CONSTRUCTION OF SIX SIGMA BASED FUZZY CONTROL CHART FOR RANGE UNDER MODERATE DISTRIBUTION

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Abstract
Fuzzy sets theory is an impressive mathematical methodology to evaluate the vagueness related uncertainty that can linguistically express data in these situations. Control charts are the effective and quietest form of statistical process control methods. Many of the times, data are obtained in quantitative form; however there are many quality characteristics that cannot be expressed in numerical measure, such as characteristics for appearance, smoothness and colour, etc. In this paper, we construct a six sigma based fuzzy control limits for range under moderate distribution.

Keywords: **Fuzzy control chart, Moderate distribution, Process capability and Six sigma.**

1. Introduction
Control charts are widely used for monitoring and examining a production process. The power of control charts lies in their ability to detect process shifts and to identify abnormal conditions in the process (Amirzadeh, 2008). If w be a sample statistic that measures some quality characteristic of interest the mean of w is μ_w and the standard deviation of w is σ_w, then the control limits are defined as $\mu_w \pm A\sigma_w$, where A is the "distance" of the control limits from the centre line, expressed in standard deviation units (Shewhart, 1924). In many situations, control limits could not be so precise. Uncertainty comes from the measurement system including operators and gauges, and environmental conditions. In this context, fuzzy set theory is a useful tool to handle this uncertainty. Numeric control limits can be transformed to fuzzy control limits by using membership functions (Sevil Senturk and Nihal Erginel, 2009). They compared with the conventional chart for fraction defective. It is seen that Fuzzy Multinomial chart with Variable Sample Size performs better than the conventional chart. An extension of standard control chart to deal with linguistic categories and the variable sampling size (VSS), it is named as fuzzy multinomial charts (FM-chart), and they compared FM-chart with the conventional p – chart and EWMA Control Chart (Kawa Jamal Rashid and Suzan Haydar, 2014). It is seen that FM chart with VSS performs better than the conventional charts, this method is more sensitive, accurate and more economic for assisting decision maker to control the operation system as early time, especially when there is a change in sample sizes. In this research article we introduce the six sigma based fuzzy control chart for range under moderate distribution by numerical example.

2. Methods and materials
The average of range for the trapezoidal fuzzy numbers are represented as $\left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right)$.

$$\left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right) = \left(\frac{\sum_{j=1}^{m} R_{aj}}{m}, \frac{\sum_{j=1}^{m} R_{bj}}{m}, \frac{\sum_{j=1}^{m} R_{cj}}{m}, \frac{\sum_{j=1}^{m} R_{dj}}{m} \right), \; j = 1, 2, \ldots m$$

where

$$\overline{R}_r = \frac{\sum_{j=1}^{m} R_{rj}}{m}, r = a,b,c,d \text{ and } j = 1,2,...m$$

$$R_{rj} = \left(X_{\max.aj} - X_{\min.dj}, X_{\max.bj} - X_{\min.cj}, X_{\max.cj} - X_{\min.bj}, X_{\max.dj} - X_{\min.aj} \right), \ j = 1,2,...m$$

$\left(X_{\max.aj}, X_{\max.bj}, X_{\max.cj}, X_{\max.dj} \right)$ are the maximum trapezoidal fuzzy numbers for each sample and

$\left(X_{\min.aj}, X_{\min.bj}, X_{\min.cj}, X_{\min.dj} \right)$ are the minimum trapezoidal fuzzy numbers for each sample.

The Shewhart (1924) control limits for range (R) are given below:

$$UCL_{\overline{R}} = \overline{R} + 3d_3 \left(\frac{\overline{R}}{d_2} \right)$$

$$CL_{\overline{R}} = \overline{R}$$

$$LCL_{\overline{R}} = \overline{R} - 3d_3 \left(\frac{\overline{R}}{d_2} \right)$$

where d_2 and d_3 are a control chart co-efficients.

The fuzzy control limits for range are as follows:

$$\begin{aligned} U\tilde{C}L_{\overline{R}} &= \left(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d \right) + \left(\frac{3d_3}{d_2} \right) \left(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d \right) \\ &= \left(\overline{R}_a + \frac{3d_3}{d_2} \overline{R}_a, \overline{R}_b + \frac{3d_3}{d_2} \overline{R}_b, \overline{R}_c + \frac{3d_3}{d_2} \overline{R}_c, \overline{R}_d + \frac{3d_3}{d_2} \overline{R}_d \right) \\ &= \left(U\tilde{C}L_{a.\overline{R}}, U\tilde{C}L_{b.\overline{R}}, U\tilde{C}L_{c.\overline{R}}, U\tilde{C}L_{d.\overline{R}} \right) \end{aligned}$$

$$C\tilde{L}_{\overline{R}} = \left(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d \right)$$

$$\begin{aligned} L\tilde{C}L_{\overline{R}} &= \left(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d \right) - \left(\frac{3d_3}{d_2} \right) \left(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d \right) \\ &= \left(\overline{R}_a - \frac{3d_3}{d_2} \overline{R}_a, \overline{R}_b - \frac{3d_3}{d_2} \overline{R}_b, \overline{R}_c - \frac{3d_3}{d_2} \overline{R}_c, \overline{R}_d - \frac{3d_3}{d_2} \overline{R}_d \right) \\ &= \left(L\tilde{C}L_{a.\overline{R}}, L\tilde{C}L_{b.\overline{R}}, L\tilde{C}L_{c.\overline{R}}, L\tilde{C}L_{d.\overline{R}} \right) \end{aligned}$$

The proposed standard deviation $\left(\sigma_{r.FMD:6\sigma}, \ r = a,b,c,d \right)$ for six sigma based fuzzy control chart under moderate distribution with the help of process capability $C_p = \frac{USL_{r.RFC_p} - LSL_{r.RFC_p}}{6\sigma}, r = a,b,c,d$ using a JAVA script (Radhakrishnan and Balamurugan,

2012) under moderate distribution is to calculate by the specified tolerance level from the relation $\frac{\left(\frac{\sum_{j=1}^{m} R_{rj}}{m}\right) d_3}{d_2}$, $r = a,b,c,d$ and $j = 1,2,...m.$

Apply the value of $\sigma_{FMD:6\sigma}$ in the control limits, to get the six sigma based fuzzy control limits for mean using range under Moderate distribution. The value of $A_{FMD:6\sigma}$ is obtained using $p(z \le z_{6\sigma}) = 1 - \alpha_1, \alpha_1 = 3.4 \text{ x } 10^{-6}$ and z is a standard moderate variate.

Therefore the resultant of proposed six sigma based fuzzy control limits for range under moderate distribution is given below:

$$U\tilde{C}L_{\bar{R}:C_p} = \left(\bar{R}_a + A_{FMD:6\sigma}\tilde{\sigma}_{aR.FC_p}, \bar{R}_b + A_{FMD:6\sigma}\tilde{\sigma}_{bR.FC_p}, \bar{R}_c + A_{FMD:6\sigma}\tilde{\sigma}_{cR.FC_p}, \bar{R}_d + A_{FMD:6\sigma}\tilde{\sigma}_{dR.FC_p}\right)$$

$$= \left(U\tilde{C}L_{a.\bar{R}:C_p}, U\tilde{C}L_{b.\bar{R}:C_p}, U\tilde{C}L_{c.\bar{R}:C_p}, U\tilde{C}L_{d.\bar{R}:C_p}\right)$$

$$C\tilde{L}_{\bar{R}:C_p} = \left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right)$$

$$L\tilde{C}L_{\bar{R}:C_p} = \left(\bar{R}_a - A_{FMD:6\sigma}\tilde{\sigma}_{aR.FC_p}, \bar{R}_b - A_{FMD:6\sigma}\tilde{\sigma}_{bR.FC_p}, \bar{R}_c - A_{FMD:6\sigma}\tilde{\sigma}_{cR.FC_p}, \bar{R}_d - A_{FMD:6\sigma}\tilde{\sigma}_{dR.FC_p}\right)$$

$$= \left(L\tilde{C}L_{a.\bar{R}:C_p}, L\tilde{C}L_{b.\bar{R}:C_p}, L\tilde{C}L_{c.\bar{R}:C_p}, L\tilde{C}L_{d.\bar{R}:C_p}\right)$$

3. Application

Consider a process by which coils are manufactured by a company in Salem District. The primary data collected and presented in Table-1 have been used the samples of size 5 are randomly selected from the process, and the 'between' measurements resistance values (in ohms) of the coils are measured for the application. These measurements are then converted into trapezoidal fuzzy numbers (TFN) using computer program and are given in Table-2.

Table 1: Resistance values (in ohms) of coils

Sample No.	X_1		X_2		X_3		X_4		X_5	
1	0.52	0.55	0.49	0.51	0.56	0.57	0.49	0.50	0.52	0.53
2	0.52	0.53	0.51	0.52	0.52	0.54	0.51	0.53	0.45	0.47
3	0.51	0.52	0.53	0.54	0.54	0.55	0.49	0.55	0.50	0.51
4	0.42	0.45	0.42	0.44	0.46	0.56	0.51	0.54	0.53	0.54
5	0.48	0.50	0.47	0.51	0.50	0.50	0.57	0.58	0.52	0.53
6	0.55	0.56	0.49	0.50	0.50	0.52	0.48	0.49	0.50	0.51
7	0.49	0.53	0.52	0.54	0.49	0.55	0.46	0.48	0.49	0.50
8	0.43	0.46	0.49	0.52	0.49	0.51	0.50	0.53	0.50	0.51
9	0.54	0.55	0.48	0.53	0.51	0.53	0.47	0.49	0.48	0.50
10	0.50	0.52	0.49	0.51	0.48	0.49	0.48	0.52	0.46	0.47
11	0.47	0.50	0.55	0.57	0.50	0.52	0.49	0.50	0.48	0.50

12	0.50	0.54	0.56	0.57	0.47	0.51	0.48	0.51	0.48	0.50
13	0.46	0.48	0.52	0.54	0.50	0.51	0.53	0.54	0.50	0.51
14	0.50	0.51	0.48	0.51	0.48	0.52	0.51	0.53	0.44	0.45
15	0.49	0.51	0.50	0.51	0.54	0.55	0.48	0.52	0.49	0.50

Table 2: Trapezoidal Fuzzy measurement resistance valuesof coils

Sample No.	**X_1**				**X_2**				**X_3**			
1	0.45	0.52	0.55	0.58	0.48	0.49	0.51	0.53	0.55	0.56	0.57	0.58
2	0.50	0.52	0.53	0.55	0.45	0.51	0.52	0.55	0.50	0.52	0.54	0.56
3	0.47	0.51	0.52	0.56	0.52	0.53	0.54	0.58	0.53	0.54	0.55	0.56
4	0.40	0.42	0.45	0.48	0.41	0.42	0.44	0.49	0.41	0.46	0.56	0.57
5	0.45	0.48	0.50	0.53	0.45	0.47	0.51	0.54	0.46	0.50	0.50	0.52
6	0.52	0.55	0.56	0.57	0.47	0.49	0.50	0.53	0.47	0.50	0.52	0.53
7	0.46	0.49	0.53	0.56	0.51	0.52	0.54	0.55	0.45	0.49	0.55	0.56
8	0.40	0.43	0.46	0.47	0.44	0.49	0.52	0.55	0.46	0.49	0.51	0.54
9	0.53	0.54	0.55	0.57	0.41	0.48	0.53	0.56	0.46	0.51	0.53	0.55
10	0.49	0.50	0.52	0.54	0.44	0.49	0.51	0.54	0.45	0.48	0.49	0.52
11	0.41	0.47	0.50	0.53	0.54	0.55	0.57	0.58	0.46	0.50	0.52	0.55
12	0.48	0.50	0.54	0.56	0.54	0.56	0.57	0.59	0.42	0.47	0.51	0.53
13	0.44	0.46	0.48	0.50	0.51	0.52	0.54	0.55	0.48	0.50	0.51	0.56
14	0.47	0.50	0.51	0.53	0.45	0.48	0.51	0.57	0.46	0.48	0.52	0.58
15	0.46	0.49	0.51	0.55	0.48	0.50	0.51	0.56	0.53	0.54	0.55	0.56

Continued…

Sample No.	**X_4**				**X_5**			
1	0.47	0.49	0.50	0.52	0.49	0.52	0.53	0.54
2	0.49	0.51	0.53	0.55	0.42	0.45	0.47	0.49
3	0.48	0.49	0.55	0.58	0.48	0.50	0.51	0.52
4	0.46	0.51	0.54	0.57	0.50	0.53	0.54	0.55
5	0.56	0.57	0.58	0.59	0.49	0.52	0.53	0.54
6	0.46	0.48	0.49	0.50	0.48	0.50	0.51	0.52
7	0.44	0.46	0.48	0.51	0.47	0.49	0.50	0.55
8	0.48	0.50	0.53	0.55	0.49	0.50	0.51	0.53
9	0.46	0.47	0.49	0.54	0.45	0.48	0.50	0.52
10	0.45	0.48	0.52	0.56	0.42	0.46	0.47	0.48
11	0.41	0.49	0.50	0.54	0.45	0.48	0.50	0.52
12	0.45	0.48	0.51	0.55	0.47	0.48	0.50	0.52
13	0.48	0.53	0.54	0.55	0.48	0.50	0.51	0.53
14	0.50	0.51	0.53	0.54	0.42	0.44	0.45	0.48
15	0.46	0.48	0.52	0.55	0.41	0.49	0.50	0.51

The average of range for the trapezoidal fuzzy numbers are represented as $\left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right)$.

$$R_{a1} = \left(X_{\max.aj} - X_{\min.dj}, X_{\max.bj} - X_{\min.cj}, X_{\max.cj} - X_{\min.bj}, X_{\max.dj} - X_{\min.aj}\right)$$
$$= \left(0.55 - 0.52, 0.56 - 0.50, 0.57 - 0.49, 0.58 - 0.45\right)$$
$$R_{a2} = \left(0.50 - 0.49, 0.52 - 0.47, 0.54 - 0.45, 0.56 - 0.42\right)$$

and so on....

$$\bar{R}_a = \frac{R_{a1} + R_{a1} + ... + R_{a15}}{15} = \frac{0.03 + 0.01 + ... + 0.02}{15} = 0.0173$$
$$\bar{R}_b = \frac{R_{b1} + R_{b1} + ... + R_{b15}}{15} = \frac{0.06 + 0.05 + ... + 0.04}{15} = 0.0520$$
$$\bar{R}_c = \frac{R_{c1} + R_{c1} + ... + R_{c15}}{15} = \frac{0.08 + 0.09 + ... + 0.07}{15} = 0.0887$$
$$\bar{R}_d = \frac{R_{d1} + R_{d1} + ... + R_{d15}}{15} = \frac{0.13 + 0.14 + ... + 0.15}{15} = 0.1427$$
$$\left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right) = \left(0.0173, 0.520, 0.0887, 0.1427\right)$$

The fuzzy control limits for range are as follows:

$$U\tilde{C}L_{\bar{R}} = \left(0.0173, 0.0520, 0.0887, 0.1427\right) + \left(\frac{3 \times 0.864}{2.326}\right)\left(0.0173, 0.0520, 0.0887, 0.1427\right)$$
$$= \begin{pmatrix} 0.0173 + \frac{3 \times 0.864}{2.326} 0.0173, 0.0520 + \frac{3 \times 0.864}{2.326} 0.0520, \\ 0.0887 + \frac{3 \times 0.864}{2.326} 0.0887, 0.1427 + \frac{3 \times 0.864}{2.326} 0.01427 \end{pmatrix}$$
$$= \left(0.0366, 0.1099, 0.1875, 0.3016\right)$$
$$C\tilde{L}_{\bar{R}} = \left(0.0173, 0.0520, 0.0887, 0.1427\right)$$
$$L\tilde{C}L_{\bar{R}} = \left(0.0173, 0.0520, 0.0887, 0.1427\right) - \left(\frac{3 \times 0.864}{2.326}\right)\left(0.0173, 0.0520, 0.0887, 0.1427\right)$$
$$= \begin{pmatrix} 0.0173 - \frac{3 \times 0.864}{2.326} 0.0173, 0.0520 - \frac{3 \times 0.864}{2.326} 0.0520, \\ 0.0887 - \frac{3 \times 0.864}{2.326} 0.0887, 0.1427 - \frac{3 \times 0.864}{2.326} 0.01427 \end{pmatrix}$$
$$= \left(-0.0020 \square 0, -0.0059 \square 0, -0.0101 \square 0, -0.0163 \square 0\right)$$

The proposed standard deviation

$$USL_{aR.FC_p} - LSL_{aR.FC_p} = 0.01486 - 0.00000 \Rightarrow \tilde{\sigma}_{aR.FC_p} = 0.00124$$
$$USL_{bR.FC_p} - LSL_{bR.FC_p} = 0.03343 - 0.01114 \Rightarrow \tilde{\sigma}_{bR.FC_p} = 0.00186$$
$$USL_{cR.FC_p} - LSL_{cR.FC_p} = 0.05200 - 0.02229 \Rightarrow \tilde{\sigma}_{cR.FC_p} = 0.00248$$
$$USL_{dR.FC_p} - LSL_{dR.FC_p} = 0.06315 - 0.04086 \Rightarrow \tilde{\sigma}_{dR.FC_p} = 0.00186$$

Therefore the resultant of proposed fuzzy control limits for range using process capability is given below:

$$\tilde{UCL}_{\bar{\bar{R}}:C_p} = \left(\bar{R}_a + A_{FMD:6\sigma}\tilde{\sigma}_{aR.FC_p}, \bar{R}_b + A_{FMD:6\sigma}\tilde{\sigma}_{bR.FC_p}, \bar{R}_c + A_{FMD:6\sigma}\tilde{\sigma}_{cR.FC_p}, \bar{R}_d + A_{FMD:6\sigma}\tilde{\sigma}_{dR.FC_p}\right)$$

$$= \begin{pmatrix} [0.0173 + 4.5\times 0.00124], [0.0520 + 4.5\times 0.00186], \\ [0.0887 + 4.5\times 0.00248], [0.1427 + 4.5\times 0.00186] \end{pmatrix}$$

$$= (0.0229, 0.0604, 0.0998, 0.1510)$$

$$\tilde{CL}_{\bar{\bar{R}}:C_p} = \left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right) = (0.0173, 0.0520, 0.0887, 0.1427)$$

$$\tilde{LCL}_{\bar{\bar{R}}:C_p} = \left(\bar{R}_a - A_{FMD:6\sigma}\tilde{\sigma}_{aR.FC_p}, \bar{R}_b - A_{FMD:6\sigma}\tilde{\sigma}_{bR.FC_p}, \bar{R}_c - A_{FMD:6\sigma}\tilde{\sigma}_{cR.FC_p}, \bar{R}_d - A_{FMD:6\sigma}\tilde{\sigma}_{dR.FC_p}\right)$$

$$= \begin{pmatrix} [0.0173 - 4.5\times 0.00124], [0.0520 - 4.5\times 0.00186], \\ [0.0887 - 4.5\times 0.00248], [0.1427 - 4.5\times 0.00186] \end{pmatrix}$$

$$= (0.0118, 0.0436, 0.0775, 0.1343)$$

In reveals that the expected number of samples needed to detect a shift of 'multiple of σ' under the six sigma based control chart for fuzzy range using moderate distribution is more agile than the existing fuzzy range control limits.

4. Conclusion

The constructed six sigma based fuzzy control chart for range under moderate distribution, procedures adopted and discussed in the research article by taking the process capability (C_p) as the base only. In this article offers the possibility of using fuzzy control chart under moderate distribution, which rules out the weaknesses compared to the existing control charts. Specifically, it presents one of the six sigma based fuzzy control chart and on the real-life data illustrates the simplicity of its usage in practice.

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