The Stress-intensity factors for four Griffith cracks further opened by body forces in stress-free orthotropic infinite strip

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Abstract- In this Paper, we study the problem of stress-intensity factors for four Griffith cracks further opened by body forces in stress-free orthotropic infinite strip. The permeability of the porous medium decreases exponentially with time about a constant mean. Using perturbation technique the expressions for the velocity distribution, the mean angular velocity of rotation of particles and skin friction are obtained. The effects of couple stresses and permeability parameter and other parameters entering into the problem on velocity distribution and skin friction are shown graphically and discussed numerically.

Keywords – Watermarking, Haar Wavelet, DWT, PSNR

I. Introduction

In chapter-v we discussed further opening of two Griffith-cracks by body forces whiles cracks were opened by an interior wedge. The present analysis is an obvious extension, of a wedge to two similar interior wedge an it’s effect upon physical properties like crack-opening and stress-intensity factors.

Another part of analysis is mathematical difficulty, if any, coming in the way. In chapter v, the physical problem was reduced to triple series relation. The present research endeovour will reduce the problem to quintuple series relations.

All the assumption of previous chapters are valid here also. The cracks of occupy the region $y=0, b_1<|x|<b_2, b_3<|x|<b_1$, The cracks are lengths (b_2-b_1) and (b_4-b_3).

The physical problem is translated to mathematical model through mixed-boundary conditions as given below :

$$\sigma_{xx}(\pm a, y) - \sigma_{xy}(\pm a, y) = 0 \quad 0 \le |y| < \infty, \qquad \ldots(1)$$

$$\sigma_{xy}(x, 0^2) = 0 \quad 0 \le |x| \le a, \qquad \ldots(2)$$

and mixed-boundary conditions, where symmetry of geometry has been used,

$$u,(x,0) = \begin{cases} 0, & x \in I_1 \\ u(x) & x \in I_3 \\ 0, & x \in I_5 \end{cases} \qquad \ldots(3)$$

$$\sigma_{xy}(x,0) = 0, \, x \in I_2 \cup I_4, \, b_3 < |x| < b_4, \qquad \ldots(4)$$

where

$$\left.\begin{aligned} & I_1 = [0, b_1], \; I_2 = (b_1, b_2), \, I_3 = [b_2, b_3], \\ & I_4 = (b_3, b_4), \, I_5 = [b_4, a], \end{aligned}\right\} \qquad \ldots(5)$$

We have checked throughout that,

$$u_y(x, 0*) > 0, \, b_1 < |x| < b_2, \, b_3 < |x| < b_4 \qquad \ldots(6)$$

Which means that the cracks are really opened out and the crack faces do not meet each other except at crack tips. The plan of the chapter is as follows : In section 1 we gave the introduction of the problem. Section 2 will reduce to and give solution quintuple series equation.

Solution of Fredholm integral equation will be given insection 3. Physical quantities will be given in section 4.

II. REDUCTION TO AND SOLUTION OF QUINTUPLE SERIES RELATIONS REDUCTION

The method of formulation of present problem is same as used in previous chapters. The displacement $u_y^{(e)}(x,0)$ and normal stress-component, $\sigma_{yy}^{(e)}(x,0)$, after using mixed-boundary conditions (7.1.4), will give the solution of the problem.

The physical problem is reduced to the solution of the following quintuple series relation.

$$\frac{\phi_0}{2}+\sum_{n=1}^{\infty} \phi_n \cos(\alpha_n x)=\begin{cases}0, & x\in I_1\cup I_5 \\ u_0(x), & x\in I_0\end{cases} \qquad \ldots(7)$$

$$\sum_{n=1}^{\infty} \alpha_n \phi_n \cos(\alpha_n x)=P(x),\ x\in I_2\cup I_4, \qquad \ldots(8)$$

with,

$$u_0(x)=\frac{u(x)}{d},\ \alpha_n B_n=\phi_n, \phi_0=\frac{A_0}{d}, \qquad \ldots(9)$$

$$P(x)=\sigma_{yy}^{(b)}(x,0)+\sum_{n=1}^{\infty} (-1)^n \alpha_n^2 N(\alpha_n,x)\phi_n \cos(\alpha_n,x) \qquad \ldots(10)$$

$$M(\alpha_n,x)-\int_0^{\infty} f_3(s\alpha_n,x)ds \qquad \ldots(11)$$

$$f_3(\alpha_n,x)=\frac{f_2(\alpha_n s)F_2(\alpha_n s,x)}{F_1(s\alpha)} \qquad \ldots(12)$$

Where F_1, F_2& f_2 are given by (3.3.19). (3.3.20)

III SOLUTION

The solution of above quintuple series relation will be obtained by the method of Kushwaha [129] or it can be derived by Kushwaha et.al. [138]. We take trial solution as

$$\alpha_n \phi_n = 2\left[\left\langle \int_{a_1}^{b_2} g(t) - \int_{b_2}^{b_3} \frac{u_0'(t)}{a} + \int_{b_3}^{b_4} h(t) \right\rangle \sin(\alpha_n t)\, dt \right] \qquad \ldots(13)$$

$$\phi_0 = 2\left[\int_{b_1}^{b_2} t g(t)\, dt - \int_{b_2}^{b_3} \frac{t u_0'(t)}{a}\, dt + \int_{b_3}^{b_4} t h(t)\, dt\right], \qquad \ldots(14)$$

Then substituting (7) . (8) in (1) this will be satisfied identically if

$$\int_{b_1}^{b_2} g(t)\, dt = \frac{1}{a} u_0(b_2), \qquad \ldots(15)$$

$$\int_{b_1}^{b_4} h(t)\, dt = \frac{1}{a} u_0(b_3), \qquad \ldots(16)$$

And then substituting 7) into (2) and inverting twice.

$$g(t) = \frac{1}{a^2 \delta(t)}\left[\Delta_0(t) + \left\langle \int_{b_1}^{b_2} g(\alpha) \int_{b_1}^{b_4} h(\alpha) \right\rangle K(\alpha, t)\, d\alpha\right],$$

$$b_1 \le t \le b_2, \qquad \ldots(17)$$

$$h(t) = \frac{1}{a^2 \delta(t)}\left[\Delta_0(t) + \left\langle \int_{b_1}^{b_2} g(\alpha) \int_{b_1}^{b_4} h(\alpha) \right\rangle K(\alpha, t)\, d\alpha\right],$$

$$b_3 \le t \le b_4, \qquad \ldots(18)$$

$$\left.\begin{aligned}&\delta(t)=\delta_1(t)\delta_2(t)\\&\delta_1(t)=\left[\left|\cos(qb_1)-\cos(qt)\right|\left|\cos(qt)-\cos(qb_2)\right|\right]^{1/2}\\&\delta_2(t)=\left[\left|\cos(qb_3)-\cos(qt)\right|\left|\cos(qt)-\cos(qb_4)\right|\right]^{1/2}\end{aligned}\right\}\quad ...(19)$$

$$\Delta_0(t)=\left(\int_{b_1}^{b_2}\ \int_{b_1}^{b_4}\right)\frac{\sin(qx)\delta(x)\sigma_{yy}^{(b)}(x,0)}{G(x,t)}\,dx$$

$$+\Delta_{01}(t)+D_1\cos(qt)+D_2,\quad \ldots(20)$$

$$\Delta_{01}(t)=\left(\int_{b_1}^{b_2}\ \int_{b_3}^{b_4}\right)\frac{\sin(qx)\delta(x)P_1(x)}{G(x,t)}\,dx\quad \ldots(21)$$

$$P_1(x)=\int_{b_2}^{b_3}\frac{u_0'(\alpha)\sin(q\alpha)}{G(\alpha,x)}\,d\alpha\quad \ldots(22)$$

D_1 and $D_2$are two arbitrary constants which will be determined by (11) -(12) and (9) – (10). The equations (11) - (12) are called coupled Fredholm integral equation of second type.

IV SOLUTION SOLUTION OF FREDHOL INTEGRAL EQUATION EVALUATING CONSTANTS

The solution of Coupled Fredholm integral equation is obtained by numerical methods. Before using the numerical method we must know the constant D_1 and D_2

For the evaluation D_1 and D_2 we assume,

$$\left.\begin{aligned} g(t) &= \frac{\Delta_0(t)}{a^2\delta(t)}, b_1 \le t \le b_2 \\ h(t) &= \frac{\Delta_0(t)}{a^2\delta(t)}, b_1 \le t \le b_4 \end{aligned}\right\} \quad \ldots(23)$$

Then we use the conditions (9) – (10) with (1) we get,

$$D_1 = \frac{c_6}{c_5}, D_2 \frac{c_7}{c_5} \tag{24}$$

$$\left.\begin{aligned} & c_7 = \beta_4 c_3, c_6 = \beta_4 c_4 - \beta_6 c_2 \\ & c_6 = c_1 c_4 - c_2 c_3 \\ & c_1 = \int_{b_2}^{b_1} \frac{\cos(qt)\,dt}{\delta(t)}, c_2 = \int_{b_1}^{b_2} \frac{dt}{\delta(t)} \\ & c_3 = \int_{b_4}^{b_4} \frac{\cos(qt)\,dt}{\delta(t)}, c_4 = \int_{b_3}^{b_4} \frac{dt}{\delta(t)} \end{aligned}\right\} \tag{25}$$

$$\left.\begin{aligned} \beta_4 &= au_0(b_2) + \beta_1 - \beta_0 \\ \beta_5 &= -au_0(b_3) + \beta_3 - \beta_3 \end{aligned}\right\} \tag{26}$$

$$\left.\begin{aligned} \beta_0 &= \int_{b_3}^{b_2} \frac{Q_0^+(t)\,dt}{\delta(t)}, \beta_1 = \int_{b_1}^{b_2} \frac{Q_1^+(t)\,dt}{\delta(t)} \\ \beta_2 &= \int_{b_3}^{b_4} \frac{Q_3^+(t)\,dt}{\delta(t)}, \beta_3 = \int_{b_1}^{b_4} \frac{Q_1^+(t)\,dt}{\delta(t)} \end{aligned}\right\} \tag{27}$$

$$Q_0^{\pm} = \int_{b_1}^{b_2} \frac{P_3(x)\,dx}{G(x,t)}, \; Q_1^{\pm}(t) = \int_{b_3}^{b_4} \frac{P_3(x)\,dx}{G(x,t)}, \tag{28}$$

(+) and (-) over Q_1 (t) means $b_1 \le t \le b_2$ for (+) and $b_3 \le t < b_4$ for (-) with I = 0,1

$$\left.\begin{aligned} P_3(x) &= \sin(qx)\left\{\sigma_{yy}^{(b)}(x,0) + P_1(x)\right\} \\ P_1(x) &= \int_{b_2}^{b_3} \frac{u_0'(\alpha)\sin(q\alpha)}{G(x,\alpha)}\,d\alpha \end{aligned}\right\} \tag{29}$$

V SOLUTION

Now we solve coupled Fredholm integral equations (11).- (12). We divide the intervals (b_1,b_2) and (b_3,b_4) into (m+1) equal part as.

$$\left.\begin{aligned} t_i &= b_1 + \frac{b_2 - b_1}{m+1} i, \; i = 1, 2 \ldots m \\ t_j &= b_3 + \frac{b_4 - b_3}{m+1} j, \; j = 1, 2 \ldots m \end{aligned}\right\} \tag{30}$$

Then substituting t1 and t3 in (11) – (10), with the notation,

$$g(t) = g(t_i), h_j(t) = h(t_j); t, j = 1, 2, \ldots m \tag{31}$$

We get,

$$\begin{bmatrix} A_1 : B_1 \\ A_2 : B_2 \end{bmatrix} \begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} D_3 \\ D_4 \end{bmatrix} \tag{32}$$

Where A_1, A_2; B_1, B_2,;G,H; D_3,D_4 are matrices defined below

$$\left.\begin{aligned}&A_1=\left(a_{ij}\right),\ i,\ j=1,2,...m\\&a_{ij}=1-\frac{H_1}{2m}K_1\left(\alpha_j,t_i\right),t=1,...m\\&a_{ij}=-\frac{H_1}{2m}K_1\left(\alpha_i,t_j\right),i\neq j=1,2,...m\\&with\ b_1<\alpha_i,t_j<b_2\end{aligned}\right\}\qquad(33)$$

$$\left.\begin{aligned}&A_2=\left(a'_{ij}\right),a'_{ij}=-\frac{H_2}{2m}K_1\left(\alpha_i,t_i\right)\\&with\ b_1<\alpha_i<b_2,b_3<t_j<b_4\end{aligned}\right\}\qquad(34)$$

$$\left.\begin{aligned}&B_1=\left(b_{ij}\right),\ i,\ j=1,...m,\\&b_{ij}=-\frac{H_1}{2m}K_j\left(\alpha_i,t_n\right)\\&b_3<\alpha_i<b_4,b<t_n<b_1\end{aligned}\right\}\qquad(35)$$

$$\left.\begin{aligned}&B_1=\left(b'_{ij}\right),\\&b'_{ij}=-\frac{H_2}{m}K_1\left(\alpha_i,t_n\right),\ i\neq j=1,...m\\&b'_{ij}=1-\frac{H_2}{2m}K_1\left(\alpha_i,t_j\right)\\&with\\&b_3<\alpha_i,t_j<b_4,\ i,\ j=1,2,...m\end{aligned}\right\}\qquad(36)$$

$$\left.\begin{array}{l} D_3 = \left(d_{ij}\right), i = 1, i = 1, 2 \ldots m \\ \\ d_{ji} = \Delta_3\left(t_i\right) = a^2 \dfrac{\Delta_0\left(t_i\right)}{\delta\left(t_j\right)}, b_1 < t_i < b_2 \end{array}\right\} \tag{37}$$

$$\left.\begin{array}{l} D_4 = \left(d'_{ij}\right), i = 1, i = \ldots m \\ \\ d_{ji} = \Delta_3\left(t_i\right) = a^2 \dfrac{\Delta_0\left(t_i\right)}{a^2\delta\left(t_i\right)}, b_2 < t_i < b_4 \end{array}\right\} \tag{38}$$

$$H_1 = b_2 - b_1,\ H_2 = b_4 - b_3 \tag{39}$$

Thus we can solve (10) which is matrix equation with 2m variables.

VI SOLUTION

The crack opening displacement, $u_y^{(e)}\left(x,0\right),$ is given as,

$$u_y^{(e)}\left(x,0\right) = \frac{2d}{a}\begin{cases} \int_x^{b_2} g\left(t\right)dt \\ \\ \int_x^{b_1} h\left(t\right)dt \end{cases} \tag{40}$$

Where $g(t)$ and $h(t)$ are to be obtained from (11) – (12).

NORMAL STRESS

$$\sigma_{yy}^{(e)}(x,0)=\frac{1}{\pi r_1}\begin{cases}\dfrac{\Delta_1(x)}{\delta(x)}+F(x),\ 0\le x<b_1\\ \dfrac{\Delta_1(x)}{\delta(x)}+F(x),\ b_2<x<b_2\\ -\dfrac{\Delta_1(x)}{\delta(x)}+F(x),\ b_4<x\le a\end{cases} \tag{41}$$

$$F(x)=\left(\int_{b_1}^{b_2} g(\alpha)-\int_{b_1}^{b_4} h(\alpha)\right)F_4(\alpha,x)\,d\alpha \tag{42}$$

$$F_4(\alpha,x)=\sum_{n=1}^{\infty}(-1)^n\alpha_n\sin(\alpha_n\alpha)\int_0^{\infty} f_3(s\alpha_n,x)\,ds \tag{43}$$

$$\Delta_t(x)\left[\Delta_0(x)+\left\langle\int_{b_1}^{b_2} g(\alpha)-\int_{b_3}^{b_4} h(\alpha)\right\rangle K(\alpha,j)\,d\alpha\right] \tag{44}$$

STRESS-INTENSITY FACTORS

This stress-intensity factors at crack tips (b1,00, i=1,2,3,4 are defined as,

$$\left.\begin{aligned}K_{b_1}&=\lim_{x\to b_1}\sqrt{b_1-x}\,\sigma_{yy}^{(e)}(x,0)\\ K_{b_2}&=\lim_{x\to b_2}\sqrt{x-b_2}\,\sigma_{yy}^{(e)}(x,0)\\ K_{b_3}&=\lim_{x\to b_3}\sqrt{b_3-x}\,\sigma_{yy}^{(e)}(x,0)\\ K_{b_4}&=\lim_{x\to b_4}\sqrt{x-b_4}\,\sigma_{yy}^{(e)}(x,0)\end{aligned}\right\} \tag{45}$$

Now using (2) into definitions (7) and evaluating limits.

$$K_{b_1} = \frac{\Delta_1(b_1)}{\delta_3(b_1)} \tag{46}$$

$$K_{b_2} = \frac{\Delta_1(b_1)}{\delta_3(b_1)} \tag{47}$$

$$K_{b_3} = \frac{\Delta_1(b_3)}{\delta_4(b_3)} \tag{48}$$

$$K_{b_4} = \frac{\Delta_1(b_4)}{\delta_4(b_4)} \tag{49}$$

$$\delta_3(x) = \left[q \sin(qx) G(b_1, b_2)\right]^{1/2} \delta_2(x), \; x = b_1, b_2 \tag{50}$$

$$\delta_4(x) = \left[q \sin(qx) G(b_3, b_4)\right]^{1/2} \delta_1(x), \; x = b_3, b_4 \tag{51}$$

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