FUZZY STRONGLY sg-CONTINUOUS MAPPINGS

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Abstract

In this paper we introduce a new class of fuzzy mappings called fuzzy strongly sg-continuous mappings in fuzzy topological spaces and obtain some of its basic properties and characterizations.

Keywords: fuzzy g-closed, fuzzy sg-closed, fuzzy rg-closed, fuzzy sg-open, and fuzzy rg-open sets, fuzzy strongly sg-continuous and fuzzy sg-continuous mappings.

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1. Introduction

In 1965, Zadeh introduced the concept of fuzzy sets and fuzzy set operation in his classical paper [15]. Subsequently several authors generalize various concepts from general topology to fuzzy sets and developed the theory of fuzzy topological spaces. The concept of fuzzy sg-closed sets and sg- continuous mappings were studied [10]. Recently Many author [4, 7, 9, 13] have been done work in different topological space. Semi generalized closed set in topological space studied by Bhattacharya et.al [2]. In the present paper we introduce and study the concept of fuzzy strongly sg-continuous mappings in fuzzy topological spaces.

2. Preliminaries

Let X be a non-empty set and I = [0,1]. A fuzzy set A in X is a mapping from X to I. The null fuzzy set 0 is the mapping from X into I which assumes only the value 0 and the whole fuzzy set 1 is a mapping from X into I which takes value 1 only. A family τ of fuzzy sets of X is called a fuzzy topology [5] on X, If 0 and 1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their compliments are fuzzy closed sets. If A is a fuzzy set, then closure of A (denoted by cl(A)) is the intersection of all fuzzy closed super sets of A and interior of A (denoted by int(A)) is the union of all fuzzy open subsets of A. A fuzzy point x_β in X is a fuzzy set defined by

$$x_\beta(y) = \begin{cases} \beta, \beta \in [0,1] \ \text{for } y = x, y \in X \\ 0, \text{otherwise} \end{cases}$$

where x and β are respectively called the support and the value of x_β. A fuzzy point $x_\beta \in A$ if and only if $\beta \leq A(x)$. A fuzzy set A is the union of all fuzzy points belongs to A. A fuzzy point $x_\beta \in A$ is said to be quasi-coincident with the fuzzy set A denoted by $x_\beta qA$ if and only if $\beta + A(x) > 1$.

Definition 2.1. [1] A fuzzy set A of a fuzzy topological space (X,τ) is said to be:

(a) fuzzy semi open if $A \leq cl(int(A))$.

(b) fuzzy semi closed if 1-A is fuzzy semi open.

(c) fuzzy regular open if A= int (cl(A)).

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(d) fuzzy regular closed if 1-A is fuzzy regular open.

Definition 2.2. [14] The semi interior sint(A) and semi closure scl(A) of a fuzzy set A are respectively defined as

$$\text{sint}(A) = \cup\{G : G \leq A, G \text{ is fuzzy semi open}\}$$

$$\text{scl}(A) = \cap\{F : F \geq A, F \text{ is fuzzy semi closed}\}.$$

Definition 2.3. A fuzzy set A of a fuzzy topological space (X,τ) is said to be:

(a) fuzzy g-closed if $cl(A) \leq G$ whenever $A \leq G$ and G is fuzzy open [11].

(b) fuzzy g-open if 1-A is fuzzy g-closed [11].

(c) fuzzy sg-closed set if $scl(A) \leq G$ whenever $A \leq G$ and G is fuzzy semi open.[10]

(d) fuzzy sg-open if 1-A is fuzzy sg-closed [10].

(e) fuzzy rg-closed if $cl(A) \leq G$ whenever $A \leq G$ and G is fuzzy regular open [12].

(f) fuzzy rg-open if 1-A is fuzzy rg-closed [12]

Remark 2.1. [11, 12] Every fuzzy closed set is fuzzy g-closed and every fuzzy g-closed set is fuzzy rg-closed (resp. sg-closed) but the converse may not be true.

Definition 2.4. [12] A collection $\{A_\alpha : \alpha \in \wedge\}$ of a fuzzy rg-open (resp. sg-open) sets in a fuzzy topological space (X,τ) is called a fuzzy rg-open (resp. sg-open) cover of a fuzzy set G of X if $G \leq \cup\{A_\alpha : \alpha \in \wedge\}$.

Definition 2.5. [14] A fuzzy topological space (X, τ) is said to be fuzzy rg-compact (resp. sg-compact) if every fuzzy rg-open (resp. sg-open) cover of X has finite sub cover.

Definition 2.6. [12] A fuzzy set A of a fuzzy topological space (X,τ) is said to be fuzzy rg-compact (resp. sg- compact) relative to X if every collection$\{A\alpha : \alpha \in \wedge\}$ of fuzzy rg-open(resp. sg-open) subsets of X such that $A \leq \cup\{A_\alpha : \alpha \in \wedge\}$, there exist a finite subset $\wedge_0$ such that $A \leq \cup\{A_{\alpha j} : \alpha_j \in \wedge_0\}$.

Definition 2.7. [8, 12] A fuzzy topological space (X,τ) is fuzzy rg-connected (resp sg-connected),if there is no proper fuzzy sets of X which is both fuzzy rg-open and fuzzy rg-closed (resp. fuzzy sg-open and fuzzy sg-closed).

Definition 2.8. A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be :

(a) fuzzy continuous if the inverse image of each fuzzy open set of Y is fuzzy open in X. [5].

(b) fuzzy semi continuous if $f^{-1}(A)$is fuzzy semi open in X for every fuzzy open set A of Y. [1].

(c) g-continuous if inverse image of every fuzzy closed set of Y is fuzzy g-closed in X. [6].

(d) fuzzy rg-continuous if the inverse image of every fuzzy closed set of Y is fuzzy rg-closed in X. [14].

(e) fuzzy sg-continuous if the inverse image of every fuzzy closed set of Y is fuzzy sg-closed in X. [10].

Remark 2.2. [8,14] Every fuzzy continuous mapping is fuzzy g-continuous and every g-continuous mapping is fuzzy rg-continuous (resp. sg-continuous) but the converse may not be true.

The concept of generalized continuous maps in topological space are given by Balchandran et.al. [3].

3. Main Result (Fuzzy Strongly sg-Continuous Mappings)

Definition 3.1. A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be fuzzy strongly sg-continuous mapping if the inverse image of every fuzzy rg-closed set of Y is fuzzy sg-closed set in X.

Theorem 3.1. A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy strongly sg-continuous mapping if and only if the inverse image of fuzzy rg-open set of Y is fuzzy sg-open in X.

Proof: It is obvious because $f^{-1}(1-A) = 1-f^{-1}(A)$ for every fuzzy set A of Y.

Remark 3.1. Every fuzzy strongly sg-continuous mapping is fuzzy sg-continuous, but the converse may not be true.

Example 3.1. Let X = {a,b}, Y={x,y} and the fuzzy set U and V are defined as follows:

$U(a) = 0.5 \qquad U(b) = 0.3$

$V(x) = 0.5 \qquad V(y) = 0.4.$

Let $\tau = \{0,U,1\}$ and $\sigma = \{0,1\}$ be fuzzy topologies on X and Y respectively. Then the mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ defined by f(a) = x and f(b) = y is fuzzy sg- continuous but not fuzzy strongly sg-continuous mapping.

Theorem 3.2. If $f:(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy strongly sg-continuous then for each fuzzy point x_β of X and each fuzzy rg-open set V of Y such that $f(x_\beta)\in V$ there exist a fuzzy sg-open set U of X such that $x_\beta \in U$ and $f(U) \leq V$.

Proof: Let x_β be a fuzzy point of X and V be a fuzzy rg-open set of Y such that $f(x_\beta)\in V$. Then by hypothesis U is a fuzzy sg-open set of X such that $x_\beta \in U$ and $f(U) = f(f^{-1}(V)) \leq V$.

Theorem 3.3. If $f : (X,\tau) \rightarrow (Y,\sigma)$ is fuzzy strongly sg-continuous then for each fuzzy point x_β of X and each fuzzy rg-open set V of Y such that $f(x_\beta) \in V$, there exists a fuzzy sg-open set U of X such that $x_\beta\in U$ and $f(U)\leq V$.

Proof: Let x_β be a fuzzy point of X and V be a fuzzy rg-open set such that $f(x_\beta) \in V$ put $U= f^{-1}(V)$. Then by hypothesis U is a fuzzy sg-open set of X such that $x_\beta\in U$ and $f(U) = f(f^{-1}(V)\leq V$.

Definition 3.2. Let (X,τ) be a fuzzy topological space. The semi generalized closure (resp. regular generalized closure) of a fuzzy set A of X denoted by sgcl(A) (resp. rgcl(A)) is defined as follows:

$$sgcl(A) = \inf\{B:B \geq A, B \text{ is fuzzy sg-closed set of } (X,\tau)\}.$$

$$rgcl(A) = \inf\{B:B \geq A, B \text{ is fuzzy rg-closed set of } (X,\tau)\}.$$

Theorem 3.4. If $f:(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy strongly sg-continuous, then $f(sgcl(A)) \leq rgcl(f(A))$ for every fuzzy set A of X.

Proof: Let A be a fuzzy set of X. Then rgcl(f(A)) is a fuzzy rg-closed set of Y. Since f is fuzzy sg-continuous $f^{-1}(rgcl(f(A)))$ is fuzzy sg-closed in X clearly$A\leq f^{-1}(rgcl(f(A)))$. Therefore $sgcl(A) \leq sgcl(f^{-1}(rgcl(f(A))) = f^{-1}(rgcl(f(A)))$. Hence $f(sgcl(A)) \leq rgcl(f(A))$.

Definition 3.3. A fuzzy topological space (X,τ) is said to be fuzzy regular $T_{1/2}$ (resp. fuzzy semi $T_{1/2}$.) if every fuzzy g-closed (fuzzy sg-closed) set in X is fuzzy closed (resp. fuzzy semi closed).

Theorem 3.5. A mapping f from a fuzzy regular $T_{1/2}$ space (X,τ) to a fuzzy topological space (Y,σ) is fuzzy semi continuous if and only if it is fuzzy strongly sg-continuous.

Proof: Obvious.

Theorem 3.6. If f:(X,τ)→(Y,σ) is fuzzy strongly sg-continuous and g:(Y,σ)→(Z,μ) is fuzzy rg-irresolute. Then gof : (Y,σ)→(Z,μ) is fuzzy strongly sg-continuous.

Proof: Let A be fuzzy rg-closed in Z, then $g^{-1}(A)$ is fuzzy rg-closed in Y because g is fuzzy rg-irresolute. Therefore $(gof)^{-1}(A)=f^{-1}(g^{-1}(A))$ is fuzzy sg-closed in X. Hence gof is fuzzy strongly sg - continuous.

Theorem 3.7. A fuzzy strongly sg-continuous image of a fuzzy sg- compact space is fuzzy rg-compact.

Proof: Let f:(X,τ)→(Y,σ) is fuzzy strongly sg-continuous mapping from a fuzzy sg-compact space(X,τ) onto a fuzzy topological space (Y, σ). Let $\{A_i : i\in\wedge\}$ be a fuzzy rg-open cover of Y then$\{f^{-1}(A_i): i\in\wedge\}$is fuzzy sg-open cover of X, since X is sg-compact it has finite fuzzy sub cover say $\{f^{-1}(A_1),\ldots,f^{-1}(A_n)\}$. Since f is onto $\{A_1, A_2,\ldots, A_n\}$is an fuzzy rg-open cover of Y and so (Y,σ) is fuzzy rg-compact.

Theorem 3.8. If f:(X,τ)→(Y,σ) is fuzzy strongly sg-continuous surjection and X is fuzzy sg-connected, then Y is fuzzy rg-connected.

Proof: Suppose Y is not fuzzy rg-connected. Then there exists a proper fuzzy set G of Y which is both fuzzy rg-open and fuzzy rg- closed. Therefore $f^{-1}(G)$ is a proper fuzzy set of X, which is both fuzzy sg-open and sg-closed, because f is fuzzy strongly sg-continuous surjection. Hence X is not fuzzy sg-connected, which is a contradiction.

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