A Comparative Analysis for the Solution of Transportation Model by Various Methods

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Abstract- In Operations Research, an important application that has a special case of linear programming is known as the transport problem. Transportation problems regarded as a significant characteristic that has been investigated in a variety of operations as well as research provinces. In this study, various newly alternate methods such as; mean proposed method (AMM, GMM, HMM, QMM), ATM method, and second minimum value method in recent years are applied to solve real-world problems of the transportation problems. In the present study, an attempt has made to comparative investigation and graphical representation of IBF solution obtained by various methods.

Keywords –Transportation problem, Mean proposed method, Allocation Table method ATM, Initial Basic Feasible solution, IBFS

I. Introduction

The transportation problem is a calculated issue for associations particularly for assembling and transport businesses which is a valuable tool in the dynamic and procedure of allotting issues in these associations. The transportation problem is a particular sort of linear programming, which is related to our everyday, exercises, in actuality, and for the most part, manages coordination. The purpose of a transport problem is to meet destination requirements within the operating production capacity at the lowest possible cost. The transportation problem has an application in industry, communication networks, planning, scheduling transport, and allocation, etc.
Gaspard Monge, a French mathematician in 1781 developed the transportation problem which has intriguing highlights with regards to genuine issues. Frank Lauren Hitchcock 1941 was the foremost who researched Transportation problems which assumes a significant work in the field of businesses for reducing the transportation cost when the sources and goals are given and requirements and supplies are fulfilled. Charnes, and Cooper (1954), have investigated a new method known as the stepping stone method of elucidating LPP in transportation problems. Hammer (1969), Szwarc (1971), Garinkel and Rao(1971), Bhatia, KantiSwaroop and Puri (1976) worked on reducing transportation problems. Furthermore, numerous researchers have been worked on the Vogel's approximation method (Juman and Hoque, 2013; Juman and Hoque, 2014).
To check the optimality of the initial basic solution, stepping stones, and modified distribution (MODI) methods are usually given preference. Various methods have been proposed in the literature to find an initial solution to the transport problem. Since, an initial basic possible solution to the transport problem can be calculated by applying the Northwest corner rule, Least-Cost, or Vogel's Approximation method.Adlakha and Kowalski (2003) have been developed a new method dependent on small fixed charges of transport goods which not associated with the amount transported. The author also suggested heuristic algorithms for the solution. Kulkarni and Datar (2010)formulated a heuristic method to test optimality of modified unbalanced transportation problem which was also useful to reduce

number of iteration. Korukoglu and Balli (2011) have developed modifications in VAM to find the optimal solution for transportation problems whereas Ramadan and Ramadan 2012) have been investigated a hybrid two-stage algorithm for the optimal solution of the problem. Juman, et al. (2013) developed a sensitivity investigation for the solution of an unbalanced transportation problem by VAM method. Das et al. (2014)have developed advanced VAM method which was the extension of the VAM method to remove the complexity occurs when the magnitude of least cost and the next least cost was the identical by Ahmed et al. (2016) have been employed the allocation table method, ATM to find an initial basic feasible solution of transportation problems and gave compare results from existing method. Mishra (2017) has investigated the problem of Millennium Herbal Company ship by North-West corner rule, Minimum Cost Method, and Vogel's Approximation Method. The author also compares his results from the North-West corner rule and VAM.

Davda and Patel (2019) have investigated the very effective the second minimum value method which required less iteration for optimality test.Aaliya et al. (2019) have been given the comparative study of results of initial basic solution and the optimal solution obtained by NWCR, LCM, and VAM method by taking the real-life example of OBU cement factory. Sathyavathy and Shalini (2019) has been developed a novel proposed mean method which to find the most favorable solution of transportation problem which dependent on easy statistical computation. The authors also gave numerical illustrations to check the validity of proposed mean methods.

On the basis of critical review, the present study finds a sufficient research gap to investigate the comparative study for the solution of transportation problems by the existing method and newly some statistical methods for solution. This study has been taken a real-life of BUA cement factories to minimize the cost of shipping cement from BUA cement factories to the various warehouses and applied various methods to find the initial basic feasible solutions as well as the optimal solutions. Some new methods are applied to find the solution of the transportation model which plays a vital role in explaining the nature of the distribution in the transportation model.

Many researchers have investigated alternative methods to modify the initial basic possible solutions in previous years. The transportation problem can be classified into two parts: Balanced transportation problem and unbalanced transportation problem, if the total number of supply is same as the total number of demand then the problem is called as balanced transportation problem i.e.$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$, Otherwise it is called unbalanced transportation problem i.e.$\sum_{i=1}^{m} S_i \neq \sum_{j=1}^{n} D_j$.

II.METHODOLOGY

As we know that the Transportation model deals with the minimum cost plan to transport a product from (m) number of sources to (n) number of destinations.

Let the number of supply units required/needed at source i isS_i, $(i = 1, 2, 3 \ldots\ldots m)$, the number of demand units required/needed at destination j is D_i, $(i = 1, 2, 3 \ldots\ldots m)$, and C_{ij} represent the unit transportation cost for transporting the units from sources i to destination j. We need to determine the value of objective function which minimize the transportation cost and determine how many numbers of the unit can be transported from source i to destination j.

The objective function, Minimize $Z = \sum_{i=1}^{m} c_{ij} \sum_{j=1}^{n} x_{ij}$,

Subject to constraints

$\sum_{j=1}^{n} x_{ij} = S_i$for $i = 1, 2 \ldots m$.

$\sum_{i=1}^{m} C_{ij} = D_i$for $j = 1, 2 \ldots n$.

Where, x_{ij}number of units shipped from source i to destination$j$,$x_{ij} \geq 0$ for all i to$j$.

2.1 Transportation problem by Existing methods –

The optimal cost has desirable in the movement of raw materials and goods from the sources to destinations. The mathematical model known as a transportation problem tries to provide optimal costs in the transportation system. Some well-known and long use algorithms to solve transportation problems are Vogel's Approximation Method (VAM), North West Corner (NWC) method, and Matrix Minima method (MMM), row minima method (RMM) and Colum Minima method (CMM) always provide IBFS of a transportation problem. Afterward, many researchers provide many methods and algorithms to solve transportation problems.

2.2. Transportation problem by newly proposed methods –

Arithmetic Mean Method:

Arithmetic Mean is defined as equal to the sum of numerical values of each & every observation divided by the total number of observations. It can be calculated by the following formula:

Arithmetic Mean (AM) $= \frac{\sum_{i=1}^{n} X_n}{n}$

Harmonic Mean Method:

The Harmonic mean is defined as the total number of observations divided by the sum of the reciprocal of the numbers.

Harmonic mean (HM) $= \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots\ldots\ldots+\frac{1}{x_n}}$

Geometric Mean Method:

The geometric mean is the nth root when you multiply n numbers together. It is not the same as the arithmetic mean, or average, that we know. The geometric mean uses multiplication and roots. For example, for the product of two numbers, we would take the square root. For the product of three numbers, we take the third root.

Geometric Mean (GM) $= \sqrt[n]{(X_1.X_2.X_3 \ldots\ldots..X_n)} = \left(\prod_{i=1}^{n} X_i\right)^{1/n}$

Quadratic Mean Method:

The Quadratic mean is also known by the root mean square because it is the square root of the mean of the squares of the numbers. The quadratic mean is equal to the square root of the mean of the squared values. The formula is:

Quadratic mean (QM)= $\sqrt{\frac{1}{n}(x_1^2 + x_2^2 + \cdots + x_n^2)}$

We follow the solution process of transportation problem by Arithmetic Mean, Geometric Mean, Harmonic Mean, and Quadratic mean method given by Sathyavathyand Shalini (2019).

Second Minimum Value Method (SMVM):

We follow the solution process of transportation problems as given by Davda and Patel (2019). In this method, the Author uses the Second minimum Value cell which has a minimum cost coefficient.

Allocation Table Method (ATM):

In this method, for the transportation problem, an allocation table is formed to find the solution. That’s why this method is named as the Allocation Table Method (ATM). We follow the solution process of transportation problems as given by Aliyu et al. (2019).

III. Application: Numerical Example of Real-Life Problem

Find the minimum cost of transporting manufactured goods from factories to warehouses to (distributors). CCNN transportation data and OBU cement transportation data collected from the BUA group of Companies were utilized by Aliya et al.(2019). The data were modelled as a Linear Programming model of transportation type and represented as transportation tableau i.e. (Table 1).Find its initial basic feasible solution and optimal solution

Table -1 Transportation Tableof the Secondary Data Collected from BUA Cement Company

	Sokoto	**Kebbi**	**Kebbi**	**Kano**	**Kaduna**	**Katsina**	**Niger republic**	**Supply**
CCNN	**5**	**19**	**12**	**70**	**66**	**74**	**283**	**40000**
OBU	**103**	**89**	**81**	**26**	**23**	**62**	**97**	**47700**
Demand	**21600**	**15600**	**15600**	**19500**	**16800**	**10500**	**8100**	

Table 1, shows an individually associated cost of transporting a piece of the bag from the individual supply center or plant to the various demand destinations. The table also shows the demands from various destinations and the supply capacity of the plants.

Now solving the transportation problem by the above methods. So, when we are dealing with the transportation problem our first call is to determine whether the given problem is a balanced transportation problem or an unbalanced transportation problem. If

1. Total demand = Total supply, thus the problem is balanced
2. Total demand $\neq$ Total supply, thus the problem is unbalanced

From the Table 1, the Total Demand=107,700 units and Total Supply= 87,700 units.So, the given transportation problem is unbalanced. Therefore, we need to balance the problem by adding a dummy row to the transportation problem.

The supply from the dummy plant is calculated by the difference between the Total demand and Total Supply i.e. 107700 – 87700=20000, thus 20000 units will be supplied by the Dummy Plant.

Table 2 shows the balanced transportation problem where the Dummy Plant takes the 20000 units for supply, with associated cost zero (0), and after doing that Total Demand is equal to the Total supply, hence the problem is balanced, therefore we can start our procedure of finding the IBFS (Initial Basic Feasible Solution) and optimal solution.

Table -2The Balanced Transportation Table

	Sokoto	Kebbi	Kebbi	Kano	Kaduna	Katsina	Niger republic	Supply
CCNN	**5**	**19**	**12**	**70**	**66**	**74**	**283**	**40000**
OBU	**103**	**89**	**81**	**26**	**23**	**62**	**97**	**47,700**
Dummy Plant	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**20,000**
Demand	**21600**	**15600**	**15600**	**19500**	**16800**	**10500**	**8100**	**107,700**

IV.Results and Discussion

Aliya et al. (2019) have been given the solution of transportation problems solved by existing method NCWR, LCEM, and VAM method. The transportation costs are 2,336,000 solved by NWCR, 4,160,900 by LCEM method, and 2,331,800 by VAM method, 1,972,900 by Row Minima Method, and 4,160,900 Column Minima Method, optimal solution from VAM, 1,905,300, optimal solution from LCEM, 1,972,000.

Table 3 shows the result obtained as the initial basic feasible solution (IBFS) by applying the arithmetic mean method.

Table -3Thearithmetic mean method

	Sokoto	Kebbi	Kebbi	Kano	Kaduna	Katsina	Niger republic	Supply
CCNN	**5** **21600**	**19** **2800**	**12** **15600**	**70**	**66**	**74**	**283**	**40000**
OBU	**103**	**89** **900**	**81**	**26** **19500**	**23** **16800**	**62** **10500**	**97**	**47,700**
Dummy Plant	**0**	**0** **11900**	**0**	**0**	**0**	**0**	**0** **8100**	**20,000**
Demand	**21600**	**15600**	**15600**	**19500**	**16800**	**10500**	**8100**	**107,700**

The transportation cost is
5 * 21600 + 19 * 2800 + 12 * 15600 + 89 * 900 + 26 * 19500 + 23 * 16800 + 62* 10500 + 0 * 11900 + 0 * 8100 = 1,972,900

Explanation:

Here we will find the initial basic feasible solution for the given data or we say given transportation problem from table 1 using statistical tools. By calculating Arithmetic Mean of each row and column, the values are

$$R_1 = 75.57\ ;\ R_2 = 68.71\ ;\ R_3 = 0\ ;\ C_1 = 36\ ;\ C_2 = 36\ ;\ C_3 = 31\ ;\ C_4 = 32\ ;$$
$$C_5 = 29.66\ ;\ C_6 = 19.42\ ;\ C_7 = 126.66$$

From the above calculated value the maximum Arithmetic Mean is$R_1 = 75.57$, so we choose R_1. In row R_1 the cell with minimum cost value is (1,1). The cell is allocated with maximum supply and the remaining cell in that row is deleted after all the supply is fulfilled. The steps are repeated until the demand and supply are satisfied.

Table 4 shows the result obtained as the initial basic feasible solution (IBFS) by applying the Harmonic Mean Method.

Table -4TheHarmonicMean Method

	Sokoto	Kebbi	Kebbi	Kano	Kaduna	Katsina	Niger republic	Supply
CCNN	**5** **21600**	**19** **3700**	**12** **14700**	**70**	**66**	**74**	**283**	**40000**
OBU	**103**	**89**	**81** **900**	**26** **19500**	**23** **16800**	**62** **10500**	**97**	**47,700**
Dummy Plant	**0**	**0** **11900**	**0**	**0**	**0**	**0**	**0** **8100**	**20,000**
Demand	**21600**	**15600**	**15600**	**19500**	**16800**	**10500**	**8100**	**107,700**

The transportation cost is
5 * 21600 + 19 * 3700 + 12 * 14700 + 81 * 900 + 26 * 19500 + 23 * 16800 + 62* 10500 + 0 * 11900 + 0 * 8100 = 1,972,000

Explanation:

In this explanation, we find the initial basic feasible solution of for the given data, or we can say given transportation problems from table 1 using statistical tools. By calculating Harmonic Mean of each row and column, the values are

$$R_1 = 18.30\ ;\ R_2 = 49.41\ ;\ R_3 = 0\ ;\ C_1 = 14.30\ ;\ C_2 = 46.97;\ C_3 = 31.35\ ;\ C_4 = 56.87\ ;$$
$$C_5 = 51.16\ ;\ C_6 = 101.20\ ;\ C_7 = 216.71$$

From the above calculated value, the maximum Harmonic Mean is $R_2 = 49.41$, so we choose R_2. In row R_2 the cell with minimum cost value is (2,5). The cell is allocated with maximum supply and the remaining cell in that row is deleted after all the supply is fulfilled. The steps are repeated until the demand and supply are satisfied.

Table 5shows the result obtained as initial basic feasible solution (IBFS) by applying the Quadratic Mean Method.

Table -5 TheQuadratic Mean Method

	Sokoto	Kebbi	Kebbi	Kano	Kaduna	Katsina	Niger republic	Supply
CCNN	**5** **21600**	**19** **2800**	**12** **15600**	**70**	**66**	**74**	**283**	**40000**
OBU	**103**	**89** **900**	**81**	**26** **19500**	**23** **16800**	**62** **10500**	**97**	**47,700**
Dummy Plant	**0**	**0** **11900**	**0**	**0**	**0**	**0**	**0** **8100**	**20,000**
Demand	**21600**	**15600**	**15600**	**19500**	**16800**	**10500**	**8100**	**107,700**

The transportation cost is
5 * 21600 + 19 * 2800 + 12 * 15600 + 89 * 900 + 26 * 19500 + 23 * 16800 + 62* 10500 + 0 * 11900 + 0 * 8100 = 1,972,900

Explanation:

Here we will find the initial basic feasible solution of for the given data or we say given transportation problem from table 1 using statistical tools. By calculating Quadratic Mean of each row and column, the values are

$$R_1 = 75.57\ ;\ R_2 = 68.71\ ;\ R_3 = 0\ ;\ C_1 = 36\ ;\ C_2 = 36;\ C_3 = 31\ ;\ C_4 = 32\ ;$$
$$C_5 = 29.66\ ;\ C_6 = 45.33\ ;\ C_7 = 126.66$$

From the above-calculated value the maximumQuadraticMean is $R_1 = 75.57$, so we choose R_1. In row R_1 the cell with minimum cost value is (1,1). The cell is allocated with maximum supply and the remaining cell in that row is deleted after all the supply is fulfilled. The steps are repeated until the demand and supply are satisfied.

Table 6 shows the result obtained as initial basic feasible solution (IBFS) by applying the Geometric MeanMethod.
The transportation cost is
5 * 21600 + 19 * 3700 + 12 * 14700 + 81 * 900 + 26 * 19500 + 23 * 16800 + 62* 10500 + 0 * 11900 + 0 * 8100 = 1,972,000

Explanation:

Here we will find the initial basic feasible solution for the given data or we say given transportation problem from table 1 using statistical tools. By calculating Geometric Mean of each row and column, the values are

$$R_1 = 37.80;\ R_2 = 59.59\ ;\ R_3 = 0\ ;\ C_1 = 0\ ;\ C_2 = 0;\ C_3 = 0\ ;\ C_4 = 0\ ;$$

$$C_5 = 0\ ;\ C_6 = 0\ ;\ C_7 = 0$$

Table -6 TheGeometric Mean Method

	Sokoto	Kebbi	Kebbi	Kano	Kaduna	Katsina	Niger republic	Supply
CCNN	5 21600	19 3700	12 14700	70	66	74	283	40000
OBU	103	89	81 900	26 19500	23 16800	62 10500	97	47,700
Dummy Plant	0	0 11900	0	0	0	0	0 8100	20,000
Demand	21600	15600	15600	19500	16800	10500	8100	107,700

From the above calculated value the maximumGeometric Mean is$R_2 = 59.59$, so we choose R_2. In row R_2 the cell with minimum cost value is (2,5). The cell is allocated with maximum supply and the remaining cell in that row is deleted after all the supply is fulfilled. The steps are repeated until the demand and supply are satisfied.

Table 7 shows the result obtained as initial basic feasible solution (IBFS) by applying the Second minimum value method

Table -7 Second minimum value method

	Sokoto	Kebbi	Kebbi	Kano	Kaduna	Katsina	Niger republic	Supply
CCNN	5 8800	19 15600	12 15600	70	66	74	283	40000
OBU	103	89	81	26 19500	23 9600	62 10500	97 8100	47,700
Dummy Plant	0 12800	0	0	0	0 7200	0	0	20,000
Demand	21600	15600	15600	19500	16800	10500	8100	107,700

The transportation cost is

5 * 8800 + 19 * 15600 + 12 * 15600 + 26 * 19500 + 23 * 9600 + 62* 10500 + 0 * 12800 + 97* 8100 + 0 * 7200 = 2,692,100

Explanation:

Here, in the first step, we have maximum value of 'second minimum value' is 97, which appears in the second row and seventh column. So we will give allocation to cell (2, 7) of 8100 and eliminate column 7.In 2^{nd}step, we have a maximum value of 'second minimum value' is 62, which appears in the second row and sixth column. So we will give allocation to cell (2, 6) of 10500 and eliminate column 6.In 3^{rd}step, we have a maximum value of 'second minimum value' is 26, which appears in the second row and fourth column. So we will give allocation to cell (2, 4) of 19500 and eliminate column 4.In 4^{th}step, we have a maximum value of 'second minimum value' is 23, which appears in the second row and fifth column. So we will give allocation to cell (2, 5) of 9600 and eliminate row 2.In 5^{th}step, we have a maximum value of 'second minimum value' is 19, which appears in the first row and second column. So we will give allocation to cell (1, 2) of 15600 and eliminate column 2.In 6^{th}step, we have maximum value of 'second minimum value' is 12, which appears in the first row and third column. So we will give allocation to cell (1, 3) of 15600 and eliminate column 3.In final step, for 2*2 matrix, we will take difference of row and column , we get maximum value in column 5^{th}, so we will give allocation to cell (3, 5) of 7200 and by fulfilling supply and demand constraints we will allocate cell (1, 1) of 8800 and cell (3, 1) of 12800.

Table 8 shows the result obtained as the initial basic feasible solution (IBFS) by applying the Allocation Table method (ATM)

The transportation cost is

5 * 21600 + 19 * 15600 + 81 * 900 + 26 * 19500 + 23 * 16800 + 62* 10500 + 283* 2800 + 0* 14700 + 0 * 5300 = 2104200

Explanation:

As per step-2 minimum, the odd cost is 5 in the cell (1, 1) among all the cells of the transportation table. According to a step-3 minimum, the odd cost in the cell (1, 1) remains the same, but this odd cost is subtracted from all other odd valued cost cells of the transportation table. Here minimum supply & demand is 21600 that is allocated in the cell (1, 1). After allocating this value the demand is satisfied. For which the C_1 column is exhausted.

Table -8Allocation Table method

	Sokoto	Kebbi	Kebbi	Kano	Kaduna	Katsina	Niger republic	Supply
CCNN	**5** **21600**	**19** **15600**	**12**	**70**	**66**	**74**	**283** **2800**	**40000**
OBU	**103**	**89**	**81** **900**	**26** **19500**	**23** **16800**	**62** **10500**	**97**	**47,700**
Dummy Plant	**0**	**0**	**0** **14700**	**0**	**0**	**0**	**0** **5300**	**20,000**
Demand	**21600**	**15600**	**15600**	**19500**	**16800**	**10500**	**8100**	**107,700**

Table 9 shows the basic feasible solution for the given transportation problem of table 1 by the various methods

Table 9: Solution of the numerical results by various methods

Serial Number	Name of Methods	Transportation Costs	Serial Number	Name of Methods	Transportation Costs
1	**NWCM**	**2336000**	**7.**	**HMM**	**1972000**
2	**LCM**	**4160900**	**8.**	**QMM**	**1972900**
3	**VAM**	**2331800**	**9.**	**GMM**	**1972000**
4	**RMM**	**1972900**	**10.**	**SMVM**	**2,692,100**
5	**CMM**	**4160900**	**11.**	**ATM**	**2104200**
6	**AMM**	**1972900**			

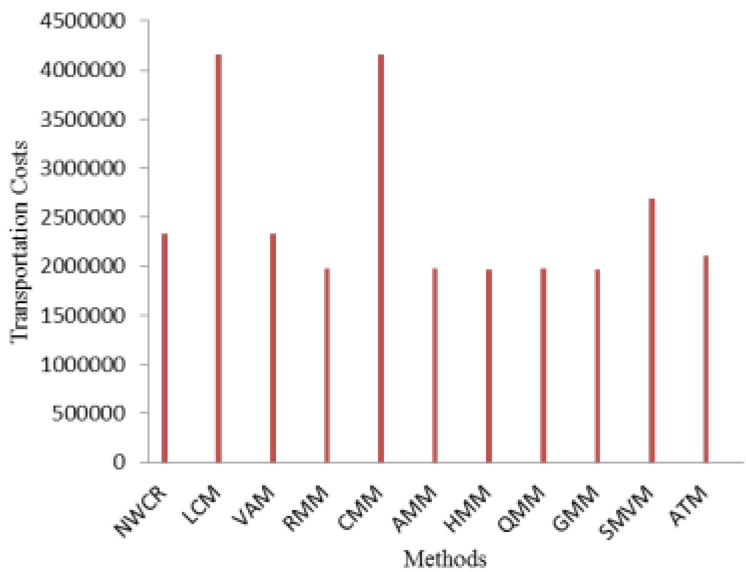

Figure 1. Graphical representation of solution of the Transportation problemby various methods

We concluded from this comparative study that obtained Initial basic feasible solution of BUA cement company by HMM method and GMM method are 1,972,000 which less than all other methods. Obtained results of these two methods are nearly equal to the optimal solution of the BUA group of cement Company.

V.CONCLUSION

Most organizations wish to give their commodities to consumers in a cost-effective way, with the intention that the marketplace turns into a highly bloodthirsty marketplace. To remove these types of difficulty, the transportation model gives a dominant framework to settle on the most admirable ways to transport commodities to the consumers. All previous researchers have not given any comparative analysis of the real-life problems by various methods including statistical methods. Therefore, this study has been identified the research gap for the comparative study of real-life transportation model by the existing method and newly some other methods which play a vital role in explaining the nature of the distribution in the transportation model.We concluded that obtained Initial basic feasible solutions of BUA cement company by HMM and GMM methods are 1,972,000 which less than all other methods. The present study has been taken a real-life example of BUA Cement Company to minimize the cost of shipping cement to the various warehouses and applied various methods to find the initial basic feasible solution as well as the optimal solution. This study has been used the graphical representation of the solution for a clear crystal picture of their comparison.

REFERENCES

[1] A. Charnes and W.W. Cooper "The stepping-stone method for explaining linear programming: Calculation in transportation problems", *Management Science*, Vol. 1, No. 1, pp. 49-69, (1954).

[2] F.L. Hitchcock "The Distribution of a Product from Several Sources to Numerous Localities" *Journal of Mathematics and Physics*, Vol. 20, pp. 224-230, (1941).

[3] G. Monge "Memoire sur la theorie des deblais et des remblais",*in: Histoire de l'Academie Royale des sciencas, Annee M. DCCLXXXI avec les Memoires de Mathematique et de Physique, pour la meme Annee, Tires des Registers de cette Academie, Paris*, pp. 666–704, (1781).

[4] H. L. Bhatia, K. Swaroop and M. C. Puri "A procedure for time minimization transportation problem", *Indian Journal of Pure and Applied Mathematics*, Vol. 8, No. 8, pp. 920-929, (1974).

[5] M. M. Ahmed, A.R. Khan, M. S. Uddin andF. Ahmed,"A new approach to solve transportation problems",*Open Journal of Optimization*, Vol. 5,pp. 22-30, (2016).

[6] M.L. Aliyu, U. Usman, Z. Babayaro and M.K. Aminu"A minimization of the cost of transportation", American Journal of Operational Research,Vol. 9, No. 1, pp. 1-7, (2019).

[7] M. Sathyavathy and M. Shalini "Solving transportation problem with four different proposed mean method and comparison with existing methods for optimum solution, *Journal of Physics: Conference Series*,1362:012088, (2019).

[8] M. Palanivel, and M. Suganya "A new method to solve transportation problem- Harmonic Mean Approach", *Engineering TechnologyOpen Access Journal*, Vol. 2, No. 3, pp. 1-3, (2018).

[9] P. Davda and J. Patel "Developed method for optimal solution of transportation problem", *International Research Journal of Engineering and Technology*, Vol. 06, Issue 3, pp. 1880-1882,(2019).

[10] P. L. Hammer "The time minimizing transportation problem", Naval Research Logistics Quartely, Vol. 18, pp. 487-490, (1969).

[11] R.S. Garfinkel and M.R. Rao " The bollleneck transportation problem", *Naval Research Logistics Quartely*, Vol. 18, pp 465-472, (1971).

[12] S.S. Kulkarni and H.G. Datar "On solution to modiied unbalanced transportation problem",*Bulletin of the Marathwada Mathematical Society*,Vol. 11, No. 2, pp. 20-26,(2010).

[13] S. Korukoglu and S. Balli "A improved Vogel's approximation method for the transportation problem",*Mathematical and Computational Applications*, Vol. 16, No. 2, pp. 370-381, (2011).

[14] S. Mishra "Solving transportation problem by various methods and their comparison", *International Journal of Mathematics Trends and Technology*,Vol. 44, No. 4, pp.270-275, (2017).

[15] S.Z. Ramadan and I.Z. Ramadan"Hybrid two- stage algorithm for solving transportation problem",*International Journal of Physics and Mathematical Sciences*,Vol.6, No. 4, pp. 12-22, (2012).

[16] T. Imam,G. Elsharawy, M. Gomah and I. Samy "Solving transportation problem using object-oriented model",*IJCSNS International Journal of Computer Science and Network Security*, Vol. 9, No. 2, pp. 353-361 (2009).

[17] U. K. Das, M. A. Babu, A. R. Khan and M. S. Uddin "Advanced Vogel's Approximation Method (AVAM): A new approach to determine penalty cost for better feasible solution of transportation problem",*International Journal of Engineering Research & Technology*, Vol. 3. pp.182-187, (2014).

[18] V. Adlakha and K. Kowalski"A simple heuristicfor solving small fixed-charge transportation problem",*Omega*,Vol.31, No. 3, pp. 205-211,(2003).

[19] W. Szwarc "Some remarks on the transportation problem", *Naval Research Logistics Quarterly*, Vol. 18, pp. 473 –485, (1971).

[20] Z.A.M.S Juman, M.A. Hoque andM.I.A. Buhari, "Sensitivity analysis and an implementation of the well-known Vogel's approximation method for solving unbalanced transportation problems", *Malaysian Journal of Science*,Vol. 32, No. 1, pp. 66-72(2013).

[21] Z.A.M.S. Juman and M.A. Hoque"A heuristic solution technique to attain the minimal total cost bounds of transporting a homogeneous product with varying demands and supplies",*European Journal of Operational Research*,Vol. 239, pp. 146-156, (2014).