Cotangent Similarity Measures of Interval Rough Fuzzy Sets and Their Applications

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Abstract - In this paper, we expose an interval rough fuzzy cotangent similarity measure and weighted interval rough fuzzy cotangent similarity measure. Then the proposed methods are applied for solving multi attribute decision making problems. Finally, a numerical example is solved to show the feasibility, applicability and effectiveness of the proposed strategies.

Keywords - Cotangent similarity measure, Interval fuzzy sets, rough interval fuzzy sets, MADM problems.

I.INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [6] in his classic paper in 1965 and has been applied to many branches in mathematics. Later on Zadeh[5] also introduced the concept of interval valued fuzzy set by considering the values of membership functions as the intervals of numbers instead of the numbers alone. The notion of rough set theory was proposed by Z.Pawlak[4]. The concept of rough set theory is an extension of crisp set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. The notion of rough fuzzy sets and fuzzy sets are studied by Dubois and Prade[1]. In recent years the concept of rough fuzzy set was found to be more useful in decision making and medical diagnosis problems. A similarity measure is an important tool for determining the degree of similarity between two objects. Measures of similarity between fuzzy sets as an important content in fuzzy mathematics.

II.PRELIMINARIES

2.1.Definition Let X be a nonempty set. A mapping $\widetilde{\Lambda}: x \to D[0,1]$ is called an interval fuzzy subset of X, where $\widetilde{\Lambda}(x) = [\Lambda^-(x), \Lambda^+(x)], x \in X,$ and Λ^- and Λ^+ are the fuzzy subets in X such that $\Lambda^-(x) \leq \Lambda^+(x)\ x \in X.\ D[0,1]$ denotes the set of closed subsets of [0,1].

2.2.Definition Let R be an congruence relation on X. Let Λ be any nonempty subset of X. The sets $\underline{R}(\Lambda) = \{x \in X/[x]_R \subseteq \Lambda\}$ and $\overline{R}(\Lambda) = \{x \in X/[x]_R \cap R \neq \emptyset\}$ are called the lower and upper approximations of Λ. Then $R(\Lambda) = (\underline{R}(\Lambda), \overline{R}(\Lambda))$ is called rough set in $(X, R) \Leftrightarrow \underline{R}(\Lambda) \neq \overline{R}(\Lambda)$.

2.3.Definition Let R be an congruence relation on X. Let Λ fuzzy subset of X. The upper and lower approximations of Λ defined by $\overline{R}(\Lambda)(x) = \underset{a\in[x]_R}{\vee} \Lambda(a)$ and $\underline{R}(\Lambda)(x) = \underset{a\in[x]_R}{\wedge} \Lambda(a)$. $R(\Lambda) = (\underline{R}(\Lambda), \overline{R}(\Lambda))$ is called a rough fuzzy set of Λ with respect to R if $\underline{R}(\Lambda) \neq \overline{R}(\Lambda)$.

2.4.Definition Let $\widetilde{\Lambda}$ be an interval fuzzy subset of X and let R be the complete congruence relation on X. Let $\underline{R}(\widetilde{\Lambda})$ and $\overline{R}(\widetilde{\Lambda})$ be the interval valued fuzzy subset of X defined by,

$$\underline{R}(\widetilde{\Lambda})(x) = [\wedge \Lambda^{-}(y), \wedge \Lambda^{+}(y); y \in [x]_R]$$

$$\overline{R}(\widetilde{\Lambda})(x) = [\vee \Lambda^{-}(y), \vee \Lambda^{+}(y); y \in [x]_R]$$

Then $R(\widetilde{\Lambda}) = \left(\underline{R}(\widetilde{\Lambda}), \overline{R}(\widetilde{\Lambda})\right)$ is called an interval rough fuzzy subset of X if $\underline{R}(\widetilde{\Lambda}) \neq \overline{R}(\widetilde{\Lambda})$.

III. COTANGENT SIMILARITY MEASURE OF INTERVAL ROUGH FUZZY SET

In this section, we discuss cotangent similarity between two interval rough fuzzy sets. Also, discuss properties of proposed method.

3.1.Definition Let $R(\widetilde{\Lambda_1}) = \left(\underline{R}(\widetilde{\Lambda_1})(x_i), \overline{R}(\widetilde{\Lambda_1})(x_i)\right)$ and $R(\widetilde{\Lambda_2}) = (\underline{R}(\widetilde{\Lambda_2})(x_i), \overline{R}(\widetilde{\Lambda_2})(x_i))$ be two interval rough fuzzy sets in $X = \{x_1, x_2 \dots\dots x_n\}$. Then we define the cotangent similarity between $R(\widetilde{\Lambda_1})$ and $R(\widetilde{\Lambda_2})$as follows,

$$COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right) = \frac{1}{n}\sum_{i=1}^{n} \cot\left(\frac{\pi}{4} + \frac{\pi}{4}\left(\left|\delta R(\widetilde{\Lambda_1})(x_i) - \delta R(\widetilde{\Lambda_2})(x_i)\right|\right)\right) \text{---------(1)}$$

where

$$\delta R(\widetilde{\Lambda_1})(x_i) = \left(\frac{\underline{R}(\widetilde{\Lambda_1})(x_i) + \overline{R}(\widetilde{\Lambda_1})(x_i)}{2}\right) \text{ and } \delta R(\widetilde{\Lambda_2})(x_i) = \left(\frac{\underline{R}(\widetilde{\Lambda_2})(x_i) + \overline{R}(\widetilde{\Lambda_2})(x_i)}{2}\right)$$

3.2.Proposition Let X be a nonempty set. The cotangent similarity measure $COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right)$ between $R(\widetilde{\Lambda_1})$ and $R(\widetilde{\Lambda_2})$ satisfies the following properties:

1. $0 \leq COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right) \leq 1$
2. $COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right) = 1 \Leftrightarrow R(\widetilde{\Lambda_1}) = R(\widetilde{\Lambda_2})$
3. $COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right) = COT_{IRF}\left(R(\widetilde{\Lambda_2}), R(\widetilde{\Lambda_1})\right)$
4. If $R(\widetilde{\Lambda_3})$ is a interval rough fuzzy set and $R(\widetilde{\Lambda_1}) \subset R(\widetilde{\Lambda_2}) \subset R(\widetilde{\Lambda_3})$ then, $COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_3})\right) \leq COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right)$ and $COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_3})\right) \leq COT_{IRF}\left(R(\widetilde{\Lambda_2}), R(\widetilde{\Lambda_3})\right)$.

Proof:

1. Since, $\frac{\pi}{4} \leq \left(\frac{\pi}{4} + \frac{\pi}{4}\left(\left|\delta R(\widetilde{\Lambda_1})(x_i) - \delta R(\widetilde{\Lambda_2})(x_i)\right|\right)\right) \leq \frac{\pi}{2}$ so that cotangent function $COT_{IRF}(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2}))$ are with in 0 and 1.
2. If $R(\widetilde{\Lambda_1}) = R(\widetilde{\Lambda_2})$ then $COT_{IRF}(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})) = 1$. Conversely if $COT_{IRF}(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})) = 1$ then $R(\widetilde{\Lambda_1})(x_i) = \delta R(\widetilde{\Lambda_2})(x_i)$ implies $\underline{R}(\widetilde{\Lambda_1})(x_i) = \underline{R}(\widetilde{\Lambda_2})(x_i)$ and $\overline{R}(\widetilde{\Lambda_1})(x_i) = \overline{R}(\widetilde{\Lambda_2})(x_i)$. Hence $R(\widetilde{\Lambda_1}) = R(\widetilde{\Lambda_2})$.
3. It is obvious
4. Assume that $R(\widetilde{\Lambda_1}) \subset R(\widetilde{\Lambda_2}) \subset R(\widetilde{\Lambda_3})$then $\underline{R}(\widetilde{\Lambda_1})(x_i) \leq \underline{R}(\widetilde{\Lambda_2})(x_i) \leq \underline{R}(\widetilde{\Lambda_3})(x_i), \overline{R}(\widetilde{\Lambda_1})(x_i) \leq \overline{R}(\widetilde{\Lambda_2})(x_i) \leq \overline{R}(\widetilde{\Lambda_3})(x_i)$. The cotangent function is decreasing function within the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$. Hence we can write $COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_3})\right) \leq COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2}\right)$ and $COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_3})\right) \leq COT_{IRF}\left(R(\widetilde{\Lambda_2}), R(\widetilde{\Lambda_3})\right)$.

3.1 Weighted cotangent similarity measure of interval rough fuzzy set-

If we consider weights of each elements as x_i, a weighted interval rough fuzzy cotangent similarity

measure between two interval rough fuzzy sets $R(\widetilde{\Lambda_1})$ and $R(\widetilde{\Lambda_2})$ can be defined as follows:
$COT_{WIRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right) = \frac{1}{n}\sum_{i=1}^{n} w_i \cot\left(\frac{\pi}{4} + \frac{\pi}{4}\left(\left|\delta R(\widetilde{\Lambda_1})(x_i) - \delta R(\widetilde{\Lambda_2})(x_i)\right|\right)\right)$----(2)
$w_i \in [0,1], i = 1,2,3 \ldots n$ and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}, i = 1,2, \ldots . n$ then

$COT_{WIRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right) = COT_{IRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right)$.

3.1.1.Proposition The weighted cotangent similarity measure between two interval rough fuzzy sets $R(\widetilde{\Lambda_1})$ and $R(\widetilde{\Lambda_2})$ satisfies the following properties:

1. $0 \leq COT_{WIRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right) \leq 1$
2. $COT_{WIRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right) = 1 \Leftrightarrow R(\widetilde{\Lambda_1}) = R(\widetilde{\Lambda_2})$
3. $COT_{WIRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right) = COT_{WIRF}\left(R(\widetilde{\Lambda_2}), R(\widetilde{\Lambda_1})\right)$
4. If $R(\widetilde{\Lambda_3})$ is a weighted interval rough fuzzy set and $R(\widetilde{\Lambda_1}) \subset R(\widetilde{\Lambda_2}) \subset R(\widetilde{\Lambda_3})$ then, $COT_{WIRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_3})\right) \leq COT_{WIRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_2})\right)$ and $COT_{WIRF}\left(R(\widetilde{\Lambda_1}), R(\widetilde{\Lambda_3})\right) \leq COT_{WIRF}\left(R(\widetilde{\Lambda_2}), R(\widetilde{\Lambda_3})\right)$.

3.2. Algorithm for weighted interval rough fuzzy cotangent similarity measure-

In this section we present an algorithm of weighted interval rough fuzzy similarity measure in interval rough fuzzy environment to diagnosis disease of patient.
Let $\mathcal{S} = \{\mathcal{S}_1, \mathcal{S}_2, \ldots \ldots . \mathcal{S}_r\}$ be set of symptoms and $T = \{\mathcal{T}_1, \mathcal{T}_2, \ldots \ldots . \mathcal{T}_n\}$ be set of periods and $D = \{D_1, D_2, \ldots \ldots . D_p\}$ be set of disease.
For a patient P with various symptoms, we give relation between a patient and symptoms in Table1 and relation between symptoms and disease in Table 2.

Table 1

	$\mathcal{T}_k$	$\mathcal{S}_1$	$\mathcal{S}_2$	…	$\mathcal{S}_r$
	$\mathcal{T}_1$	$\mathcal{V}_1(\mathcal{T}_1)$	$\mathcal{V}_2(\mathcal{T}_1)$		$\mathcal{V}_r(\mathcal{T}_1)$
$\mathcal{P}$	$\mathcal{T}_2$	$\mathcal{V}_1(\mathcal{T}_2)$	$\mathcal{V}_2(\mathcal{T}_2)$		$\mathcal{V}_r(\mathcal{T}_2)$
	…	…..	…..		….
	$\mathcal{T}_n$	$\mathcal{V}_1(\mathcal{T}_n)$	$\mathcal{V}_2(\mathcal{T}_n)$		$\mathcal{V}_r(\mathcal{T}_n)$

In the above table $V_j(\mathcal{T}_k)$ denotes the values between patient P and j th symptom $S_j (j = 1,2, \ldots r)$ in the k th period $\mathcal{T}_k$ for k=1,2,.n.

Table 2

	S_1	S_2	…..	S_r
D_1	V_{11}	V_{12}	..	V_{1r}
D_2	V_{21}	V_{22}	..	V_{2r}
..				
D_P	V_{p1}	V_{p2}	…..	V_{pr}

From the above table V_{ij}denotes the relation between the j th symptom $S_j (j = 1,2 \ldots r)$ and disease $D_i (i = 1,2..p)$. For a multi-period diagnosis with interval rough fuzzy sets, the relation between a patient and symptoms, symptoms and a disease are denoted by the form of interval rough fuzzy value for convenience. Let us take the weight vector of the symptoms as $\omega = (\omega_1, \omega_2 \ldots \ldots \omega_n)^T$with $\omega_j \in [0,1]$ and the weight vector of each period is $\omega = (\omega(T_1), \omega(T_2) \ldots \ldots \omega(T_n))$ with $\omega(T_k) \in [0,1]$ and $\sum_{k=1}^{n} \omega(T_k) = 1$.

The steps are given by:

1. Calculate the similarity measure between patient and disease $D_i(i = 1,2..p)$ in each period (k=1,2,..q) by using the formula:
   $COT_{WIRF}(\widetilde{\Lambda_1},\widetilde{\Lambda_2}) = \frac{1}{n}\sum_{i=1}^{n} w_i \cot\left(\frac{\pi}{4} + \frac{\pi}{4}\left(\left|\delta\widetilde{\Lambda_1}(x_i) - \delta\widetilde{\Lambda_2}(x_i)\right|\right)\right)$.
2. Obtain the weighted measure $M_i\ for\ i = 1,2,..n$ by the formula
   $M_i = \sum_{k=1}^{n} W_i(T_k)\omega(T_k)$.
3. Rank all the weighted measure of M_i for P with respect to $D_i(i = 1,2..n)$ in a decreasing order.
4. Give a proper diagnosis according to the maximum weighted measure value.

IV. ILLUSTRATIVE EXAMPLE

In this section we present an example for cotangent similarity measure and weighted similarity measure in interval rough fuzzy environment.

4.1 Example of interval rough fuzzy cotangent similarity measure-

We consider a problem of buying the best branded smart phone. This is used to communicate over long distances. Mobiles provide us the best way of communication in our life. Mobile phones make our life easy and convenient. We can also surf the internet using the phone. Most importantly we can take photograph and record videos. It can be used to manage our important documents. Since there are many advantages and disadvantages. It is a difficult task to buy the best branded mobile. Here we consider the interval rough fuzzy cotangent similarity measure as a suitable tool for the decision making. The proposed similarity measure among the buyers and brands will provide the proper decision. Now we present the example as follows. Let $U = \{U_1, U_2, U_3\}$ be a set of buyers and $F = \{F_1, F_2, F_3, F_4, F_5\}$ be a set of features and $B = \{B_1, B_2, B_3, B_4, B_5\}$ be a set of brands. Our aim is to determine the buyers and the best brand in interval rough fuzzy set environment.

Table 3 Relation 1

	Battery	**Cost**	**RAM**	**ROM**	**Camera**
U_1	**([.10,.40], [.50,.60])**	**([.03,.40],[.65,.70])**	**([.43,.55],[.80,.90])**	**([.30,.40],[.50,.70])**	**([.20,.30],[.40,.70])**
U_2	**([.40,.50],[.40,.60])**	**([.20,.70],[.40,.90])**	**([.40,.50],[.71,.79])**	**([.16,.23],[.42,.51])**	**([.10,.12],[.19,.23])**
U_3	**([.39,.44],[.42,.51])**	**([.40,.50],[.70,.80])**	**([.50,.70],[.60,.80])**	**([.16,.37],[.24,.35])**	**([.03,.05],[.30,.40])**
U_4	**([.30,.45],[.46,.50])**	**([.50,.60],[.65,.70])**	**([.55,.61],[.60,.70])**	**([.22,.36],[.33,.51])**	**([.13,.31],[.37,.49])**
U_5	**([.40,.60],[.40,.80])**	**([.12,.50],[.26,.60])**	**([.40,.50],[.60,.70])**	**([.10,.30],[.20,.40])**	**([.11,.21],[.33,.49])**

Table 4 Relation 2

	B_1	B_2	B_3	B_4	B_5
Battery	**([.50,.40],[.75,.80])**	**([.20,.50],[.60,.40])**	**([.15,.70],[.15,.70])**	**([.40,.60],[.40,.70])**	**([.03,.20],[.80,.90])**
Cost	**([.60,.70],[.80,.90])**	**([.30,.50],[.40,.90])**	**([.05,.70],[.1,.75])**	**([.20,.40],[.40,.50])**	**([.06,.05],[.69,.80])**
RAM	**([.61,.78],[.69,.81])**	**([.27,.60],[.40,.71])**	**([.02,.14],[.20,.19])**	**([.26,.42],[.54,.70])**	**([.25,.48],[.80,.95])**
ROM	**([.32,.44],[.48,.50])**	**([.26,.36],[.40,.52])**	**([.20,.50],[.30,.71])**	**([.53,.63],[.65,.75])**	**([.33,.52],[.40,.60])**
Camera	**([.04,.25],[.41,.50])**	**([.19,.31],[.49,.59])**	**([.10,.30],[.40,.40])**	**([.22,.31],[.53,.61])**	**([.31,.80],[.40,.90])**

Table 5 Relation between the buyers and brands

	B_1	B_2	B_3	B_4	B_5
U_1	0.6184	0.8383	0.8593	0.7051	0.7794
U_2	0.6333	0.7551	0.5786	0.5603	0.7117
U_3	0.6568	0.6376	0.5837	0.4491	0.5848
U_4	0.6751	0.8061	0.6463	0.5823	0.7250
U_5	0.8226	0.4677	0.4471	0.7959	0.8886

From above table we see that $U_1$and U_2select $B_2$and U_3,U_4, U_5 select B_1, B_2, B_5 respectively.

4.2 Example of weighted interval cotangent similarity measure-

Let D = {Malaria, Typhoid, Viral fever, Gastritis} be the disease and S = {Temperature, Headache, Cough, Chest pain, Stomach pain} be the symptoms. The relation between symptoms and the disease are given by form of interval rough fuzzy set.

Table 6 Relation between disease and symptoms

	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_3$	$\mathcal{S}_4$	$\mathcal{S}_5$
$\mathcal{D}_1$	([.2,.4],[.6,.8])	([.3,.5],[.3,.5])	([.2,.3],[.4,.5])	([.3,.4],[.5,.6])	([.2,.4],[.3,.6])
$\mathcal{D}_2$	([.2,.5],[.4,.7])	([.3,.5],[.4,.6])	([.2,.2],[.4,.4])	([.2,.3],[.4,.5])	([.1,.2],[.3,.3])
$\mathcal{D}_3$	([.1,.2],[.3,.4])	([.1,.1],[.3,.3])	([.2,.4],[.4,.6])	([.1,.2],[.3,.6])	([.2,.4],[.4,.6])
$\mathcal{D}_4$	([.2,.7],[.6,.8])	([.1,.3],[.4,.5])	([.4,.6],[.5,.7])	([.2,.4],[.6,.6])	([.1,.4],[.3,.6])

Table 7 Relation between patient and symptoms

	T_k	S_1	S_2	S_3	S_4	S_5
P	T_1	([.1,.3], [.6,.8])	([.3,.4],[.5,.6])	([.2,.5],[.5,.7])	([.1,.3],[.6,.8])	([.2,.4],[.5,.5])
	T_2	([.1,.2],[.7,.9])	([.2,.4],[.6,.8])	([.6,.8],[.7,.9])	([.3,.5],[.4,.6])	([.1,.4],[.3,.6])

Let us consider weight vector of the five symptoms is $\omega = (\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5})$ and the weight vector of the two periods is $\omega = (.4,.6)$. Then we get the table which is a relation between the patient P and the disease D_i in each period T_k for i=1,2,3,4,5 and k=1,2.

Table 8

	T_1	T_2
$W_1(T_k)$	0.8601	0.9150
$W_2(T_k)$	0.7693	0.8054
$W_3(T_k)$	0.6648	0.7139
$W_4(T_k)$	0.7956	0.7592

calculate the weighted measure M_i for 1=1,2,3,4 for the period k=1,2. We get $M_1 > M_2 > M_4 > M_3$. The patient P suffers from Malaria according to maximum weighted measure value M_i .

V. CONCLUSION

In this paper, we have defined cotangent similarity measure of interval rough fuzzy sets. We have also proved their basic properties. We have developed MADM strategies based on the proposed measure.

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