CONSTRUCTION OF SIX SIGMA BASED CONTROL CHART FOR STANDARD DEVIATION UNDER MODERATENESS

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Abstract

Control charts play a significant role to monitor the performance of a process. In parametric control charts, the parent distribution of the process production is usually known and commonly assumed to be a normal. If the distribution of the process production is unknown, the traditional control limits no longer remain effective and the detection ability of parametric control charts can be negatively affected, Muhammad Riaz (2019). This leads us to the development of control charts that are not specifically designed under the assumption of normality or any other parametric distribution. In SPC literature, the nonparametric control charts are widely employed and have numerous advantages for the monitoring of real processes, Chakraborti et al. (2001). The large shifts are detected efficiently by Shewhart's (1931) control chart, whereas for small and medium instabilities, cumulative sum and exponentially weighted moving average control charts are used. Now the companies in developed and developing countries started applying Six Sigma initiatives in their manufacturing process, which results in lesser number of defects. The companies practicing six sigma initiatives are expected to produce 3.4 or less number of defects per million opportunities, a concept suggested by Motorola. By exploring the literature, we found that no study as of yet, an attempt is made in this research paper to construct a six sigma based control chart for standard deviation under Moderate distribution specially designed for the companies applying six sigma initiatives in their organization.
Keywords: Control Chart, Moderate distribution, Process control and Six Sigma.

1. Introduction

Statistical process control (SPC) is a collection of tools for the monitoring of process parameters. The most valuable of these tools is control chart, Montgomery (2009). Control charts are useful for analysing and controlling repetitive processes because they help to determine when corrective actions are needed (www.cpajournal.com). Because they display running records of performance, control charts provide numerous types of information to management. For example, control charts are useful for:

- Pinpointing erratic or unpredictable processes;
- Obtaining warning of impending trouble, such as an unexpected change in a process;
- Evaluating product (service) consistency over time;
- Decreasing performance variability in a process, thereby decreasing the level of post-process inspection of the output generated by the process;
- Determining the cause of trouble when a process is generating output which has errors and mistakes; and
- Knowing when a process is doing the best that can be expected from it.

A control chart is a graph that contains a centerline, and upper and lower control limits. The centerline represents the process average. The control limits represent the upper and lower boundaries of acceptability around the centerline. The horizontal axis represents sample numbers or points in time, and the vertical axis represents measurements from samples.

Control charts are usually based on data collected from samples of a process. After a sufficient number of samples are drawn and the data is plotted on a control chart, the stability of the process is evaluated (www.cpajournal.com). A process which is stable is deemed to be "in control" whereas an "out of control" process is unstable, and therefore, unpredictable. The companies, which are practicing Six Sigma, are expected to produce 3.4 or less number of defects per million opportunities. Radhakrishnan and Sivakumaran (2008) used the concept of six sigma in the construction of sampling plans such as single, double and repetitive group sampling plans indexed through Six Sigma Quality Levels (SSQLs) with Poisson distribution as the base line distribution. Radhakrishnan (2009) suggested single sampling plan indexed through Six Sigma quality levels (SSQLs) based on Intervened Random Effect Poisson Distribution and Weighted Poisson Distribution as the base line distributions. Radhakrishnan and Balamurugan (2016) constructed control chart for standard deviation based on Six Sigma. The control charts originated by W.A. Shewhart (1931) was based on 3 sigma control limits. If the same charts are used for the products of the companies which adopt six sigma initiatives in the process, then no point will fall outside the control limits because of the improvement in the quality. So a separate control chart is required to monitor the outcomes of the companies, which adopt six sigma initiatives. In this research article an attempt is made to construct a six sigma based control chart for standard deviation under Moderate distribution.

2. Assumptions for the study

- Production managers involved in the study will be willing and able to learn the principles of evaluating control charts.
- Production managers involved in the study will have adequate knowledge and experience to make adjustments to an activity to improve the productivity of a process based on the data conveyed in the control charts.
- The activities to be studied will feature crews comprised of the same laborers and operators during the pre-intervention and intervention periods.

3. Conditions for application

- Six sigma based control limits will be used if the distribution is found to be Moderate.
- Companies adopt the concept of Six sigma based control limits using process capability under Moderate distribution in its processes.

4. Moderate distribution

Desai (2011) has developed moderate distribution, an alternative of normal distribution. In this distribution mean μ and mean deviation δ are parameters. Suppose the probability distribution function of a distribution of a random variable X is defined as,

$$f(x) = \frac{1}{\pi\delta} e^{-\frac{1}{\pi}\left(\frac{X-\mu}{\delta}\right)^2}, \; -\infty < X < \infty, \; \delta > 0$$

Suppose, X~ M (μ, δ), then the variable Z is defined as,

$$Z = \frac{X-\mu}{\delta},$$

has the probability distribution function defined as,

$$g(Z) = \frac{1}{\pi} e^{-\frac{1}{\pi}Z^2}, \; -\infty < Z < \infty,$$

And this variable Z may be called standard moderate variate. Also, its distribution may be called standard moderate distribution.

5. Construction of six sigma based control chart for standard deviation under Moderate distribution

Fix the tolerance level (TL) and process capability (C_P) to determine the process standard deviation ($\sigma_{MD:6\sigma}$).

Apply the value of $\sigma_{MD:6\sigma}$ in the control limits $\bar{S} \pm \left[B_{MD:6\sigma} \sqrt{\left(1-c_4^{\,2}\right)} \right] \sigma_{MD:6\sigma}$, to get the six sigma based control limits for standard deviation under Moderate distribution.

The value of $A_{MD:6\sigma}$ is obtained using $p(z \le z_{6\sigma}) = 1-\alpha_1, \alpha_1 = 3.4 \text{ x } 10^{-6}$ and z is a standard moderate variate.

For a specified TL and C_P of the process, the value of σ (termed as $\sigma_{MD:6\sigma}$) is calculated from $c_p = \frac{TL}{6\sigma}$ using a JAVA Script for various combinations of TL and C_P.

The six sigma based control limits for standard deviation under Moderate distribution are

$$\text{UCL}_{MD:6\sigma} = \bar{S} + \left[B_{MD:6\sigma} \sqrt{\left(1-c_4^{\,2}\right)} \right] \sigma_{MD:6\sigma}$$

$$\text{CL}_{MD:6\sigma} = \bar{S}$$

$$\text{LCL}_{MD:6\sigma} = \bar{S} - \left[B_{MD:6\sigma} \sqrt{\left(1-c_4^{\,2}\right)} \right] \sigma_{MD:6\sigma}$$

where $\sigma_{MD:6\sigma}$ is in place of $\frac{\bar{S}}{c_4}$ in 3 – Sigma control limits

6. Example

The example provided by Gupta (2001, Page No. 122) is considered here. The following data (mm) is indicated the quality characteristic was the depth of the keyway in shafting.

Table 1: Measurements of Quality Characteristics for Depth keyway in Shafting

Product: Shafts **Dept. No: M/c-999,** **Order No: 1A/PEC: 118**
Characteristic: Depth of keyway **Special Instructions: Measurements to be Coded from 6.00 mm.**
Unit of measure: mm.

Sub group Number	Date	Time	Measurements				Average	Range	Standard deviation	Comments
			X_1	X_2	X_3	X_4				
1	23/6	8:50	35	40	32	33	6.35	0.08	0.031	
2		11:30	46	37	36	41	6.40	0.10	0.042	
3		1:45	34	34	40	34	6.30	0.06	0.023	New Operator

4		3:45	69	64	68	59	6.65	0.10	0.045	Temporary Operator
5		4:20	38	34	44	40	6.39	0.10	0.042	
6	27/6	8:35	42	41	43	34	6.40	0.09	0.042	New Operator
7		9:00	44	41	41	46	6.43	0.05	0.021	
8		9:40	33	41	38	36	6.37	0.08	0.032	
9		1:30	48	52	49	51	6.50	0.04	0.016	
10		2:50	47	43	36	42	6.42	0.11	0.043	
11	28/6	8:30	38	41	39	38	6.39	0.03	0.012	
12		1:35	37	37	41	37	6.38	0.04	0.017	
13		2:25	40	38	47	35	6.40	0.12	0.044	
14		2:35	38	39	45	42	6.41	0.07	0.025	
15		3:55	50	42	43	45	6.45	0.08	0.031	
16	29/6	8:25	33	35	29	39	6.34	0.10	0.041	
17		9:25	41	40	29	34	6.36	0.12	0.048	
18		11:00	38	44	28	58	6.42	0.30	0.109	Damaged oil Line
19		2:35	33	32	37	38	6.35	0.06	0.025	
20		3:15	56	55	45	48	6.51	0.11	0.044	Bad Material
21	30/6	9:35	38	40	45	37	6.40	0.08	0.031	
22		10:20	39	42	35	40	6.39	0.07	0.025	
23		11:35	42	39	39	36	3.39	0.06	0.021	
24		2:00	43	36	35	38	6.38	0.08	0.031	
25		4:25	39	38	43	44	6.41	0.06	0.025	
Total							160.25	2.19	1.006	
$\sum \bar{X}_i = 160.25$ $\sum R = 2.19$ $\sum s = 1.006$										

$$\bar{S} = \frac{\sum s_i}{N} = \frac{1.006}{25} = 0.041$$

6.1 Construction of control limits (3σ) for standard deviation

The 3–Sigma control limits suggested by Shewhart (1931) for standard deviation are

$$\bar{S} \pm \left[3 \times \sqrt{\left(1 - c_4^{\,2}\right)}\right]\left(\frac{\bar{S}}{c_4}\right)$$

$$UCL_{3\sigma} = \bar{S} + \left[3 \times \sqrt{\left(1 - c_4^{\,2}\right)}\right]\left(\frac{\bar{S}}{c_4}\right)$$

$$= 0.041 + \left[3 \times \sqrt{\left(1 - 0.9213^2\right)}\right]\left(\frac{0.041}{0.9213}\right)$$

$$= 0.093mm$$

$$\text{Central line } CL_{3\sigma} = \bar{S}$$

$$= 0.041mm$$

$$LCL_{3\sigma} = \bar{S} - \left[3 \times \sqrt{\left(1 - c_4^{\,2}\right)}\right] \left(\frac{\bar{S}}{c_4}\right)$$

$$= 0.041 - \left[3 \times \sqrt{\left(1 - 0.9213^2\right)}\right] \left(\frac{0.041}{0.9213}\right)$$

$$= 0$$

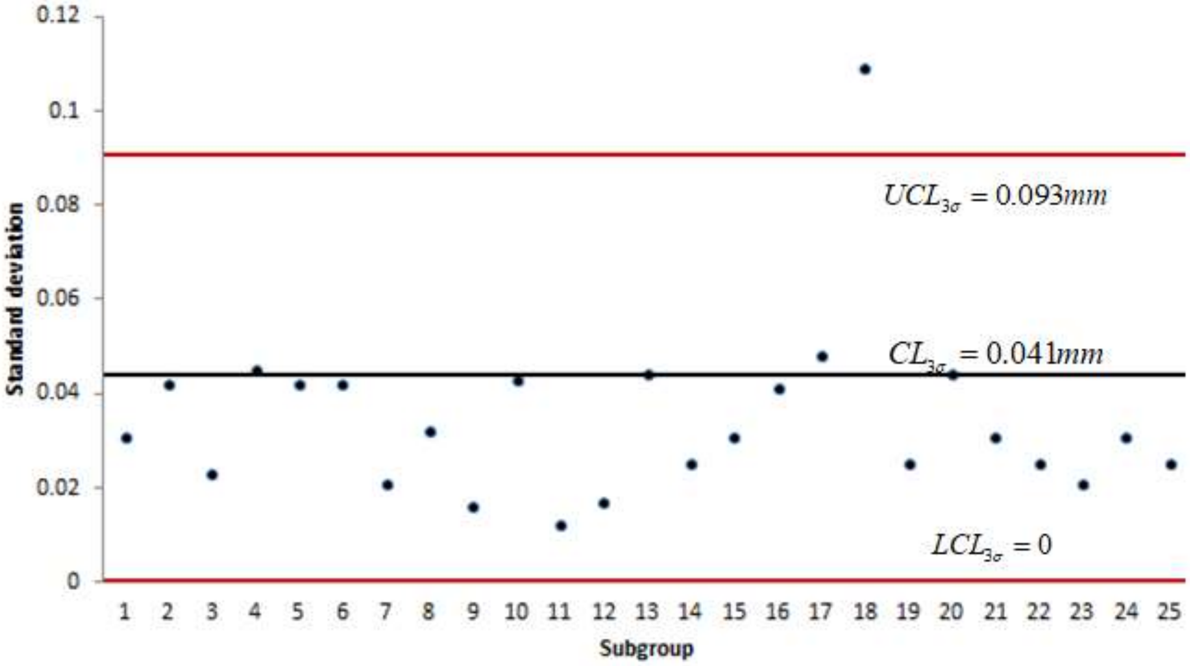

Figure 1: 3σ based control limits for standard deviation

From the resulting Figure 1, it is clear that the process does not exhibit statistical control since the sub group number 18 goes above the upper control limit.

6.2 Six sigma based control limits for standard deviation under Moderate distribution

For a given TL=0.097 and C_p=1.5, the value of $\sigma_{MD:6\sigma}$ can be obtained as 0.011 and the value of $z_{6\sigma}$ obtained from the Moderate distribution as 5.64 and hence, the value of $B_{MD:6\sigma}$ is obtained as 5.64.

The six sigma based standard deviation chart under Moderate distribution for a specified TL, $A_{MD:6\sigma}$ and n is $\bar{S} \pm \left[B_{MD:6\sigma}\sqrt{\left(1 - c_4^{\,2}\right)}\right]\sigma_{MD:6\sigma}$ with

$$\mathrm{UCL}_{MD:6\sigma} = \bar{S} + \left[B_{MD:6\sigma}\sqrt{\left(1 - c_4^{\,2}\right)}\right]\sigma_{MD:6\sigma}$$

$$\mathrm{UCL}_{MD:6\sigma} = 0.041 + \left[5.64 \times \sqrt{\left(1 - 0.9213^2\right)}\right]0.011 = 0.065$$

$$\mathrm{CL}_{MD:6\sigma} = \bar{S} = 0.041$$

$$\mathrm{LCL}_{MD:6\sigma} = \bar{S} - \left[B_{MD:6\sigma}\sqrt{\left(1 - c_4^{\,2}\right)}\right]\sigma_{MD:6\sigma}$$

$$\mathrm{LCL}_{MD:6\sigma} = 0.041 - \left[5.64 \times \sqrt{\left(1 - 0.9213^2\right)}\right]0.011 = 0.017$$

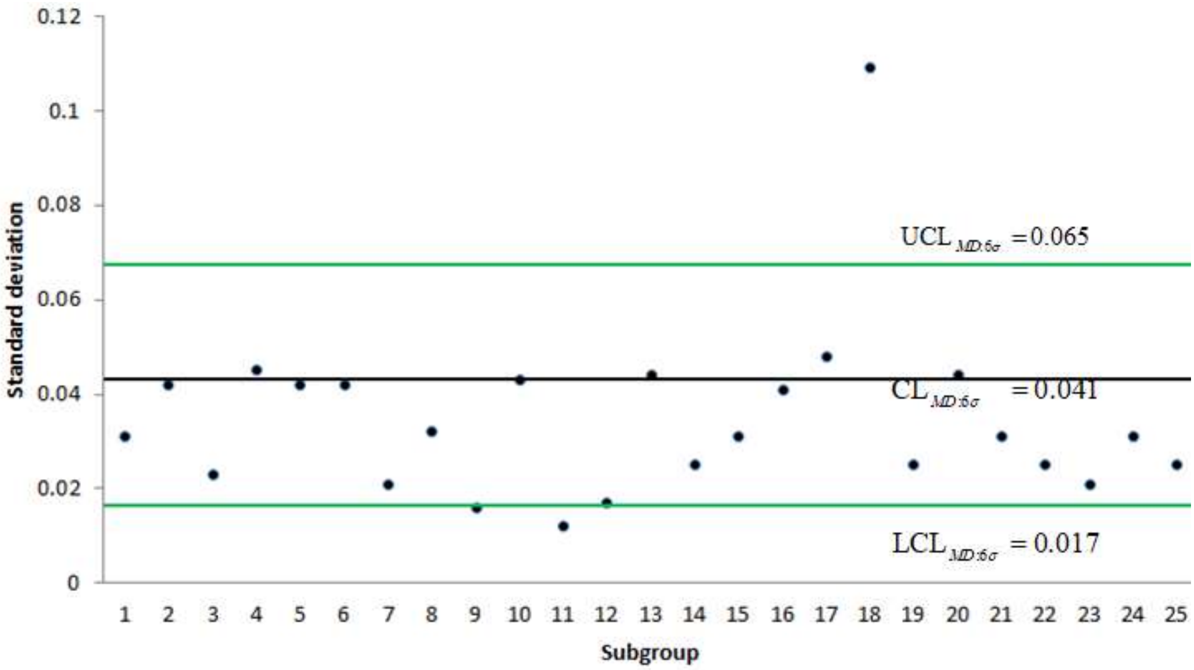

Figure 2: Six sigma based control limits for standard deviation under MD

From the resulting Figure 2, it is clear that the process does not exhibit statistical control since the sub group numbers 9 and 11 goes below the lower control limit. It is found from the Figures 1 and 2, the process is in Statistical control when 3σ control limits are used and the process is not in statistical control when Six Sigma based control limits are adopted. It is clear that the product/service is not in good quality as expected using Six Sigma initiatives, so a correction is needed in the process/system.

7. Conclusion

In this paper, a procedure is given to construct a six sigma based control chart for standard deviation under Moderate distribution with an example. It is found that the process is in control even when six sigma initiatives are adopted but it is very clear from the comparison that when the process is centered with reduced variation than the 3σ control limits, which indicate that the process is not in the level it was expected. So a correction in the process is very much required to reduce the variations. The charts suggested in this research paper will be very useful for the companies practicing six sigma initiatives under Moderate distribution in their process. These charts will replace the existing Shewhart (1931) control charts in future when all the companies started implementing six sigma initiatives under Moderate distribution in their organization.

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