Fuzzy totally somewhat completely pre-irresolute mappings

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Abstract

In this article, we study the concept of fuzzy totally somewhat completely pre-irresolute and fuzzy totally somewhat R-map. In addition, some interesting properties of those mappings are given.

Keywords: Fuzzy totally somewhat completely pre-irresolute; Fuzzy totally somewhat R-map; Fuzzy strongly compact; Fuzzy almost regular; Fuzzy almost normal; Fuzzy r-closed.

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1 Introduction

For the past five decades the notion of fuzzy sets by Zadeh[14] has been a prominent role in various areas of science and engineering. As a result of this Chang [4] first introduced the concept of fuzzy topology in 1968. The notions of fuzzy totally somewhat continuous mapping and fuzzy totally somewhat semicontinuous mapping were investigated by Vadivel and Swaminathan in [13]. In this paper we introduce the ideas of fuzzy totally somewhat completely pre-irresolute and fuzzy totally somewhat R-map and since some examples and relationships among these new classes with other classes of fuzzy mappings are obtained. In section 4, the compositions of these new mappings are given. In section 5, fuzzy totally somewhat completely pre-irresolute preopen mapping, fuzzy totally somewhat open R-map and some properties are studied. Finally, in section 6, some preservation of these mappings are discussed.

2 Preliminaries

Definition 2.1. *Let η be a fuzzy subset of a fts $(\mathscr{X},\mathcal{F})$, then*
(i) η is called fuzzy regular open [1](resp. fuzzy regular closed) if $\eta = Int(Cl(\eta))$ (resp. $\eta = Cl(Int(\eta))$).
(ii) η is called fuzzy preopen([8] [3])(resp. fuzzy preclosed) if $\eta \leq Int(Cl(\eta))$ (resp. $\eta \geq Cl(Int(\eta))$).

Definition 2.2. *Let $f : (\mathscr{X},\mathcal{F}) \to (\mathscr{Y},\mathcal{F}')$ be a function from fts $(\mathscr{X},\mathcal{F})$ to another fts $(\mathscr{Y},\mathcal{F}')$, then*
(i) f is called fuzzy completely pre-irresolute [6] if $f^{-1}(\vartheta)$ is fuzzy regular open set on $\mathscr{X}$ for any fuzzy preopen set ϑ on $\mathscr{Y}$.
(ii) f is called fuzzy completely continuous[7] if $f^{-1}(\vartheta)$ is a fuzzy regular open set on $\mathscr{X}$ for any fuzzy open set ϑ on $\mathscr{Y}$.
(iii) f is called fuzzy R-map[2] if $f^{-1}(\vartheta)$ is a fuzzy regular open set on $\mathscr{X}$ for any fuzzy regular open set ϑ on $\mathscr{Y}$.

Definition 2.3. *Let $f : (\mathscr{X},\mathcal{F}) \to (\mathscr{Y},\mathcal{F}')$ be a function from fts $(\mathscr{X},\mathcal{F})$ to another fts $(\mathscr{Y},\mathcal{F}')$, then*
(i) f is called fuzzy totally completely pre-irresolute [11] if $f^{-1}(\vartheta)$ is fuzzy regular clopen set on $\mathscr{X}$ for any fuzzy preopen set ϑ on $\mathscr{Y}$.
(ii) f is called fuzzy totally completely continuous[11] if $f^{-1}(\vartheta)$ is a fuzzy regular clopen

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set on $\mathscr{X}$ *for any fuzzy open set* ϑ *on* $\mathscr{Y}$.
(iii) f *is called fuzzy totally* R*-map[11] if* $f^{-1}(\vartheta)$ *is a fuzzy regular clopen set on* $\mathscr{X}$ *for any fuzzy regular open set* ϑ *on* $\mathscr{Y}$.

Definition 2.4. *Let* $f : (\mathscr{X}, \mathcal{F}) \to (\mathscr{Y}, \mathcal{F}')$ *be a function from fts* $(\mathscr{X}, \mathcal{F})$ *to another fts* $(\mathscr{Y}, \mathcal{F}')$*, then*
(i) f *is called fuzzy totally completely pre-irresolute preopen[11] if* $f(\vartheta)$ *is a fuzzy regular clopen set on* $\mathscr{Y}$ *for any fuzzy preopen set* ϑ *on* $\mathscr{X}$.
(ii) f *is called fuzzy totally completely open[11] if* $f(\vartheta)$ *is a fuzzy regular clopen set on* $\mathscr{Y}$ *for any fuzzy open set* ϑ *on* $\mathscr{X}$.
(iii) f *is called fuzzy open* R*-map[11] if* $f(\vartheta)$ *is a fuzzy regular clopen set on* $\mathscr{Y}$ *for any fuzzy regular open set* ϑ *on* $\mathscr{X}$.

Definition 2.5. *A mapping* $f : \mathscr{X} \to \mathscr{Y}$ *is called fuzzy totally somewhat completely continuous[12] if there exists a fuzzy regular clopen set* $\sigma \neq 0_{\mathscr{X}}$ *on* $\mathscr{X}$ *such that* $\sigma \leq f^{-1}(\vartheta) \neq 0_{\mathscr{X}}$ *for any fuzzy open set* $\vartheta \neq 0_{\mathscr{Y}}$ *on* $\mathscr{Y}$.

3 Fuzzy totally somewhat completely pre-irresolute mappings

In this section, we introduce fuzzy totally somewhat completely pre-irresolute mapping and we characterize these mappings.

Definition 3.1. *A mapping* $f : \mathscr{X} \to \mathscr{Y}$ *is called fuzzy totally somewhat completely pre-irresolute if there exists a fuzzy regular clopen set* $\sigma \neq 0_{\mathscr{X}}$ *on* $\mathscr{X}$ *such that* $\sigma \leq f^{-1}(\vartheta) \neq 0_{\mathscr{X}}$ *for any fuzzy preopen set* $\vartheta \neq 0_{\mathscr{Y}}$ *on* $\mathscr{Y}$.

Definition 3.2. *A mapping* $f : \mathscr{X} \to \mathscr{Y}$ *is called fuzzy totally somewhat* R*-map if there exists a fuzzy regular clopen set* $\sigma \neq 0_{\mathscr{X}}$ *on* $\mathscr{X}$ *such that* $\sigma \leq f^{-1}(\vartheta) \neq 0_{\mathscr{X}}$ *for any fuzzy regular open set* $\vartheta \neq 0_{\mathscr{Y}}$ *on* $\mathscr{Y}$.

Remark 3.1. Every fuzzy totally completely pre-irresolute mapping is a fuzzy totally completely continuous[11].

From the above definitions the following reverse implications are false:
(i)Every fuzzy totally completely pre-irresolute is a fuzzy totally somewhat completely pre-irresolute .
(ii)Every fuzzy totally somewhat completely pre-irresolute is a fuzzy totally somewhat completely continuous.
(iii)Every fuzzy totally R-map is a fuzzy totally somewhat R-map.

Example 3.2. Let $\mathcal{K}_1(x)$ and $\mathcal{K}_2(x)$ be fuzzy sets on $I = [0,1]$ defined as follows:

$$\mathcal{K}_1(x) = \begin{cases} \frac{1}{4}, & 0 \leq x \leq \frac{1}{2}, \\ \frac{1-x}{2}, & \frac{1}{2} \leq x \leq 1. \end{cases}$$

$$\mathcal{K}_2(x) = \begin{cases} \frac{1}{2}, & 0 \leq x \leq \frac{1}{2}, \\ 1-x, & \frac{1}{2} \leq x \leq 1. \end{cases}$$

$$\mathcal{K}_3(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x-1, & \frac{1}{2} \leq x \leq 1 \end{cases}$$

and

$$\mathcal{K}_4(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x+2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq 1 \end{cases}$$

$$\mathcal{K}_5(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{4} \\ \frac{1}{3}(4x-1), & \frac{1}{4} \leq x \leq 1 \end{cases}$$

Let $\mathcal{J}_1 = \{0, \mathcal{K}_1, \mathcal{K}_1^c, \mathcal{K}_2, 1\}$ and $\mathcal{J}_2 = \{0, \mathcal{K}_1, \mathcal{K}_2, 1\}$ be a fuzzy topologies on I. Let $f : (I, \mathcal{J}_1) \to (I, \mathcal{J}_2)$ defined by $f(x) = x$ for each $x \in I$. It is observed that, for the fuzzy regular clopen set $\mathcal{K}_1$ on $(I, \mathcal{J}_1)$, $\mathcal{K}_1 \leq f^{-1}(\mathcal{K}_1) = \mathcal{K}_1$ and $\mathcal{K}_1 \leq f^{-1}(\mathcal{K}_2) = \mathcal{K}_2$. Therefore, f is fuzzy totally somewhat completely pre-irresolute mapping. But for a fuzzy preopen set $\mathcal{K}_2$ on $(I, \mathcal{J}_2)$, $f^{-1}(\mathcal{K}_2) = \mathcal{K}_2$ which is not fuzzy regular clopen set on $(I, \mathcal{J}_1)$. Hence f is not fuzzy totally completely pre-irresolute mapping.

Example 3.3. Consider the fuzzy topologies $\mathcal{J}_3 = \{0, \mathcal{K}_3, \mathcal{K}_3^c, \mathcal{K}_3 \vee \mathcal{K}_3^c, \mathcal{K}_3 \wedge \mathcal{K}_3^c, 1\}$ and $\mathcal{J}_4 = \{0, \mathcal{K}_3, \mathcal{K}_4^c, 1\}$ a mapping $f : (I, \mathcal{J}_3) \to (I, \mathcal{J}_4)$ defined by $f(x) = \frac{x}{2}$ for each $x \in I$. It is observed that, for the fuzzy open sets $\mathcal{K}_3$ and $\mathcal{K}_4^c$ on $(I, \mathcal{J}_4)$, $f^{-1}(\mathcal{K}_3) = 0$ and $\mathcal{K}_3 \leq f^{-1}(\mathcal{K}_4^c) = \mathcal{K}_3$. Therefore, f is fuzzy totally somewhat completely continuous mapping. But for a fuzzy preopen set $\mathcal{K}_5$ on $(I, \mathcal{J}_4)$, $f^{-1}(\mathcal{K}_5) = \mathcal{K}_5 f(\frac{x}{2}) = M(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ \frac{1}{3}(2x-1), & \frac{1}{2} \leq x \leq 1 \end{cases}$ for each $x \in I$. Hence there is no non-zero fuzzy regular clopen set smaller than $f^{-1}(\mathcal{K}_5) = \mathcal{K}_5 f(\frac{x}{2}) = M(x)$ on $(I, \mathcal{J}_3)$. Therefore f is not fuzzy totally somewhat completely pre-irresolute mapping.

Example 3.4. As described in example 3.1, consider the fuzzy topology $\mathcal{J}_5 = \{0, \mathcal{K}_2, 1\}$ and a mapping $f : (I, \mathcal{J}_1) \to (I, \mathcal{J}_5)$ defined by $f(x) = x$ for each $x \in I$. It is observed that, for the fuzzy regular open set $\mathcal{K}_2$ on $(I, \mathcal{J}_5)$, $\mathcal{K}_2 \leq f^{-1}(\mathcal{K}_2) = \mathcal{K}_2$. Therefore, f is fuzzy totally somewhat R-map. But for a fuzzy regular open set $\mathcal{K}_2$ on $(I, \mathcal{J}_5)$, $f^{-1}(\mathcal{K}_2) = \mathcal{K}_2$ which is fuzzy regular open but not regular closed on $(I, \mathcal{J}_1)$. Hence f is not fuzzy totally R-map.

Theorem 3.5. Let $\mathscr{X}_1$ be product related to $\mathscr{X}_2$ and let $\mathscr{Y}_1$ be product related to $\mathscr{Y}_2$. If $f_1 : \mathscr{X}_1 \to \mathscr{Y}_1$ and $f_2 : \mathscr{X}_2 \to \mathscr{Y}_2$ is fuzzy totally somewhat completely pre-irresolute mappings, then the product $f_1 \times f_2 : \mathscr{X}_1 \times \mathscr{X}_2 \to \mathscr{Y}_1 \times \mathscr{Y}_2$ is also fuzzy totally somewhat completely pre-irresolute mappings.

Proof. Let $\lambda = \vee_{i,j}(\mu_i \times \nu_j)$ be a fuzzy preopen set on $\mathscr{Y}_1 \times \mathscr{Y}_2$ where $\mu_i \neq 0_{\mathscr{Y}_1}$ and $\nu_j \neq 0_{\mathscr{Y}_2}$ are fuzzy preopen sets on $\mathscr{Y}_1$ and $\mathscr{Y}_2$ respectively. Then $(f_1 \times f_2)^{-1}(\lambda) = \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j))$. Since f_1 is fuzzy totally somewhat completely pre-irresolute, there exists a fuzzy regular clopen set $\delta_i \neq 0_{\mathscr{X}_1}$ such that $\delta_i \leq f_1^{-1}(\mu_i) \neq 0_{\mathscr{X}_1}$. And, since f_2 is fuzzy totally somewhat completely pre-irresolute, there exists a fuzzy regular clopen set $\eta_j \neq 0_{\mathscr{X}_2}$ such that $\eta_j \leq f_2^{-1}(\nu_j) \neq 0_{\mathscr{X}_2}$. Now $\delta_i \times \eta_j \leq f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j) = (f_1 \times f_2)^{-1}(\mu_i \times \nu_j)$ and $\delta_i \times \eta_j \neq 0_{\mathscr{X}_1} \times 0_{\mathscr{X}_2}$ is a fuzzy regular clopen set on $\mathscr{X}_1 \times \mathscr{X}_2$. Hence $\bigvee_{i,j}(\delta_i \times \eta_j) \neq 0_{\mathscr{X}_1} \times 0_{\mathscr{X}_2}$ is a fuzzy regular clopen set on $\mathscr{X}_1 \times \mathscr{X}_2$ such that $\bigvee_{i,j}(\delta_i \times \eta_j) \leq \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j)) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\mu_i \times \nu_j)) = (f_1 \times f_2)^{-1}(\lambda) \neq 0_{\mathscr{X}_1 \times \mathscr{X}_2}$. Therefore, $f_1 \times f_2$ is fuzzy totally somewhat completely pre-irresolute.

Lemma 3.6. *[1]* Let $g : \mathscr{X} \to \mathscr{X} \times \mathscr{Y}$ be the graph of a mapping $f : \mathscr{X} \to \mathscr{Y}$. If μ is a fuzzy set in $\mathscr{X}$ and ν is a fuzzy set in $\mathscr{Y}$, then $g^{-1}(\mu \times \nu) = \mu \wedge f^{-1}(\nu)$.

Theorem 3.7. Let $f : \mathscr{X} \to \mathscr{Y}$ be a mapping. If the graph $g : \mathscr{X} \to \mathscr{X} \times \mathscr{Y}$ of f is fuzzy totally somewhat completely pre-irresolute, then f is also fuzzy totally somewhat completely pre-irresolute.

Proof. Let $\nu \neq 0_{\mathscr{Y}}$ be a fuzzy preopen set on $\mathscr{Y}$. Then $f^{-1}(\nu) = 1 \wedge f^{-1}(\nu) = g^{-1}(1 \times \nu)$. Since g is fuzzy totally somewhat completely pre-irresolute and $(1 \times \nu)$ is a fuzzy preopen set on $\mathscr{X} \times \mathscr{Y}$, there exists a fuzzy regular clopen set $\mu \neq 0_{\mathscr{X}}$ on $\mathscr{X}$ such that $\mu \leq g^{-1}(1 \times \nu) = f^{-1}(\nu) \neq 0_{\mathscr{X}}$. Therefore, f is fuzzy totally somewhat completely pre-irresolute.

4 Compositions of fuzzy totally somewhat completely pre-irresolute mappings

In this section the composition of fuzzy totally somewhat completely pre-irresolute and fuzzy totally somewhat R-map with other fuzzy mappings are studied.

Theorem 4.1. (i) If $f : \mathscr{X} \to \mathscr{Y}$ is fuzzy totally somewhat completely pre-irresolute and $g : \mathscr{Y} \to \mathscr{Z}$ is fuzzy precontinuous, then $g \circ f : \mathscr{X} \to \mathscr{Z}$ is fuzzy totally somewhat completely continuous.
(ii) If $f : \mathscr{X} \to \mathscr{Y}$ is fuzzy totally somewhat completely pre-irresolute and $g : \mathscr{Y} \to \mathscr{Z}$ is fuzzy pre-irresolute, then $g \circ f : \mathscr{X} \to \mathscr{Z}$ is fuzzy totally somewhat completely pre-irresolute.

Proof. Obvious.

Theorem 4.2. (i)If $f : \mathscr{X} \to \mathscr{Y}$ is fuzzy totally R-map and $g : \mathscr{Y} \to \mathscr{Z}$ is fuzzy completely continuous , then $g \circ f : \mathscr{X} \to \mathscr{Z}$ is fuzzy totally somewhat completely continuous.
(ii)If $f : \mathscr{X} \to \mathscr{Y}$ is fuzzy totally somewhat R-map and $g : \mathscr{Y} \to \mathscr{Z}$ is fuzzy R-map, then $g \circ f : \mathscr{X} \to \mathscr{Z}$ is fuzzy totally somewhat R-map.

Proof. Straightforward.

5 Fuzzy totally somewhat completely pre-irresolute preopen mappings

In this section, we introduce a fuzzy totally somewhat completely pre-irresolute preopen mapping and fuzzy totally somewhat open R-map and we characterize these mappings.

Definition 5.1. *A mapping $f : \mathscr{X} \to \mathscr{Y}$ is called fuzzy totally somewhat completely open if there exists a fuzzy regular clopen set $\nu \neq 0_{\mathscr{Y}}$ on $\mathscr{Y}$ such that $\nu \leq f(\mu) \neq 0_{\mathscr{Y}}$ for any fuzzy open set $\mu \neq 0_{\mathscr{X}}$ on $\mathscr{X}$.*

Definition 5.2. *A mapping $f : \mathscr{X} \to \mathscr{Y}$ is called fuzzy totally somewhat completely pre-irresolute preopen if there exists a fuzzy regular clopen set $\nu \neq 0_{\mathscr{Y}}$ on $\mathscr{Y}$ such that $\nu \leq f(\mu) \neq 0_{\mathscr{Y}}$ for any fuzzy preopen set $\mu \neq 0_{\mathscr{X}}$ on $\mathscr{X}$.*

Definition 5.3. *A mapping* $f : \mathscr{X} \to \mathscr{Y}$ *is called fuzzy totally somewhat open* R*-map if there exists a fuzzy regular clopen set* $\nu \neq 0_{\mathscr{Y}}$ *on* $\mathscr{Y}$ *such that* $\nu \leq f(\mu) \neq 0_{\mathscr{Y}}$ *for any fuzzy regular open set* $\mu \neq 0_{\mathscr{X}}$ *on* $\mathscr{X}$.

From the above definitions it is clear that the following reverse implications are false:

(i)Every fuzzy totally completely pre-irresolute preopen mapping is a fuzzy totally somewhat completely pre-irresolute preopen mapping.
(ii)Every fuzzy totally somewhat completely pre-irresolute preopen mapping is a fuzzy totally somewhat completely open mapping.
(iii)Every fuzzy totally open R-map is a fuzzy totally somewhat open R-map.

Example 5.1. Consider the fuzzy mapping $f : (I, \mathcal{J}_2) \to (I, \mathcal{J}_1)$ defined by $f(x) = x$ for each $x \in I$ as described in example 3.1. From this we can obtain that $\mathcal{K}_1 \leq f(\mathcal{K}_1) = \mathcal{K}_1$ and $\mathcal{K}_1 \leq f(\mathcal{K}_2) = \mathcal{K}_2$ and hence f is fuzzy totally somewhat pre-irresolute preopen mapping. But for a fuzzy preopen set $\mathcal{K}_2$ on $(I, \mathcal{J}_1)$, $f(\mathcal{K}_2) = \mathcal{K}_2$ which is not fuzzy regular clopen set on $(I, \mathcal{J}_1)$. Hence f is not fuzzy totally completely pre-irreolute preopen mapping.

Example 5.2. From example 3.2, consider a mapping $f : (I, \mathcal{J}_4) \to (I, \mathcal{J}_3)$ defined by $f(x) = \frac{x}{2}$ for each $x \in I$. It is observed that, for the fuzzy open sets $\mathcal{K}_3$ and $\mathcal{K}_4^c$ on $(I, \mathcal{J}_4)$, $f(\mathcal{K}_3) = 0$ and $\mathcal{K}_3 \leq f(\mathcal{K}_4^c) = \mathcal{K}_4^c$. Therefore, f is fuzzy totally somewhat completely open mapping. But for a fuzzy preopen set $\mathcal{K}_5$ on $(I, \mathcal{J}_4)$, $f(\mathcal{K}_5) = \mathcal{K}_5 f(\frac{x}{2}) = M(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ \frac{1}{3}(2x-1), & \frac{1}{2} \leq x \leq 1 \end{cases}$ for each $x \in I$. Hence there is no non-zero fuzzy regular clopen set smaller than $f(\mathcal{K}_5) = \mathcal{K}_5 f(\frac{x}{2}) = M(x)$ on $(I, \mathcal{J}_3)$. Therefore f is not fuzzy totally somewhat completely pre-irresolute preopen mapping.

Example 5.3. As described in example 3.1, consider the fuzzy mapping $f : (I, \mathcal{J}_5) \to (I, \mathcal{J}_1)$ defined by $f(x) = x$ for each $x \in I$. It is observed that, for the fuzzy regular open sets $\mathcal{K}_2$ on $(I, \mathcal{J}_5)$, $\mathcal{K}_1 \leq f(\mathcal{K}_2) = \mathcal{K}_2$. Therefore, f is fuzzy totally somewhat R map. But for a fuzzy regular open set $\mathcal{K}_2$ on $(I, \mathcal{J}_5)$, $f(\mathcal{K}_2) = \mathcal{K}_2$ which is fuzzy regular open but not fuzzy regular closed set on $(I, \mathcal{J}_1)$. Hence f is not fuzzy totally open R map.

Theorem 5.4. If $f : \mathscr{X} \to \mathscr{Y}$ be a bijection. Then the following are equivalent.
(1) f is fuzzy totally somewhat completely pre-irresolute preopen.
(2) If σ is a fuzzy preclosed set on $\mathscr{X}$ such that $f(\sigma) \neq 1_{\mathscr{Y}}$, then there exists a fuzzy regular clopen set $\vartheta \neq 1_{\mathscr{Y}}$ on $\mathscr{Y}$ such that $f(\sigma) < \vartheta$.

Proof.(1) $\Rightarrow$ (2): Let σ be a fuzzy preclosed set on $\mathscr{X}$ such that $f(\sigma) \neq 1_{\mathscr{Y}}$. Since f is bijective and σ^c is a fuzzy preopen set on $\mathscr{X}$, $f(\sigma^c) = (f(\sigma))^c \neq 0_{\mathscr{Y}}$. And, since

f is fuzzy totally somewhat completely pre-irresolute preopen mapping, there exists a fuzzy regular clopen set $\delta \neq 0_{\mathscr{Y}}$ on $\mathscr{Y}$ such that $\delta < f(\sigma^c) = (f(\sigma))^c$. Consequently, $f(\sigma) < \delta^c = \vartheta \neq 1_{\mathscr{Y}}$ and ϑ is a fuzzy regular clopen set on $\mathscr{Y}$.

(2) $\Rightarrow$ (1): Let σ be a fuzzy preopen set on $\mathscr{X}$ such that $f(\sigma) \neq 0_{\mathscr{Y}}$. Then σ^c is a fuzzy preclosed set on $\mathscr{X}$ and $f(\sigma^c) \neq 1_{\mathscr{Y}}$. Hence there exists a fuzzy regular clopen set $\vartheta \neq 1_{\mathscr{Y}}$ on $\mathscr{Y}$ such that $f(\sigma^c) < \vartheta$. Since f is bijective, $f(\sigma^c) = (f(\sigma))^c < \vartheta$. Hence $\vartheta^c < f(\sigma)$ and $\vartheta^c \neq 0_{\mathscr{X}}$ is a fuzzy regular clopen set on $\mathscr{Y}$. Therefore, f is fuzzy totally somewhat completely pre-irresolute preopen mapping.

Definition 5.4. *A fuzzy set which cannot be expressed as the union of two fuzzy pre-separated[6] sets is said to be a fuzzy pre-connected set.*

Proposition 5.5. If f is fuzzy totally somewhat fuzzy pre-irresolute mapping from a fuzzy pre connected space $\mathscr{X}$ into any fuzzy topological space $\mathscr{Y}$, then $\mathscr{Y}$ is indiscrete fuzzy topological space.

Proof. If possible suppose $\mathscr{Y}$ is not indiscrete. Then $\mathscr{Y}$ has a proper ($\neq 0$ and $\neq 1$) fuzzy open set η (say). Then by hypothesis on $f, f^{-1}(\eta)$ is a proper fuzzy regular clopen set of $\mathscr{X}$, which is a contradiction to the assumption that $\mathscr{X}$ is fuzzy pre connected. Hence the proposition.

Definition 5.5. *Let $(\mathscr{X}, \mathcal{F})$ be any fuzzy topological space. $\mathscr{X}$ is called fuzzy pre T_0 [8](fuzzy almost T_0 [9]) if and only if for any pair of distinct fuzzy points x_t and y_s, there exists a fuzzy preopen(regular open) set η such that $x_t \in \eta$, $y_s \notin \eta$ or $x_t \notin \eta$, $y_s \in \eta$.*

Definition 5.6. *Let $(\mathscr{X}, \mathcal{F})$ be any fuzzy topological space. $(\mathscr{X}, T)$ is called fuzzy pre T_2 [8] if for any pair of distinct fuzzy points x_t and y_s there exist fuzzy preopen sets η and μ such that $x_t \in \eta$, $y_s \in \mu$ and $pCl\eta \leq 1 - pCl\mu$.*

Definition 5.7. *Let $(\mathscr{X}, \mathcal{F})$ be any fuzzy topological space. $(\mathscr{X}, T)$ is called fuzzy almost T_2 [9] if for any pair of distinct fuzzy points x_t and y_s, there exist fuzzy regular open sets η and μ such that $x_t \in \eta$, $y_s \in \mu$ and $rCl\eta \leq 1 - rCl\mu$.*

Proposition 5.6. Let $f : (\mathscr{X}, \mathcal{F}) \to (\mathscr{Y}, \mathcal{H})$ be an injective fuzzy totally somewhat completely pre-irresolute mapping. If $\mathscr{Y}$ is fuzzy pre T_0, then $\mathscr{X}$ is fuzzy almost T_2.

Proof. Let x_t and y_s be any two distinct fuzzy points of $\mathscr{X}$. Then $f(x_t) \neq f(y_s)$. That is $(f(x))_t \neq (f(y))_s$. Since $\mathscr{Y}$ is fuzzy pre T_0, there exists a fuzzy preopen set say $\eta \neq 0$

in $\mathscr{Y}$ such that $f(x_t) \in \eta$ and $f(y_s) \notin \eta$. This means $x_t \in f^{-1}(\eta)$ and $y_s \notin f^{-1}(\eta)$. Since f is fuzzy totally somewhat completely pre-irresolute, there exists fuzzy regular open set $\mu \neq 0$ in $\mathscr{X}$ such that $\mu \leq f^{-1}(\eta)$ is fuzzy regular open set of $\mathscr{X}$.

Also $x_t \in f^{-1}(\eta)$ and $y_s \in 1 - f^{-1}(\eta)$. Now put $\mu = 1 - f^{-1}(\eta)$. Then $f^{-1}(\eta) = rCl(\eta)$ and $rCl(1 - f^{-1}(\eta)) = rCl\mu = 1 - f^{-1}(\eta)$ and $rCl(\eta) = f^{-1}(\eta) = 1 - rCl(\mu) \leq 1 - rCl(\mu)$. Hence the Proposition.

Proposition 5.7. Let $(\mathscr{X}, \mathcal{F})$ be any fuzzy pre connected space. Then every fuzzy totally somewhat completely pre-irresolute mapping from a space $\mathscr{X}$ onto any fuzzy pre T_0-space $\mathscr{Y}$ is constant.

Proof. Given that $(\mathscr{X}, \mathcal{F})$ is fuzzy preconnected. Suppose $f : \mathscr{X} \to \mathscr{Y}$ be any fuzzy totally somewhat completely pre-irresolute mapping and we assume that $\mathscr{Y}$ is a fuzzy pre T_0 space. Then by Proposition 5.5, $\mathscr{Y}$ should be an indiscrete space. But an indiscrete fuzzy topological space containing two or more points cannot be fuzzy pre T_0. Therefore, $\mathscr{Y}$ must be singleton and this proves that f must be a constant function.

Definition 5.8. *A fuzzy space $\mathscr{X}$ is called fuzzy almost regular[9] if for each fuzzy point $x_t \notin \eta$ and a fuzzy regular closed set η, there exist two fuzzy open sets σ and δ such that $\eta \leq \sigma$ and $x_t \in \delta$.*

Definition 5.9. *A fuzzy space $\mathscr{X}$ is called fuzzy preregular[8] if for each fuzzy point $x_t \notin \eta$ and a closed set η there exist disjoint fuzzy preopen sets σ and δ such that $\eta \leq \sigma$ and $x_t \in \delta$.*

Definition 5.10. *A fuzzy space $\mathscr{X}$ is called fuzzy almost normal[9] if for every fuzzy sets sets η_1 and η_2 in $\mathscr{X}$, where η_1 is fuzzy closed and η_2 is fuzzy regular closed, there exist fuzzy open sets σ and δ such that $\eta_1 \leq \sigma$ and $\eta_2 \leq \delta$.*

Definition 5.11. *A fuzzy space $\mathscr{X}$ is called fuzzy prenormal[8] if for every pair of fuzzy closed sets η_1 and η_2 in $\mathscr{X}$, there exist fuzzy preopen sets σ and δ such that $\eta_1 \leq \sigma$ and $\eta_2 \leq \delta$.*

Theorem 5.8. If $f : (\mathscr{X}, \mathcal{F}) \to (\mathscr{Y}, \mathcal{H})$ be an injective fuzzy totally somewhat completely pre-irresolute and $\mathscr{X}$ is a fuzzy almost regular space, then $\mathscr{Y}$ is fuzzy preregular.

Proof. Let η be a fuzzy preopen set on $\mathscr{Y}$ and a fuzzy point $y_\beta \notin \eta$. Take $y_\beta = f(x_\alpha)$. Since f is fuzzy totally somewhat completely pre-irresolute such that $f^{-1}(\eta)$ is a fuzzy regular open set of $\mathscr{X}$. Take $\mu = f^{-1}(\eta)$. We have $x_\alpha \notin \mu$. Since $\mathscr{X}$ is fuzzy almost regular, there exist disjoint fuzzy open sets η and δ in $\mathscr{X}$ such that $\mu \leq \eta$ and $x_\alpha \in \delta$. We obtain that $\eta = f(\mu) \leq f(\eta)$ and $y_\beta = f(x_\alpha) \in \delta$ such that $f(\eta)$ and $f(\delta)$ are disjoint fuzzy preopen sets of $\mathscr{Y}$. This shows that $\mathscr{Y}$ is fuzzy preregular.

Theorem 5.9. If $f : (\mathscr{X}, \mathcal{F}) \to (\mathscr{Y}, \mathcal{H})$ be an injective fuzzy totally somewhat completely pre-irresolute and $\mathscr{X}$ is a fuzzy almost normal space, then $\mathscr{Y}$ is fuzzy prenormal.

Proof. Let η_1 and η_2 be disjoint fuzzy open set in $\mathscr{Y}$. Since f is fuzzy totally somewhat completely pre-irresolute, $f^{-1}(\eta_1)$ and $f^{-1}(\eta_2)$ are fuzzy regular open sets in $\mathscr{X}$. Let $\alpha = f^{-1}(\eta_1)$ and $\beta = f^{-1}(\eta_2)$. We have $\alpha \wedge \beta = 0$. Since $\mathscr{X}$ is fuzzy almost normal, there exist disjoint fuzzy open sets γ and δ such that

$\alpha \leq \gamma$ and $\beta \leq \delta$. We obtain that $\eta_1 = f(\alpha) \leq f(\gamma)$ and $\eta_2 = f(\beta) \leq f(\delta)$ such that $f(\gamma)$ and $f(\delta)$ are disjoint fuzzy open sets. Thus, $\mathscr{Y}$ is fuzzy pre normal.

6 Some preservation results

In this section by means of fuzzy totally somewhat completely pre-irresolute and fuzzy totally somewhat R-map, preservation of some fuzzy topological structures are discussed.

Definition 6.1. *A fuzzy topological space $(\mathscr{X}, \mathcal{F})$ is called*
(i) fuzzy strongly compact [10] if every fuzzy preopen cover has a finite subcover.
(ii)fuzzy nearly compact [5] if every fuzzy regular open cover has a finite subcover.
(ii) fuzzy r-closed [11] if every fuzzy regular clopen cover has a finite subcover.

Theorem 6.1. Every surjective fuzzy totally somewhat completely pre-irresolute image of a fuzzy r-closed space is fuzzy strongly compact.

Proof. Let $f : \mathscr{X} \to \mathscr{Y}$ be a fuzzy totally somewhat completely pre-irresolute mapping of a fuzzy r-closed space $(\mathscr{X}, \mathcal{F}_1)$ onto a fuzzy space $(\mathscr{Y}, \mathcal{F}_2)$. Let $\{\beta_a : a \in A\}$ be any fuzzy preopen cover of $\mathscr{Y}$. Since f is fuzzy totally somewhat completely pre-irresolute, $\left\{f^{-1}(\beta_a) : a \in A\right\}$ is a fuzzy regular clopen cover of $\mathscr{X}$. Since $\mathscr{X}$ is a fuzzy r-closed space, then there exists a finite subfamily $\left\{f^{-1}(\beta_{a_i}) : i = 1, ..., n\right\}$ of $\left\{f^{-1}(\beta)\right\}$ which covers $\mathscr{X}$. It implies that $\{\beta_{a_i} : i = 1, ..., n\}$ is a finite subcover of $\{\beta_a : a \in A\}$ which covers $\mathscr{Y}$. Hence $f(\mathscr{X}) = \mathscr{Y}$ is fuzzy strongly compact.

Theorem 6.2. Every surjective fuzzy totally somewhat R-map image of a fuzzy r-closed space is fuzzy nearly compact.

Proof. Let $f : \mathscr{X} \rightarrow \mathscr{Y}$ be a fuzzy totally somewhat R-map of a fuzzy r-closed space $(\mathscr{X}, \mathcal{F}_1)$ onto a fuzzy space $(\mathscr{Y}, \mathcal{F}_2)$. Let $\{\beta_a : a \in A\}$ be any fuzzy open cover of $\mathscr{Y}$. Since f is fuzzy totally somewhat R-map, $\left\{f^{-1}(\beta_a) : a \in A\right\}$ is a fuzzy regular clopen cover of $\mathscr{X}$. Since $\mathscr{X}$ is a fuzzy r-closed space, then there exists a finite subfamily $\left\{f^{-1}(\beta_{a_i}) : i = 1, ..., n\right\}$ of $\left\{f^{-1}(\beta)\right\}$ which covers $\mathscr{X}$. It implies that $\{\beta_{a_i} : i = 1, ..., n\}$ is a finite subcover of $\{\beta_a : a \in A\}$ which covers $\mathscr{Y}$. Hence $f(\mathscr{X}) = \mathscr{Y}$ is fuzzy nearly compact.

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