpha + 74306}{27}$$

Blocks (Unadjusted) Sum of Squares (BSS) is

$$BSS^L = \frac{198\alpha^2 - 3582\alpha + 48854}{27}$$

(Adjusted) Treatments Sum of Squares (Ad.TrtSS) is

$$Ad.TrtSS^L = \frac{93\alpha^2 - 630\alpha + 20893}{27}$$

Error sum of square (ESS) is obtained by subtraction

$$ESS^L = \frac{357\alpha^2 - 3132\alpha + 4559}{27}.$$

All these results are summarized in Table 8 is drawn.

Table 8: ANOVA Table for Fuzzy PBIBD(2) in Lower-Level Model

SV	df	SS	MSS	$\tilde{F}$ - Ratio
Varieties (Unadjusted)	8	$\frac{198\alpha^2-3582\alpha+48854}{27}$	$\frac{198\alpha^2-3582\alpha+48854}{216}$	$\frac{5}{4}\left[\frac{198\alpha^2-3582\alpha+48854}{357\alpha^2-3132\alpha+4559}\right]$
Fertilizers (Adjusted)	8	$\frac{93\alpha^2-630\alpha+20893}{27}$	$\frac{93\alpha^2-630\alpha+20893}{216}$	$\frac{5}{4}\left[\frac{93\alpha^2-630\alpha+20893}{357\alpha^2-3132\alpha+4559}\right]$
Total due to remainder	10	$\frac{357\alpha^2-3132\alpha+4559}{27}$	$\frac{357\alpha^2-3132\alpha+4559}{216}$	-
Total	26	$\frac{648\alpha^2-7344\alpha+74306}{27}$	-	-

$\tilde{F}$ -Ratio for Varieties (Unadjusted) is

$\tilde{F}_B^L = \frac{MSS_B^L}{MSS_E^L} = \frac{5}{4}\left[\frac{198\alpha^2-3582\alpha+48854}{357\alpha^2-3132\alpha+4559}\right]; \alpha \in [0,1]$ at $F^L = 3.07$. Here $\tilde{F}_B^L > F_t^L \ \forall (0 \le \alpha \le 1)$.

The null hypothesis $\tilde{H}_0^L$ is rejected. The difference between varieties is significant. Therefore, the nine varieties differ significantly with respect to the ragi crops yield.

$\tilde{F}$ -Ratio for Treatments (Adjusted) is

$\tilde{F}_{Ad.Trt}^L = \frac{MSS_{Ad.Trt}^U}{MSS_E^U} = \frac{5}{4}\left[\frac{93\alpha^2-630\alpha+20893}{357\alpha^2-3132\alpha+4559}\right]; \alpha \in [0,1]$ at $F^L = 3.07$. Here $\tilde{F}_{Ad.Trt}^L > F_t^L \ \forall (0 \le \alpha \le 1)$.

The null hypothesis $\tilde{H}_0^L$ is rejected. The difference between fertilizers is significant. Therefore, the nine fertilizers differ significantly with respect to the ragi crops yield.

Table 9: Upper-Level Model for α - Cut Interval Method using TZFNs

Fertilizers	Varieties				
	Karusuruttai	ML365	CO15	GPU67	CO14
N	-	-	[45-α]	-	-
P	-	[41-2α]	-	-	-
K	[37-2α]	-	-	-	-
Ca	[39-2α]	[44-α]	-	-	[52-α]
Mg	-	-	[37-α]	-	[42-α]
Zn	-	-	-	-	[57-2α]
Fe	-	[45-α]	[57-2α]	[62-2α]	-
B	[41-α]	-	-	[47-α]	-
Cl	-	-	-	[35-2α]	-

Conti…

Fertilizers	Varieties			
	Paiyur1	Paiyur2	CO13	CORa14
N	-	[52-2 α]	-	[64- α]
P	-	-	[62-3 α]	[70-2 α]
K	[57-2 α]	-	-	[62-2 α]
Ca	-	-	-	-
Mg	[44-2 α]	-	-	-
Zn	-	[62-2 α]	[64- α]	-
Fe	-	-	-	-
B	-	[53- α]	-	-
Cl	[52-2 α]	-	[54- α]	-

(i) $\tilde{H}_0^U$: There is no significant difference between varieties of yields in ragi crops.

(ii) $\tilde{H}_0^U$: There is no significant difference between fertilizers of yields in ragi crops.

Correction Factor is (CF) $CF^U = \frac{1849\alpha^2 - 118422\alpha + 1896129}{27}$

Total Sum of Squares (TSS) is

$$TSS^U = \frac{230\alpha^2 - 1890\alpha + 157005}{27}$$

Blocks (Unadjusted) Sum of Squares (BSS) is

$$BSS^U = \frac{32\alpha^2 - 702\alpha + 43920}{27}$$

(Adjusted) Treatments Sum of Squares (Ad.TrtSS) is

$$Ad.TrtSS^U = \frac{149\alpha^2 - 1231\alpha + 89082}{27}$$

Error sum of square (ESS) is obtained by subtraction

$$ESS^U = \frac{49\alpha^2 + 43\alpha + 2403}{27}.$$

All these results are summarized in Table 10 is drawn.

Table 10: ANOVA Table for Fuzzy PBIBD(2) in Upper-Level Model

SV	df	SS	MSS	$\tilde{F}$ - Ratio
Varieties (Unadjusted)	8	$\frac{32\alpha^2 - 702\alpha + 43920}{27}$	$\frac{32\alpha^2 - 702\alpha + 43920}{216}$	$\frac{5}{4}\left[\frac{32\alpha^2 - 702\alpha + 43920}{49\alpha^2 + 43\alpha + 24003}\right]$
Fertilizers (Adjusted)	8	$\frac{149\alpha^2 - 1231\alpha + 89082}{27}$	$\frac{149\alpha^2 - 1231\alpha + 89082}{216}$	$\frac{5}{4}\left[\frac{149\alpha^2 - 1231\alpha + 89082}{49\alpha^2 + 43\alpha + 24003}\right]$
Total due to remainder	10	$\frac{49\alpha^2 + 43\alpha + 2403}{27}$	$\frac{49\alpha^2 + 43\alpha + 24003}{216}$	-
Total	26	$\frac{230\alpha^2 - 1890\alpha + 157005}{27}$	-	-

$\tilde{F}$-Ratio for Varieties (Unadjusted) is

$\tilde{F}_B^U = \frac{MSS_B^U}{MSS_E^U} = \frac{5}{4}\left[\frac{32\alpha^2 - 702\alpha + 43920}{49\alpha^2 + 43\alpha + 24003}\right]; \alpha \in [0,1]$ at $F^U = 3.07$. Here $\tilde{F}_B^U < F_t^U \; \forall \alpha, \, (0 \le \alpha \le 1)$.

The null hypothesis $\tilde{H}_0^U$ is accepted. The difference between varieties is no significant. Therefore, the nine varieties do not differ significantly with respect to the ragi crops yield.

$\tilde{F}$-Ratio for Fertilizers (Adjusted) is

$\tilde{F}_{Ad.Trt}^U = \frac{MSS_B^U}{MSS_E^U} = \frac{5}{4}\left[\frac{149\alpha^2 - 1231\alpha + 89082}{49\alpha^2 + 43\alpha + 24003}\right]; \alpha \in [0,1]$ at $F^U = 3.07$.Here $\tilde{F}_{Ad.Trt}^U > F_t^U \; \forall \alpha, \, (0 \le \alpha \le 1)$.

The null hypothesis $\tilde{H}_0^U$ is rejected. The difference between fertilizers is significant. Therefore, the nine fertilizers differ significantly with respect to the ragi crops yield.

V. Conclusions

In the real world, sometimes observations are considered imprecise correspondingly to natural calamities. In this event, the fuzzy set theory contributes a useful method for impreciseness. The trapezoidal fuzzy numbers have numerous applications in modeling linear uncertainty in agricultural problems. In this paper a numerical illustration in which the use of the trapezoidal fuzzy PBIBD(2) statistical analysis is defined as the α-cut of the decision rule based on the crisp interval. We conclude that fuzzy PBIBD(2) with intra-block analysis observations has the potential to be used more effectively than in uncertainty. In future studies, the proposed method one can be used of this paper to extend, Lattice designs, some special designs and fuzzy ranking methods.

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