WEAKLY γ GENERALIZED CONTINUOUS MAPPINGS AND PERFECTLY γ GENERALIZED CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce weakly γ generalized continuous mappings and perfectly γ generalized continuous mappings in intuitionistic fuzzy topological spaces. Furthermore we provide some properties of the same set and discuss some fascinating theorems.

Keywords: Intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy γ generalized closed sets, intuitionistic fuzzy γ generalized continuous mappings, intuitionistic fuzzy weakly γ generalized continuous mappings and perfectly γ generalized continuous mappings.

1. INTRODUCTION

Atanassov [1] introduced the idea of intuitionistic fuzzy sets using the notion of fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Later this was followed by the introduction of intuitionistic fuzzy γ generalized closed sets by Prema, S and Jayanthi, D [5] in 2017 which was simultaneously followed by the introduction of intuitionistic fuzzy γ generalized continuous mappings [6] by the same authors. We now extend our idea towards intuitionistic fuzzy weakly γ generalized continuous mappings and perfectly γ generalized continuous mappings and discuss some of their properties.

2. PRELIMINARIES

Definition 2.1 [1]: An ***intuitionistic fuzzy set*** (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$.

Definition 2.2 [1]: Let A and B be two IFSs of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle: x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle: x \in X\}$.

The intuitionistic fuzzy sets $0\sim = \langle x, 0, 1\rangle$ and $1\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3 [2]: An ***intuitionistic fuzzy topology*** (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0\sim, 1\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called ***intuitionistic fuzzy topological space*** (IFTS in short) and any IFS in τ is known as an ***intuitionistic fuzzy open set*** (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an ***intuitionistic fuzzy closed set*** (IFCS in short) in X.

Definition 2.4 [5]: An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy γ generalized closed set (IFγGCS for short) if γcl(A) ⊆ U whenever A ⊆ U and U is an IFγOS in (X, τ).

The complement A^c of an IFγGCS A in an IFTS (X, τ) is called an intuitionistic fuzzy γ generalized open set (IFγGOS for short) in X.

Definition 2.5 [6]: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy γ generalized continuous (IFγG continuous for short) mapping if $f^{-1}(V)$ is an IFγGCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.6 [7] : A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy almost γ generalized continuous*** (IFaγG continuous for short) **mapping** if $f^{-1}(A)$ is an IFγGCS in X for every IFRCS A in Y.

Definition 2.7 [8]: A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy completely γ generalized continuous*** (IF completely γG continuous for short) ***mapping*** if $f^{-1}(V)$ is an IFRCS in X for every IFγGCS V in Y.

3. WEAKLY γ GENERALIZED CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

In this section we have introduced intuitionistic fuzzy weakly γ generalized continuous mappings and studied some of their properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy weakly γ generalized continuous*** (IFWγG continuous for short) ***mapping*** if $f^{-1}(V) \subseteq \gamma gint(f^{-1}(cl(V)))$ for each IFOS V in Y.

Example 3.2: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩, G_2 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Then, IFγC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /$0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$},

IFγO(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /$0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$},

IFγC(Y) = {0~, 1~, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$ / $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$},

IFγO(Y) = {0~, 1~, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$ / $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$}.

The IFS G_2 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩ is an IFOS in Y. Now G_2^c = ⟨y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)⟩ is an IFCS in Y.

We have γgint(f^{-1}(cl(G_2))) = ⟨x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩. Hence $f^{-1}(G_2) \subseteq$ γgint(f^{-1}(cl(G_2))). Therefore f is an IFWγG continuous mapping.

Theorem 3.3: Every IFγG continuous mapping is an IFWγG continuous mapping in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFγG continuous mapping. Let V be any IFOS in Y. Then by hypothesis, $f^{-1}(V)$ is an IFγGOS in X. Therefore γgint(f^{-1}(V)) = $f^{-1}(V)$. Now $f^{-1}(V)$ = γgint(f^{-1}(V)) ⊆ γgint(f^{-1}(cl(V))). Hence f is an IFWγG continuous mapping.

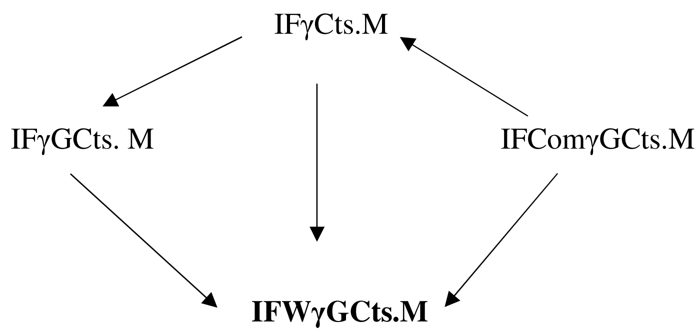
Theorem 3.4: Every IF completely γG continuous mapping is an IFWγG continuous mapping in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be any IFOS in Y. Since every IFOS is an IFγGOS, V is an IFγGOS. Then by hypothesis, $f^{-1}(V)$ is an IFROS in X and hence $f^{-1}(V)$ is an IFOS in X. Now as every IFOS is an IFγGOS, γgint(f^{-1}(V)) = $f^{-1}(V)$. Therefore $f^{-1}(V)$ = γgint(f^{-1}(V)) ⊆ γgint(f^{-1}(cl(V))).

Theorem 3.5: Every IFγ continuous mapping [4] is an IFWγG continuous mapping in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFγ continuous mapping. Since every IFγ continuous mapping is an IFγG continuous mapping [6], by Theorem 3.3, f is an IFWγG continuous mapping.

The relation between various types of intuitionistic fuzzy continuity is given in the following examples. In this diagram 'Cts.M' means continuous mapping.



The reverse implications are not true in general in the above diagram.

Theorem 3.6: For a mapping f: (X, τ) → (Y, σ), the following are equivalent:

(i) f is an IFWγG continuous mapping

(ii) $f^{-1}(int(A)) \subseteq \gamma gint(f^{-1}(A))$ for each IFCS A in Y

(iii) $f^{-1}(int(cl(A))) \subseteq \gamma gint(f^{-1}(cl(A)))$ for each IFOS A in Y

(iv) $f^{-1}(A) \subseteq \gamma gint(f^{-1}(cl(A)))$ for each IFPOS A in Y

(v) $f^{-1}(A) \subseteq \gamma gint(f^{-1}(cl(A)))$ for each IFαOS A in Y

Proof: (i) ⇒ (ii) Let A ⊆ Y be an IFCS. Then int(A) = int(cl(A)), which is an IFROS. Hence int(A) is an IFOS in Y. Therefore by (i) $f^{-1}(int(A)) \subseteq \gamma gint(f^{-1}(cl(int(A)))) \subseteq \gamma gint(f^{-1}(cl(A)))$, as cl(A) = A. Hence $f^{-1}(int(A)) \subseteq \gamma gint(f^{-1}(A))$.

(ii) ⇒ (iii) Let A ⊆ Y be an IFOS. Then int(A) = A. Now cl(A) = cl(int(A)), which is an IFRCS. Therefore cl(A) is an IFCS in Y. By (ii) $f^{-1}(int(cl(A))) \subseteq \gamma gint(f^{-1}(cl(A)))$.

(iii) ⇒ (iv) Let A be an IFPOS in Y. Then A ⊆ int(cl(A)). Now f^{-1} (A) ⊆ f^{-1} (int(cl(A))). Since int(cl(A)) is an IFROS, it is an IFOS. Hence (iii) implies f^{-1} (int(cl(int(cl(A))))) ⊆ γgint(f^{-1}(cl(int(cl(A))))) ⊆ γgint(f^{-1}(cl(A))). That is f^{-1} (int(cl(A))) ⊆ γgint(f^{-1} (cl(A))). Thus f^{-1} (A) ⊆ γgint(f^{-1} (cl(A))).

(iv) ⇒ (v) is obvious, since every IFαOS is an IFPOS.

(v) ⇒ (i) Let A ⊆ Y be an IFOS. Since every IFOS is an IFαOS, A is an IFαOS. Hence by (v), f^{-1} (A) ⊆ γgint(f^{-1} (cl(A))). Therefore f is an IFWγGCts.M.

Theorem 3.7: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then the following are equivalent:

(i) f is an IFWγG continuous mapping

(ii) γgcl(f^{-1}(A)) ⊆ f^{-1} (cl(A)) for each IFOS A in Y

(iii) γgcl(f^{-1}(int(A))) ⊆ f^{-1} (A) for each IFCS A in Y

Proof: (i) ⇒ (iii) Let A ⊆ Y be an IFCS. A^c is an IFOS in Y. By (i) f^{-1} (A^c) ⊆ γgint(f^{-1} (cl(A^c))). This implies (f^{-1} (A))^c ⊆ (γgcl(f^{-1} (int(A))))^c . Thus γgcl(f^{-1}(int(A))) ⊆ f^{-1} (A).

(iii) ⇒ (i) is obvious.

(ii) ⇒ (iii) Let A ⊆ Y be an IFCS. Then int(A) is an IFOS in Y. By (ii), γgcl(f^{-1}(int(A))) ⊆ f^{-1} (cl(int(A))) ⊆ f^{-1} (cl(A)) = f^{-1} (A), since cl(A) = A. Hence γgcl(f^{-1}(int(A))) ⊆ f^{-1} (A).

(iii) ⇒ (ii) Let A ⊆ Y be an IFOS, then int(A) = A and cl(A) is an IFCS. By (iii), γgcl(f^{-1}(int(cl(A)))) ⊆ f^{-1} (cl(A)). Now γgcl(f^{-1}(A)) = γgcl(f^{-1}(int(A))) ⊆ γgcl(f^{-1}(int(cl(A)))) ⊆ f^{-1} (cl(A)). Hence γgcl(f^{-1}(A)) ⊆ f^{-1} (cl(A)).

4. PERFECTLY γ GENERALIZED CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

In this section we have introduced intuitionistic fuzzy perfectly γgeneralized continuous mappings and studied some of their properties.

Definition 4.1: A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy perfectly γ generalized continuous*** (IFpγG continuous for short) ***mapping*** if f^{-1}(A) is clopen in (X, τ) for every IFγGCS A of (Y, σ).

Theorem 4.2: Every IFpγG continuous mapping is an IF almost γG continuous mapping in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpγG continuous mapping. Let A be an IFRCS in Y. Since every IFRCS is an IFγGCS [5], A is an IFγGCS in Y. By hypothesis, f^{-1} (A) is an intuitionistic fuzzy clopen in X. Thus f^{-1} (A) is an IFCS in X. Since every IFCS is an IFγGCS, f^{-1} (A) is an IFγGCS in X. Hence f is an IF almost γG continuous mapping.

Theorem 4.3: Every IFpγG continuous mapping is an completely γG continuous mapping in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFpγG continuous mapping. Let A be an IFγGCS in Y. By hypothesis, f^{-1} (A) is an intuitionistic fuzzy clopen in X. Thus f^{-1} (A) is an IFCS in X. Since every IFCS is an IFRCS, f^{-1} (A) is an IFRCS in X. Hence f is an IF completely γG continuous mapping.

Theorem 4.4: A mapping f: (X, τ) → (Y, σ) is an IFpγG continuous mapping if and only if the inverse image of each IFγGOS in Y is an intuitionistic fuzzy clopen in X.

Proof: Necessity: Let a mapping f: (X, τ) → (Y, σ) be IFpγG continuous mapping. Let A be an IFγGOS in Y. Then A^c is an IFγGCS in Y. Since f is an IFpγG continuous mapping, $f^{-1}(A^c)$ is IF clopen in X as $f^{-1}(A^c) = (f^{-1}(A))^c$. This implies f^{-1} (A) is IF clopen in X.

Sufficiency : Let B be an IFγGCS in Y. Then B^c is an IFγGOS in Y. By hypothesis, $f^{-1}(B^c)$ is IF clopen in X. Which implies f^{-1} (B) is IF clopen in X, as $f^{-1}(B^c) = (f^{-1}(B))^c$. Therefore f is an IFpγG continuous mapping.

Theorem 4.5: Let f: (X, τ) → (Y, σ) be an IF continuous mapping and g: (Y, σ) → (Z, δ) is IFpγG continuous mapping, then g∘ f : (X, τ) → (Z, δ) is an IFpγG continuous mapping.

Proof: Let A be an IFγGCS in Z. Since g is an IFpγG continuous mapping, g^{-1} (A) is an IF clopen in Y. Since f is an IF continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFCS in X, as well as IFOS in X. Hence g∘f is an IFpγG continuous mapping.

Theorem 4.6: The composition of two IFpγG continuous mapping is an IFpγG continuous mapping in general.

Proof: Let f: X → Y and g: Y → Z be any two IFpγG continuous mappings. Let A be an IFγGCS in Z. By hypothesis, $g^{-1}(A)$ is IF clopen in Y and hence IFCS in Y. Since every IFCS is an IFγGCS, $g^{-1}(A)$ is an IFγGCS in Y. Further, since f is an IFpγG continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is IF clopen in X. Hence $g \circ f$ is an IFpγG continuous mapping.

Theorem 4.7: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent:

(i) f is an IFpγG continuous mapping

(ii) If A is an IFγGCS in Y then $f^{-1}(A)$ is an IF clopen in X

(iii) $int(f^{-1}(\gamma gcl(B))) = f^{-1}(\gamma gcl(B)) = cl(f^{-1}(\gamma gcl(B)))$ for every B in Y

Proof:

(i) ⇒ (ii): is obviously true.

(ii) ⇒ (iii): Let B be any IFS in Y. Then γgcl(B)) is an IFγGCS in Y.By hypothesis, $f^{-1}(\gamma gcl(B))$ is an IF clopen in X. Thus $f^{-1}(\gamma gcl(B))$ is IFCS in X. Hence $f^{-1}(\gamma gcl(B))$ is an IFOS in X. That is $int(f^{-1}(\gamma gcl(B))) = f^{-1}(\gamma gcl(B))$. Also $f^{-1}(\gamma gcl(B))$ is an IFCS in X. That is $cl(f^{-1}(\gamma gcl(B))) = f^{-1}(\gamma gcl(B))$. Hence $int(f^{-1}(\gamma gcl(B))) = f^{-1}(\gamma gcl(B)) = cl(f^{-1}(\gamma gcl(B)))$ for every B in Y.

(iii) ⇒ (i): Let B be any IFS in Y. Then γgcl(B)) is an IFγGCS in Y. By hypothesis, $int(f^{-1}(\gamma gcl(B))) = f^{-1}(\gamma gcl(B)) = cl(f^{-1}(\gamma gcl(B)))$. This implies $f^{-1}(\gamma gcl(B))$ is IFOS in X and also an IFCS in X. That is $f^{-1}(\gamma gcl(B))$ is an IF clopen in X. Hence f is an IFpγG continuous mapping.

Theorem 4.8: Let f: (X, τ) → (Y, σ) be an IFpγG continuous mapping and g: (Y, σ) → (Z, δ) is an IFγG irresolute mapping [9], then $g \circ f$: (X, τ) → (Z, δ) is an IFpγG continuous mapping.

Proof: Let A be an IFγGCS in Z. By hypothesis, g^{-1} (A) is an IFγGCS in Y. Since f is an IFpγG continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}$ (A) is IF clopen in X. Hence $g \circ f$ is an IFpγG continuous mapping.

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