On Ngδs_ homeomorphism and Ngδs_irresolute in Nano Topological Spaces

Umadevi I Neeli
Department of Mathematics
KLE Technological University, Hubballi, Karnataka, India

Roopa S Algur
Department of Mathematics
KLE Technological University, Hubballi, Karnataka, India

Abstract- In this paper, we are introducing the idea of nanogeneralized delta semi cont.. functions and derive their characterizations. There is also an attempt to define nanogeneralized delta semiopen functions and nanogeneralized delta semi homeomorphisms between nano topological spaces and derive their equivalent characterizations. We have also provided a real life example of nano homeomorphism.

Keywords – **Nano generalized δ_semiclosed set, Nano cont.., Nano semi continuous, Nano open, Nano closed, Nanosemiopen, Nano semiclosed.**

I. Introduction

The class of semi open and generalized closed sets was introduced by Levine[4] in 1970 .The concept Nano topological space was introduced by Lellis Thivagar and Carmel Richard[1]. To prove our results preliminaries are recalled from the papers [2,3,5,6,7,8]

In this article continuous, inverse, image, Nano open and Nano closed are abbreviated as cont., inv, img , NO, NC respectively.

II. Nano generalized delta semi continuity

2.1 Definition- Let $(P, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces, then a mapping $f: (P, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is called nano generalized delta semi cont. (briefly Ngδs_cont.) if the inv img of every NO set B in V is Ngδs_open in U. That is, if $f^{-1}(B)$ is $Ng\delta s$_open in P for every NO set B of V.

2.2 Example - Let $P = \{a, b, c, d\}$with $P/R = \{\{a\},\{c\},\{b, d\}\}$. Let$X = \{a, b\} \subseteq P$. Then $\tau_R(X) = \{P, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$. This gives Ng$\delta$s_open (P)= { P, $\emptyset$, {a}, {b}, {c}, {d}, {a,c}, {b,c}, {b,d} ,{c,d}, {a,b,c}, {a,c,d}, {b,c,d}}
Let $V = \{x, y, z, w\}$ with$V/R' = \{\{x\},\{w\},\{y, z\}\}$. Let$Y = \{x, z\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{x\}, \{y, z\}, \{x, y, z\}\}$.
Define $f: (P, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$as $f(a) = y, f(b) = w, f(c) = z, f(d) = x$.
Then$f^{-1}(V) = P, f^{-1}(\emptyset) = \emptyset, f^{-1}(\{x\}) = \{d\}, f^{-1}(\{y, z\}) = \{a, c\}, f^{-1}(\{x, y, z\}) = \{a, c, d\}$. That is in v img of every NO set B in V is Ngδs_open in U. Therefore f is Ngδs_cont. The following theorem characterizes nano cont. functions in terms of NC sets.

2.3 Theorem- A function $f: (P, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is Ngδs_cont. iff the inv img of every NC set in V is Ngδs_ closed in U.

Proof: Let f be Ngδs_ cont. and F be NC in V. That is, $V - F$ is NO in V. Since f Ngδs_cont., $f^{-1}(V-F)$ is Ngδs_open in P. That is, $P - f^{-1}(F)$ is Ngδs_open in P. Therefore, $f^{-1}(F)$ is Ngδs_closed in P. Thus, the inv img of every NC set in V is Ngδs_closed in P, if f is Ngδs_ cont. on P. Conversely, let inv img of every NC set be Ngδs_ closed. Let G be NO in V. Then $V - G$ is NC in V. Then, $f^{-1}(V-G)$ is Ngδs_closed in P. That is, $P - f^{-1}(G)$ is Ngδs_closed in P. Therefore, $f^{-1}(G)$ is Ngδs_open in P. Thus, the inv img of every NO set in V is Ngδs_open in P. That is, f is Ngδs_cont. on P.

2.4 Theorem- Let $f:(U,\tau_R(X)) \longrightarrow (V,\tau_{R'}(Y))$ be a function from U into V then following are equivalent.(i) The function f is Ngδs_ cont..(ii) For each $x \in U$ and each NO set Y in V with $f(x) \in Y$ there is Ngδs_open set P in U such that $x \in P$ and $f(P) \subset V$

Proof: (i) ⟹(ii) If (i) holds. Let $x \in U$ and Y be a NO set in $V$with $f(x) \in Y$, then $x \in f^{-1}(Y)$. Since f is Ngδs _cont., $f^{-1}(Y)$ is Ngδs _open in U. Put $P = f^{-1}(Y)$ then $x \in P$ and $f(P) = f(f^{-1}(Y)) \subset V$. Thus for each $x \in U$ and each NO set Y in V with $f(x) \in Y$ there is Ngδs_open set P in U such that $x \in P$ and $f(P) \subset V$. Hence (ii) holds.

(ii) ⟹(i)Suppose (ii) holds. Let $x \in U$ and Y be a NO set in $V$with $f(x) \in Y$. By (ii) there exists a Ngδs _open set P_x in X such that $x \in P_x$ and $f(P_x) \subset Y$. This implies $x \in P_x \subset f^{-1}(Y)$. This implies $f^{-1}(Y)$ is Ngδs-nhd of x. Since x is arbitrary $f^{-1}(Y)$ is Ngδs-nhd of each of its points. Hence $f^{-1}(Y)$ is Ngδs-open set in U. Therefore f is Ngδs _cont.. Hence (i) holds.

2.5 Theorem- Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be nano topological spaces. Let $f:(U,\tau_R(X)) \longrightarrow (V,\tau_{R'}(Y))$be a nano cont. function then f is Ngδs_cont.

Proof: Let P be any NO set in $(V,\tau_{R'}(Y))$. Since f is Nano cont.,$f^{-1}(P)$is NO in U. But every NO set is Ngδs_open set implies $f^{-1}(P)$ is Ngδs_open set in $(U,\tau_R(X))$. Therefore$f$ is Ngδs_cont.

2.6 Remark **-** The converse of above theorem need not be true.

2.7 Example- Let $U = \{a,b,c,d\}$with $U/R = \{\{a\},\{c\},\{b,d\}\}$. Let $X = \{a,b\} \subseteq U$. Then $\tau_R(X) = \{U,\emptyset, \{a\}, \{b,d\},\{a,b,d\}$. Let $V = \{x,y,z,w\}$with $V/R' = \{\{x\},\{w\},\{y,z\}\}$. Let $Y = \{x,z\} \subseteq V$. Then $\tau_{R'}(Y) = \{V,\emptyset, \{x\}, \{y,z\},\{x,y,z\}\}$ be a nano topologies on U and Vrespectively. Define $f:(U,\tau_R(X)) \longrightarrow (V,\tau_{R'}(Y))$by $f(a) = y, f(b) = w, f(c) = z, f(d) = x$. Then for NO set $\{x,y,z\}$ in V, $f^{-1}(\{x,y,z\}) = \{a,c,d\}$is not NO in U. That is inv img of every NO set B in V is not NO in U. Therefore f is not nano cont..

2.8 Theorem- Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be nano topological spaces. Let $f:(U,\tau_R(X)) \longrightarrow (V,\tau_{R'}(Y))$ be a nano semi cont. function then f is Ngδs_cont..

Proof: Let P be any NO set in $(V,\tau_{R'}(Y))$. Since f is Nano semi cont., $f^{-1}(P)$ is Nano semi open in U. But every Nano semi open set is Ngδs_open set implies $f^{-1}(P)$ is Ngδs_open set in $(U,\tau_R(X))$. Therefore f is Ngδs_cont..

2.9 Remark- The converse of above theorem need not be true.

2.10 Example- Let $U = \{a,b,c,d\}$with $U/R = \{\{a\},\{c\},\{b,d\}\}$. Let $X = \{a,b\} \subseteq U$. Then $\tau_R(X) = \{U,\emptyset, \{a\}, \{b,d\},\{a,b,d\}$. Let $V = \{x,y,z,w\}$with $V/R' = \{\{x\},\{w\},\{y,z\}\}$. Let $Y = \{x,z\} \subseteq V$. Then $\tau_{R'}(Y) = \{V,\emptyset, \{x\}, \{y,z\},\{x,y,z\}\}$ be a nano topologies on U and V respectively. Define $f:(U,\tau_R(X)) \longrightarrow (V,\tau_{R'}(Y))$by $f(a) = y, f(b) = w, f(c) = z, f(d) = x$. Then for NO set $\{x,y,z\}$ in $V, f^{-1}(\{x,y,z\}) = \{a,c,d\}$ is not nanosemiopen in U. That is inv img of every NO set B in V is not nano semi open in U. Therefore f is not nanosemi cont..

2.11 Definition- A map $f:(U,\tau_R(X)) \longrightarrow (V,\tau_{R'}(Y))$ is called nano generalized delta semiclosed map (resp. nano generalized delta semiopen map) if $f(P)$ is Ngδs_closed (resp. Ngδs_open) in V for every NC set (resp. NO set) P of U. The nano generalized delta semiclosed map (resp. nano generalized delta semiopen map) is briefly written as Ngδs_closed map (resp. Ngδs_open map).

2.12 Example- Let $U = \{x,y,z,w\}$with $U/R = \{\{x\},\{w\},\{y,z\}\}$. Let $X = \{x,z\} \subseteq U$. Then $\tau_R(X) = \{U,\emptyset, \{x\}, \{y,z\},\{x,y,z\}\}$. Let $V = \{a,b,c,d\}$with $V/R' = \{\{a\},\{c\},\{b,d\}\}$. Let $Y = \{a,b\} \subseteq V$.

Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b,d\}, \{a,b,d\}\}$ be nano topologies on U and V respectively. Define $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ by $f(x) = d, f(y) = b, f(z) = a, f(w) = c$. Then $(U) = V, f(\emptyset) = \emptyset, f(\{x\}) = \{d\}, f(\{y,z\}) = \{a,b\}, f(\{x,y,z\}) = \{a,b,d\}$. That is img of every NO set in U is Ngδs_open in V. Hence f is Ngδs_open map.

2.13 Remark- Every NO map is Ngδs_open map, But converse is not true.

2.14 Example- Let $U = \{x,y,z,w\}$ with $U/R = \{\{x\},\{w\},\{y,z\}\}$. Let $X = \{x,z\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{x\}, \{y,z\}, \{x,y,z\}\}$. Let $V = \{a,b,c,d\}$ with $V/R' = \{\{a\},\{c\},\{b,d\}\}$. Let $Y = \{a,b\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b,d\}, \{a,b,d\}\}$ be nano topologies on U and V respectively. Define $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ by $f(x) = d, f(y) = b, f(z) = a, f(w) = c$. Then f is Ngδs_open map, but not an NO map, since for NO set {x} in $U, f(\{x\}) = \{d\}$, which is not NO in V.

2.15 Remark- Every nanosemi open map is Ngδs_open map, but converse is not true.

2.16 Example- Let $U = \{x,y,z,w\}$ with $U/R = \{\{x\},\{w\},\{y,z\}\}$. Let $X = \{x,z\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{x\}, \{y,z\}, \{x,y,z\}\}$. Let $V = \{a,b,c,d\}$ with $V/R' = \{\{a\},\{c\},\{b,d\}\}$. Let $Y = \{a,b\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b,d\}, \{a,b,d\}\}$ be nano topologies on U and V respectively. Define $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ by $f(x) = d, f(y) = b, f(z) = a, f(w) = c$. Then f is Ngδs_open map, but not an nanosemiopen map, since for NO set {x} in $U, f(\{x\}) = \{d\}$, is not nanosemi open in V.

2.17 Definition- A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is said to be nano generalized delta semi homeomorphism (briefly Ngδs_homeomorphism) if (i) f is Ngδs_cont. (ii) f is Ngδs_open (iii) f is bijective. Equivalently a bijective function f is said to be nano generalized delta semi homeomorphism (briefly Ngδs_homeomorphism) if both f and f^{-1} both are cont..

2.18 Example- Let $U = \{x,y,z,w\}$ with $U/R = \{\{x\},\{w\},\{y,z\}\}$. Let $X = \{x,z\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{x\}, \{y,z\}, \{x,y,z\}\}$. Let $V = \{a,b,c,d\}$ with $V/R' = \{\{a\},\{c\},\{b,d\}\}$. Let $Y = \{a,b\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b,d\}, \{a,b,d\}\}$ be nano topologies on U and V respectively. Define $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ by $f(x) = d, f(y) = b, f(z) = a, f(w) = c$, then f is Ngδs_homeomorphism.

2.19 Remark- Every nano homeomorphism is Ngδs_homeomorphism, but converse is not true.

2.20 Example- Let $U = \{x,y,z,w\}$ with $U/R = \{\{x\},\{w\},\{y,z\}\}$. Let $X = \{x,z\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{x\}, \{y,z\}, \{x,y,z\}\}$. Let $V = \{a,b,c,d\}$ with $V/R' = \{\{a\},\{c\},\{b,d\}\}$. Let $Y = \{a,b\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b,d\}, \{a,b,d\}\}$ be nano topologies on U and V respectively. Define $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ by $f(x) = d, f(y) = b, f(z) = a, f(w) = c$, then f is Ngδs_homeomorphism. But not nano homeomorphism, because $f^{-1}(\{a\}) = \{z\}$ is not NO in U. This implies f is not nano cont. and hence not nano homeomorphism.

2.21 Remark- Every nano semi homeomorphism is Ngδs_homeomorphism, but converse is not true.

2.22 Example- Let $U = \{x,y,z,w\}$ with $U/R = \{\{x\},\{w\},\{y,z\}\}$. Let $X = \{x,z\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{x\}, \{y,z\}, \{x,y,z\}\}$. Let $V = \{a,b,c,d\}$ with $V/R' = \{\{a\},\{c\},\{b,d\}\}$. Let $Y = \{a,b\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b,d\}, \{a,b,d\}\}$ be nano topologies on U and V respectively. Define $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ by $f(x) = d, f(y) = b, f(z) = a, f(w) = c$, then f is Ngδs_homeomorphism. But not nano homeomorphism, because $f^{-1}(\{a\}) = \{z\}$ is not nanosemiopen in U. This implies f is not nano semi cont. and hence not nanosemihomeomorphism.

2.23 Theorem - Let $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ be a bijective and is Ngδs_cont. map. Then the following are equivalent. (i) f is Ngδs_open map (ii) Ngδs_homeomorphism (iii) f is Ngδs_closed map

Proof: (i)⟺(ii) Obvious from definitions.

(i)⟺(iii) Suppose f is Ngδs_open map and F is NC set in U, then U$-F$ is an NO set in U. By (i) $f(U-F)$ is Ngδs_open in V. This implies $f(F)$ is Ngδs_closed set in V. Therefore f is Ngδs_closed map.

2.24 Remark- The composition of two Ngδs_homeomorphisms need not be Ngδs_homeomorphism in general.

2.25 Example- Let $U = \{x, y, z, w\}$with $U/R = \{\{x\}, \{w\}, \{y, z\}\}$. Let $X = \{x, z\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{x\}, \{y, z\}, \{x, y, z\}\}$. Let $V = \{a, b, c, d\}$with $V/R' = \{\{a\}, \{c\}, \{b, d\}\}$. Let $Y = \{a, b\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$. Let $W = \{a, b, c, d\}$ with $W/R'' = \{\{b\}, \{d\}, \{a, c\}\}$. Let Z= $\{a, d\} \subseteq W$. Then $\tau_{R''}(W) = \{W, \emptyset, \{d\}, \{a, c\}, \{a, c, d\}\}$ be nano topologies on U, $V$and W respectively. Define $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ by $f(x) = d, f(y) = b, f(z) = a, f(w) = c$ and $g: (V, \tau_{R'}(X)) \longrightarrow (W, \tau_{R''}(Y))$by $g(a) = c, g(b) = b, g(c) = d, g(d) = a$. Clearly f and g are Ngδs_homeomorphism. But the composition $(g \circ f)$ is not Ngδs_homeomorphism, because for NO set $\{d\}$ in W, $(g \circ f)^{-1}(\{d\}) = f^{-1}(g^{-1}(\{d\})) = f^{-1}(\{c\}) = \{W\}$, is not Ng$\delta$s_open in U.

2.26 Definition- **A** map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is said to be nano generalized delta semi irresolute (briefly, Ngδs_irresolute) if $f^{-1}(P)$ is Ngδs_open in U for every Ngδs_open P in V.

2.27 Example- Let $P = \{a, b, c, d\}$with $P/R = \{\{b\}, \{d\}, \{a, c\}\}$. Let X= $\{a, d\} \subseteq P$. Then $\tau_R(X) = \{P, \emptyset, \{d\}, \{a, c\}, \{a, c, d\}\}$. Let V= $\{a, b, c, d\}$ with $V/R' = \{\{a\}, \{c\}, \{b, d\}\}$. Let $Y = \{a, b\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$be nano topologies on U and V respectively. Define $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$by $f(a) = d, f(b) = c, f(c) = b, f(d) = a$.then $f^{-1}(V) = U, f^{-1}(\emptyset) = \emptyset, f^{-1}(\{a\}) = \{d\}, f^{-1}(\{b\}) = \{c\}, f^{-1}(\{d\}) = \{a\}, f^{-1}\{a, b\} = \{c, d\}, f^{-1}\{a, c\} = \{b, d\}, f^{-1}\{a, d\} = \{a, d\}, f^{-1}\{b, d\} = \{a, c\}, f^{-1}\{(a, b, c)\} = \{b, c, d\}, f^{-1}(\{a, b, d\}) = \{a, c, d\}, f^{-1}(\{b, c, d\}) = \{a, b, c\}$
. Clearly inv img of every Ngδs_open in$V$is Ngδs_open in U. Therefore f is Ngδs_irresolute.

2.28 Remark -Every Ngδs_irresolute is Ngδs_cont., but converse need not be true.

2.29 Example-Let $U = \{x, y, z, w\}$ with $U/R = \{\{x\}, \{w\}, \{y, z\}\}$. Let $X = \{x, z\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{x\}, \{y, z\}, \{x, y, z\}\}$. Let $V = \{a, b, c, d\}$with $V/R' = \{\{a\}, \{c\}, \{b, d\}\}$. Let $Y = \{a, b\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ be a nano topologies on U and V respectively. Define $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$by $f(x) = d, f(y) = b, f(z) = a, f(w) = c$, then f is Ngδs_cont., but not Ngδs_irresolute, because for Ngδs_open set {a,c} in V, $f^{-1}(\{a, c\}) = \{z, w\}$ is not Ngδs_open in U.

2.30 Definition- A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is called strongly nano generalized delta semiclosed map (resp. strongly nano generalized delta semiopen map) if $f(P)$ is Ngδs_closed (resp. Ngδs_open) in V for every Ngδs_closed set (resp. Ngδs_open set) P of U. The strongly nano generalized delta semiclosed map (resp. strongly nano generalized delta semiopen map) is briefly written as SNgδs_closed map (resp. SNgδs_open map).

2.31 Example- Let $U = \{a, b, c, d\}$with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$. Let X= $\{a, d\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{d\}, \{a, c\}, \{a, c, d\}\}$. LetV= $\{a, b, c, d\}$with $V/R' = \{\{a\}, \{c\}, \{b, d\}\}$. Let $Y = \{a, b\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$be nano topologies on U and V respectively. Define $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$by $f(a) = b, f(b) = c, f(c) = d, f(d) = a$. Then clearly $f(P)$ is Ngδs_open in V for every Ngδs_open set P of U. Therefore f is SNgδs_open map.

2.32 Definition- A bijective function $f: (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is called strongly Ngδs_homeomorphism (briefly SNgδs_homeomorphism) if f is both Ngδs_irresolute and SNgδs_open map. Equivalently, if both f and f^{-1} are Ngδs_irresolute. We say, nano topological spaces U and V are SNgδs_homeomorphic if there exists a SNgδs_homeomorphisms from U onto V.

2.33 Remark -i) Composition of two Ngδs_homeomorphism functions is again SNgδs_homeomorphism.
ii) Every SNgδs_homeomorphism is gδs_homeomorphism. But converse is not true.

2.34 Example- Let $U = \{x, y, z, w\}$with $U/R = \{\{x\}, \{w\}, \{y, z\}\}$. Let $X = \{x, z\} \subseteq U$. Then $\tau_R(X) = \{U, \emptyset, \{x\}, \{y, z\}, \{x, y, z\}\}$. Let $V = \{a, b, c, d\}$with $V/R' = \{\{a\}, \{c\}, \{b, d\}\}$. Let $Y = \{a, b\} \subseteq V$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ be a nano topologies on U and V respectively. Define

$f:(U,\tau_R(X)) \longrightarrow (V,\tau_{R'}(Y))$by $f(x)=d, f(y)=b,\ f(z)=a,\ f(w)=c$, then$f$ is Ngδs_homeomorphism. But not SNgδs_homeomorphism because for Ngδs_open set {c} in$V$, $f^{-1}(\{a,c\})=\{z,w\}$ is not Ngδs_open in U.

III. Application

In this section we apply the concept of Ngδs_irresolue functions in day to day problem.

3.1 Example- Consider the cost of a taxi ride as a function of distance travelled. Let $U=\{p_1,p_2,p_3,p_4,p_5\}$ be the universe of distances of five different places from bus station and $V=\{a,b,c,d,e\}$ be the universe of taxi fares to reach the five destinations in U from the bus station. We know that fares depend on the distances of places. Let $U/R\ =\{\{p_4\},\{p_1,p_2\},\{p_3,p_5\}\}$ and let $X=\{p_1,p_4\}$, a subset of U. Then$\tau_R(X)=\{U,\emptyset\{p_4\},\ \{p_1,p_2\},\{p_1,p_2,p_4\}\}$ is nano topology on U. Let $V/R'\ =\{\{d\},\{a,b\},\{c,e\}\}$ and let $Y=\{a,b\}$, a subset of V. Then $\tau_{R'}(Y)=\{V,\emptyset,\ \{d\},\ \{a,b\},\{a,b,d\}\}$ is nano topology on V. Define $f:(U,\tau_R(X)) \longrightarrow (V,\tau_{R'}(Y))$by $f(p_1)=a, f(p_2)=b,\ f(p_3)=c,\ f(p_4)=d$. Then inv img of each Ngδs_open set in V is Ngδs_open in U. Therefore f is Ngδs_irresolute function. Also we note that the img of each Ngδs_open set in V is Ngδs_open in U. Therefore f is Ngδs_open in V and f is bijection. Thus f is SNgδs_homeomorphism. Therefore, cost of taxi ride, as a distance travelled is SNgδs_homeomorphism.

3.2 Example- Consider the anti-viruses as a treatment for the disease causing viruses. Let $V\ =\ \{p1,p2,p3,p4,p5,p6\}$be the universe of viruses of three diseases namely Flu-virus,Polio-virus and Hepatitis-virus. In the sequel, p1 − A=H5N1Flu; p2 − A=HN1 − Flu;p3− Polio; p4 − Hepatitis − C; p5 − Hepatitis − B; p6 − Hepatitis − D and let $A\ =\ \{y1,y2,y3,y4,y5,y6\}$be the universe of anti-viruses for three diseases namely Flu-virus, Polio-virus and Hepatitis-virus are y1 as Arbidol, y2 as Amantadine, y3 as Sabin, y4 as Interferon, y5 as Rebetol, y6 as Alpha-Interferon. We know that antivirus depends on the disease causing virus. Let $U/R=\{\{p1,p2,p3\},\{p4;\ p5;\ p6\}\}$and let $X\ =\ \{p1,p2;\ p3\}\ \subseteq\ V$. Then $\tau_R(X)=\ \{U,\emptyset,\{p1,p2,p3\}\}$. Then the NC sets are $V;\ \emptyset;\ \{v4;\ v5;\ v6\}$. Let $U/R'=\ \{\{y1,y2\},\{y3\},\{y4,y5,y6\}\}$. Let $Y\ =\ \{y4,y5,y6\}\ \subseteq\ A$. Then$\tau_{R'}(Y)\ \ =\ \{A,\emptyset,\{y4,y5,y6\}\}$. Define $f:(V,\tau_R(X)) \longrightarrow (A,\tau_{R'}(Y))$as $f(p1)\ =\ y1, f(p2)\ =\ y2, f(p3)\ =\ y3, f(p4)\ =\ y4, f(p5)\ =\ y5, f(p6)\ =\ y6$. Then inv img of each Ngδs_open set in V is Ngδs_open in U. Therefore f is Ngδs_irresolute function. Also we note that the img of each Ngδs_open set in V is Ngδs_open in U. Therefore f is Ngδs_open in V and f is bijection. Thus f is SNgδs_homeomorphism. Therefore, the anti-viruses as a function of treatment for disease causing viruses is SNgδs_homeomorphism.

IV. RESULTS

From the above discussion we have the following table which gives the relationship between different types of functions. The symbol "T" in a cell means that a function corresponding row implies a function on corresponding column. The symbol "F" means that a function on the corresponding row does not imply a function on the corresponding column.

1. Results on Nano cont. functions

Functions	1	2	3	4
1	T	T	T	T
2	F	T	T	T
3	F	F	T	T
4	F	F	F	T

(1) Nano cont. (2) Nano semi cont.
(3) Ngδs_cont. (4) Ngδs_irresolute

2. Results on NO maps

Maps	1	2	3	4
1	T	T	T	T

2	F	T	T	T
3	F	F	T	T
4	F	F	F	T

(1)Nano open (NO) (2) Nanosemiopen
(3) Ngδs_open (4) SNgδs_open

V.CONCLUSION

In this paper we introduced Nano generalized δ semi_cont. , Nano generalized δ semiopen maps, Nano generalized δ semiclosed homeomorphism and Nano generalized δ semiclosed irresolute function in nano topological spaces and discussed the relationship between different types of functions.

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