Multiple Variables Effect on MHD Jeffrey Fluid Flow past a Vertical Plate Embedded In Porous Medium

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Abstract - In this manuscript, a study of the unsteady magneto-hydrodynamic mixed convection Jeffrey fluid flow over an inclined permeable moving plate in presence of thermal radiation, heat generation, thermophoresis effect and homogenous chemical reaction, subjected to variable suction has been undertaken. The governing equations are solved by using a regular perturbation technique. The expressions for the distributions of velocity, temperature and species concentration are obtained. With the aid of these, the expressions for skin friction, Nusselt number and Sherwood number also have been derived. The influences of various physical Parameters involved in the problem on the above-mentioned quantities are discussed with the help of graphs and tables. From the significant findings, it has been found that the velocity increases with an increase in Jeffrey fluid in the presence of permeability. It shows reverse effects in the case of a magnetic parameter, radiation parameter and chemical reaction parameter.

Keywords – Jeffrey fluid, MHD, Thermal diffusion, Chemical reaction, Radiation, Rarefaction Parameter and Heat source parameter

I. Introduction

MHD is the science of the motion of electrically conducting fluids in the presence of a magnetic field. It concerns with the interaction of the magnetic field with the fluid velocity of electrically conducting fluid. MHD generators, MHD pumps and MHD flow meters are some of the numerous examples of MHD principles. Dynamo and motor are classical examples of MHD principle. Convection problems of electrically conducting fluid in the presence of magnetic field have got much importance because of its wide applications in Geophysics, Astrophysics, Plasma Physics, Missile technology, etc. MHD principles also find its applications in Medicine and Biology. Magnetohydrodynamics has many industrial applications such as physics, chemistry and engineering, crystal growth, metal casting and liquid metal cooling blankets for fusion reactors. Unsteady MHD convective flow of Rivlin-Ericksen fluid over an infinite vertical porous plate with absorption effect and variable suction was studied by Veerasankaret al. [1]. Anuradha Punithavalli [2] studied the MHD boundary layer flow of a steady micropolar fluid along with a stretching sheet with binary chemical reaction. Rama Krishna Reddy et al. [3] have examined the MHD free convective flow past a porous plate. KarunaDwivedi et al. [4] have investigated the MHD flow through a horizontal channel containing a porous medium placed under an inclined magnetic field. Chandra Reddy et al. [5] studied the MHD natural convective heat generation/absorbing and radiating fluid past a vertical plate embedded in a porous medium – an exact solution. Obulesu et al. [6] studied the effect of radiation absorption and chemical reaction effects on MHD radiative heat source/sink fluid past a vertical porous plate. Mohapatra et al. [7] Investigated the effect of chemical reaction on MHD micropolar fluid flow on a vertical surface through porous media with a heat source. Recently researchers [8-10] shown interest in this area. Obulesu et al. [11] investigate Hall current effects on MHD convective flow past a porous plate with thermal radiation, chemical reaction and heat generation /absorption. K Raghunath et al. [12] have studied Heat and mass transfer on Unsteady MHD flow of a second-grade fluid through the porous medium between two vertical plates. Raghunath K et al. [13] Discussed Hall Effects on MHD Convective Rotating Flow of Through a Porous Medium past Infinite Vertical Plat**e**. Raghunath k, et al. [14] have discussed Heat and mass transfer on MHD flow of Non-Newtonian fluid over an infinite vertical porous plate.

II. MATHEMATICAL FORMULATION

Consider the unsteady two dimensional MHD free convective flow of a viscous incompressible, electrically conducting and radiating fluid in an optically thin environment past an infinite heated vertical porous plate embedded in a porous medium in presence of thermal and concentration buoyancy effects. Let the x-axis be taken in vertically upward direction along the plate and y-axis is normal to the plate. It is assumed that there exists a homogeneous chemical reaction of the first order with constant rate Kr between the diffusing species and the fluid. A uniform magnetic field is applied in the direction perpendicular to the plate. The viscous dissipation and the Joule heating effects are assumed to be negligible in the energy equation. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field is negligible. Also it is assumed that there is no applied voltage so that the electric field is absent. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species, which are present, and hence the Soret and Dufour effects are negligible and the temperature in the fluid flowing is governed by the energy concentration equation involving radiative heat temperature. Under the above assumptions as well as Boussinesq's approximation, the equations of conservation of mass, momentum, energy and concentration governing the free convection boundary layer flow over a vertical porous plate in the porous medium can be expressed as:

$$\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*}+V^*\frac{\partial u^*}{\partial y^*}=\left(\frac{\vartheta}{1+\lambda_1}\right)\frac{\partial^2 u^*}{\partial y^{*2}}+g\beta_T(T^*-T^*_\infty)+gB_C(C^*-C^*_\infty)-\frac{\sigma B_0^2}{\rho}u^*-\frac{\vartheta u^*}{K_p} \tag{2}$$

$$\frac{\partial T^*}{\partial t^*}+V^*\frac{\partial T^*}{\partial y^*}=\frac{K}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}}-\frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*}-\frac{Q_1}{\rho C_p}(T^*-T^*_\infty) \tag{3}$$

$$\frac{\partial C^*}{\partial t^*}+V^*\frac{\partial C^*}{\partial y^*}=D\frac{\partial^2 C^*}{\partial y^{*2}}-K_C(C^*-C^*_\infty)+D_1\frac{\partial^2 T^*}{\partial y^{*2}} \tag{4}$$

The relevant boundary conditions are given as follows

$$u^*=L^*\left(\frac{\partial u^*}{\partial y^*}\right),\quad T^*=T^*_w+(T^*_w-T^*_\infty)e^{i\omega^* t^*},\ C^*=C^*_w+(C^*_w-C^*_\infty)e^{i\omega^* t^*}\ \text{ at }\ y^*=0 \tag{5}$$

$$u^*\to 0,\qquad T^*\to T^*_\infty,\qquad C^*\to C^*_\infty\qquad \text{as}\quad y^*\to\infty \tag{6}$$

Where $T_w{}^*$ and $T_\infty{}^*$ is the temperature at the wall and infinity, $C_w{}^*$ and $C_\infty{}^*$ is the species concentration at the wall and at infinity respectively. By using Roseland approximation the radiative heat flux $q_r{}^*$ is given by $q^*{}_r=\frac{-4}{3}\frac{\sigma_s}{k_e}\frac{\partial T_w{}^{*4}}{\partial y^*}$ where σ_s is the Stephen Boltzmann constant and K_e is the main absorption coefficient.

By expanding $T_w{}^{*4}$ in to the Taylor series $T_\infty{}^*$ which after neglecting higher order terms takes the form $T_w{}^{*4}=4T^{*3}{}_\infty T^*_w-3T^{*4}{}_\infty$.

From the equation of continuity (1), it is clear that the suction velocity at the plate is either a constant or a function of time only. Hence, the suction velocity normal to the plate is assumed to be in the form

$$V^* = -V_0(1+e^{i\omega t}) \tag{7}$$

On introducing the following non-dimensional quantities,

$$u=\frac{u^*}{v_0},\ y=\frac{v_0 y^*}{\vartheta},\ \theta=\frac{T^*-T^*_\infty}{T^*_w-T^*_\infty},\ \phi=\frac{C^*-C^*_\infty}{C^*_w-C^*_\infty},\ \Pr=\frac{\mu C_p}{K},\ Sc=\frac{\vartheta}{D},\ M=\frac{\sigma B_0^2\vartheta}{\rho v_0^2},$$

$$Gr=\frac{\vartheta g\beta_T(T^*_w-T^*_\infty)}{v_0^3},\ Gc=\frac{\vartheta g\beta_c{}^*(C^*_w-C^*_\infty)}{v_0^3},\ \ K=\frac{v_0^2K_P}{\vartheta^2},\ t=\frac{t^*v_0^2}{4\vartheta},\ h=\frac{v_0L^*}{\vartheta},$$

$$Kr = \frac{\vartheta K_C}{v_0^2}, F = \frac{4I_1\vartheta^2}{Kv_0^2}, Q = \frac{Q_1 v^2}{Kv_0^2}, S_0 = \frac{D_1(T_w^* - T_\infty^*)}{\vartheta(C_w^* - C_\infty^*)}, R = \frac{KK_e}{4\sigma_s T_\infty^{*3}} \quad (8)$$

The governing equations (2) to (4) can be rewritten in the non-dimensional form as follows

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1+\varepsilon Ae^{i\omega t})\frac{\partial u}{\partial y} = (\frac{1}{1+\lambda_1})\frac{\partial^2 u}{\partial y^2} + Gr\theta + Gc\phi - M_1 u \quad (9)$$

$$\frac{\Pr}{4}\frac{\partial \theta}{\partial t} - \Pr(1+\varepsilon Ae^{i\omega t})\frac{\partial \theta}{\partial y} = \alpha\frac{\partial^2 \theta}{\partial y^2} - Q\theta \quad (10)$$

$$\frac{Sc}{4}\frac{\partial \phi}{\partial t} - Sc(1+\varepsilon Ae^{i\omega t})\frac{\partial \phi}{\partial y} = \frac{\partial^2 \phi}{\partial y^2} - ScKr\phi + So\frac{\partial^2 \theta}{\partial y^2} \quad (11)$$

The corresponding boundary conditions are given by

$$u = h(\frac{\partial u}{\partial y}),\ \theta = 1+\varepsilon e^{i\omega t}, \phi = 1+\varepsilon e^{i\omega t}, \quad at \quad y=0$$
$$u \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad as \quad y \to \infty \quad (12)$$

III. SOLUTION TO THE PROBLEM

The equations (9) to (11) are coupled, non-linear partial differential equations and these cannot be solved in closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. So this can be done, when the amplitude of oscillations (ε <<1) is very small, we can assume the solutions of flow velocity u, temperature field θ and concentration ϕ in the neighbourhood of the plate as:

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t}u_1(y)$$
$$\theta(y,t) = \theta_0(y) + \varepsilon e^{i\omega t}\theta_1(y) \quad (13)$$
$$\phi(y,t) = \phi_0(y) + \varepsilon e^{i\omega t}\phi_1(y)$$

The corresponding boundary conditions are

$$u_0 = h(\frac{\partial u_0}{\partial y}), u_1 = h(\frac{\partial u_1}{\partial y}), \quad \theta_0 = 1, \theta_1 = 1, \quad \phi_0 = 1, \phi_1 = 1 \quad at \quad y=0$$
$$u_0 \to 0, \quad u_1 \to 0, \quad \theta_0 \to 0, \theta_1 \to 0, \quad \phi_0 \to 0, \phi_1 \to 0 \quad as \quad y \to \infty \quad (14)$$

Solving equations (13) under the boundary conditions (14), the following solutions are obtained
Velocity temperature and concentration field.

$$u = b_9\exp(-m_1 y) + b_{10}\exp(-m_3 y) + b_{11}\exp(-m_5 y)$$
$$+\varepsilon(b_{12}\exp(-m_1 y) + b_{13}\exp(-m_2 y) + b_{14}\exp(-m_3 y) \quad (15)$$
$$+b_{15}\exp(-m_4 y) + b_{16}\exp(-m_5 y) + b_{17}\exp(-m_6 y))e^{i\omega t}$$

$$\theta = \exp(-m_1 y) + \varepsilon(b_1\exp(-m_1 y) + b_2\exp(-m_2 y))e^{i\omega t} \quad (16)$$

$$\phi = b_3\exp(-m_1 y) + b_4\exp(-m_3 y) + \varepsilon(b_5\exp(-m_1 y)$$
$$+b_6\exp(-m_2 y) + b_7\exp(-m_3 y) + b_8\exp(-m_4 y))e^{i\omega t} \quad (17)$$

Skin Friction:

The non-dimensional skin friction at the surface is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\tau = -(m_1 b_9 + m_3 b_{10} + m_5 b_{11}) - \varepsilon(m_1 b_{12} + m_2 b_{13} + m_3 b_{14} + m_4 b_{15} + m_5 b_{16} + m_6 b_{17}) e^{i\omega t} \qquad (18)$$

Nusselt Number :

The rate of heat transfer in terms of the Nusselt number is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$Nu = m_1 + \varepsilon(m_1 b_1 + m_2 b_2) e^{i\omega t} \qquad (19)$$

Sherwood Number :

The rate of mass transfer on the wall in terms of Sherwood number is given by

$$Sh = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$

$$Sh = (m_1 b_3 + m_3 b_4) + \varepsilon(m_1 b_5 + m_2 b_6 + m_3 b_7 + m_4 b_8) e^{i\omega t} \qquad (20)$$

III. RESULTS AND DISCUSSIONS

The test set for this evaluation experiment watermark image randomly selected from the internet. Matlab 7.0 software platform is used to perform the experiment. The PC for an experiment is equipped with an Intel P4 2.4GHz Personal laptop and 2GB memory.

In order to get a physical insight into the problem numerical calculations are carried out for the Velocity, Temperature and Concentration profiles and the following discussion is set out. Throughout the computations we employ, S_0=1, Sc=0.22, Pr=0.71, Gr=5, Gc=5, K=0.1, Kr=0.5, M=1, Q=0.5, ε=0.01, ω=20, A=0.5, h=0.2, R=2, $\omega t=\pi/3$, t=1, λ_1=2. Figs. 1 – 5 demonstrate the variations of the fluid velocity under the effects of different parameters.

In Fig.1, we represent the velocity profile for different values of Thermal Grashof number (Gr). From this figure, it is noticed that velocity increases with increases in Gr. In Fig. 2, velocity profiles are displayed with the variation in modified Grashof number (Gc). From this figure, it is noticed the velocity gets increases by the increase of modified Grashof number (Gc).Fig.3, depicts the variations in velocity profiles for different values of Porosity parameter (K) from where it is noticed that velocity increases as K increases. In Fig. 4, velocity profiles are displayed with the variation in magnetic parameter (M). From this figure, it is noticed the velocity gets reduced by the increase of magnetic parameter (M). Fig.5 depicts the variations in velocity profiles for different values of Jeffreyparameter (λ_{1}) which shows that velocity Increases as λ_1 increases.

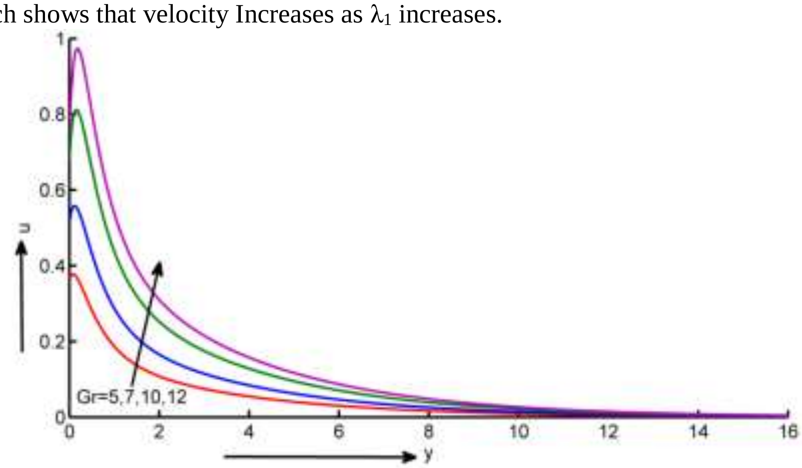

Figure 1: Effect of Grashof Number on Velocity

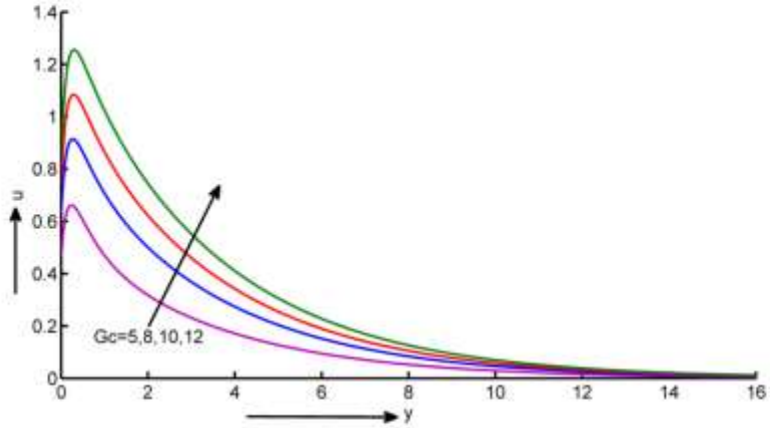

Figure 2: Effect of Modified Grashof Number on Velocity

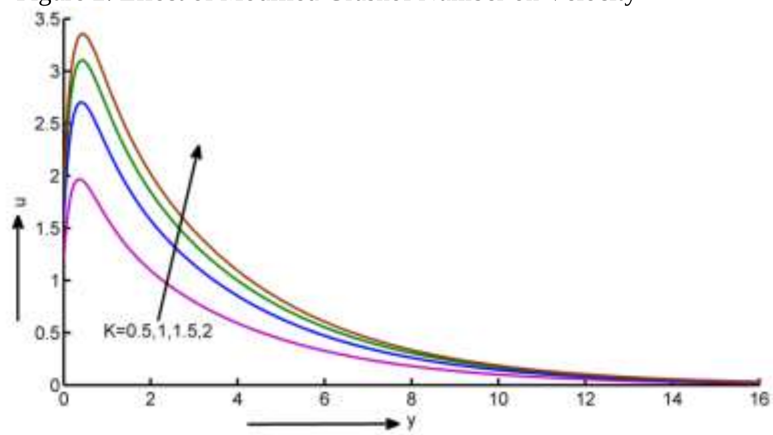

Figure 3: Effect of Porosity parameter on Velocity

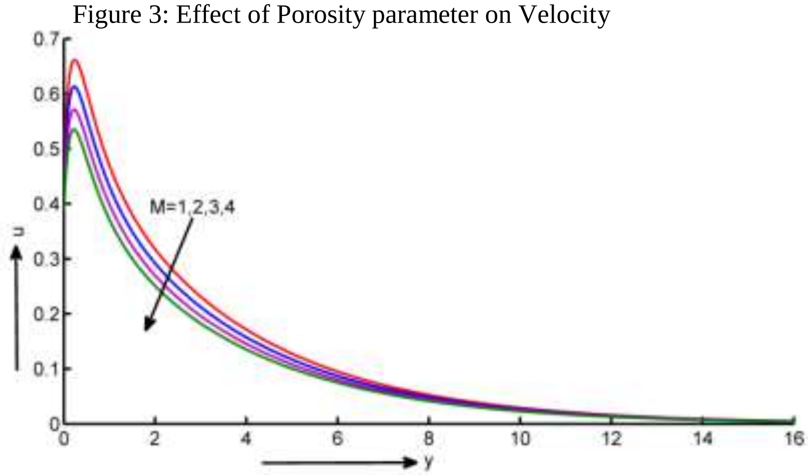

Figure 4: Effect of Magnetic parameter on Velocity

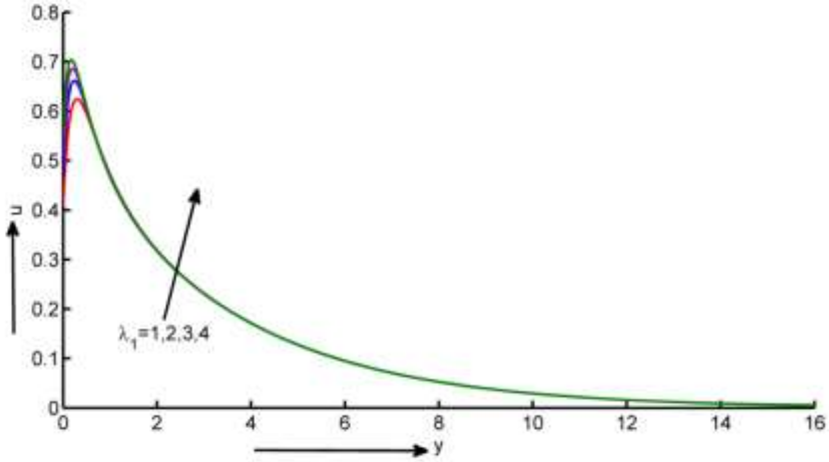

Figure 5: Effect of Jeffrey parameter on Velocity

Figures 6– 8, show the effect of Prandtl number, heat source parameter and Radiation parameter on the temperature profile. Here Chemical reaction parameter (Kr=0.5), Magnetic field parameter (M=1), Suction parameter (A=0.5), porosity parameter(K=0.1), Refraction parameter(h= 0.2), Frequency parameter (ω=20),Mass Grashof number (Gc=5), ThermalGrashof number (Gr=5), Heat source parameter (Q=0.5) and Soret parameter (So=1). From Fig6-8, it is clear that temperature decreases with the increase in Prandtl number (Pr), Heat source parameter (Q) and Radiation Parameter (R).

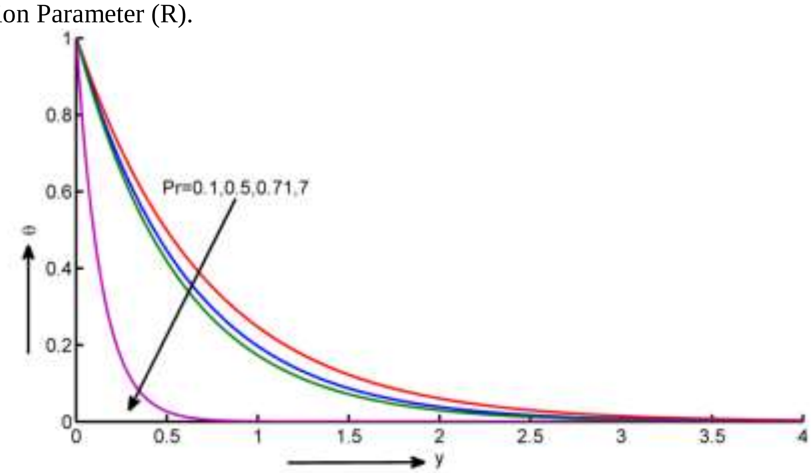

Figure 6: Effect of Prandtl Number on Temperature

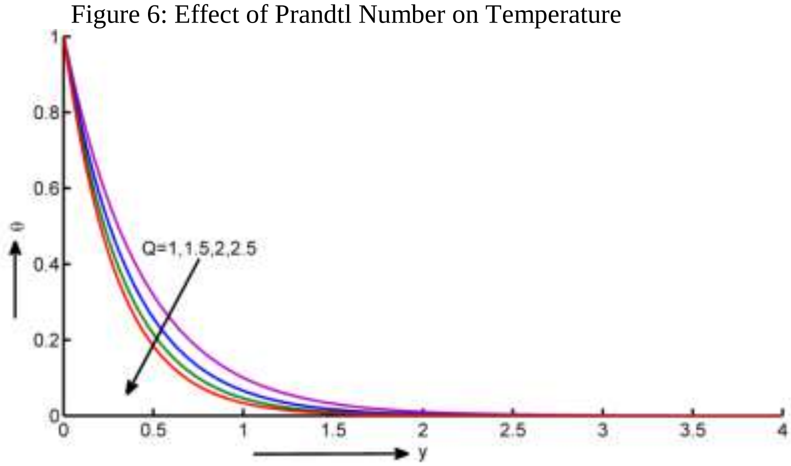

Figure 7: Effect of Heat source Parameter on Temperature

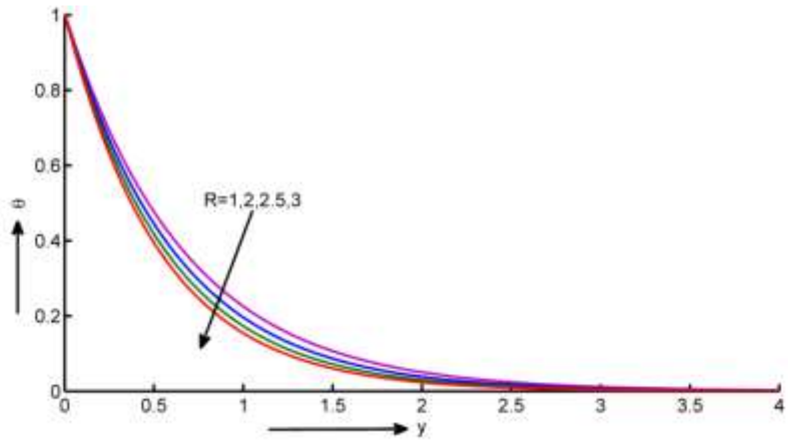

Figure 8: Effect of Radiation Parameter on Temperature

Figures 9-11, shows the effect of chemical reaction parameter (Kr), Schmidt number (Sc) and Soretparameter (S_0) on concentration profile. Here Magnetic field parameter (M=1), Radiation parameter (R=2), Suction parameter (A=0.5), Refraction parameter (h=0.2), Frequency parameter (ω=20), Mass Grashof number (Gc=5), Thermal Grashof number (Gr=5), Heat source parameter (Q=0.5) and Soret parameter (S_0=1).From Fig. 9-11, it is clear that concentration decreases with the increase in chemical reaction parameter and Schmidt number. Fig.11 depicts the variations in Concentration profile for different values of Soret parameter (S_0). From this figure, it is noticed that Concentration increases when S_0 increases.

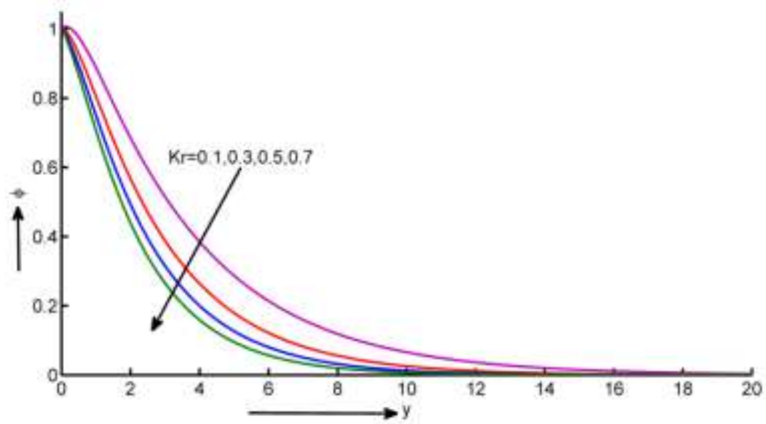

Figure 9: Effect of Chemical reaction Parameter on Concentration

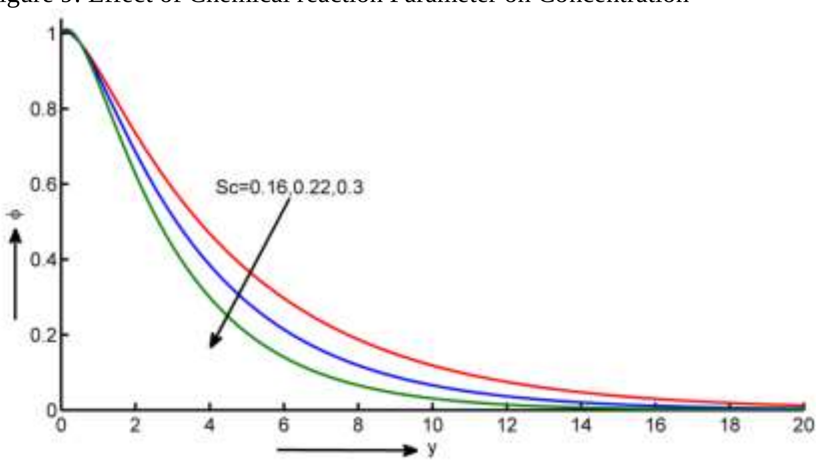

Figure 10: Effect of Schmidt Number on Concentration

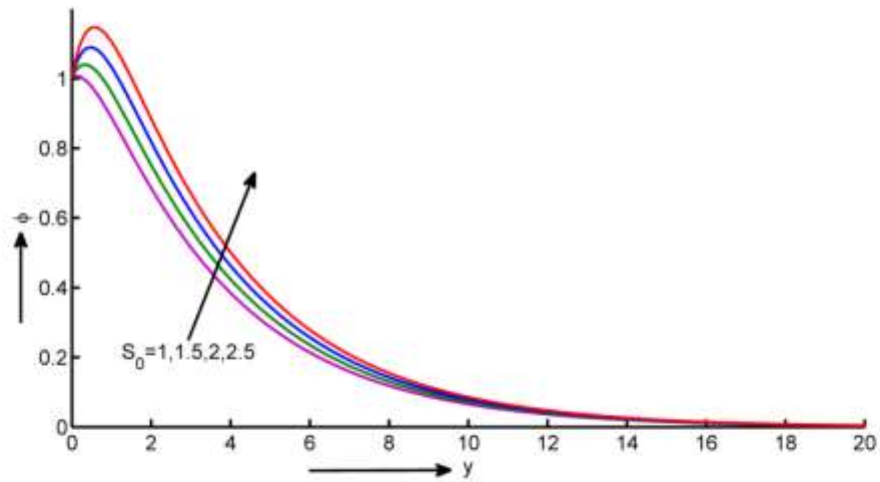

Figure 11: Effect of Soret Parameter on Concentration

Table – 1, shows numerical values of skin-friction for various of Grashof number (Gr), modified Grashof number (Gc), Magnetic parameter (M), Porosity parameter (K). From table 1, we observe that the skin-friction increases with an increase in Grashof number (Gr), modified Grashof number (Gc), Porosity parameter (K) and Jeffrey parameter(λ_1) whereas it decreases under the influence of magnetic parameter.

TABLE-1: VARIATIONS IN SKIN FRICTION

Gr	Gc	M	K	λ_1	T
5					3.3337
8					4.1722
10					4.7311
12					5.2901
	6				3.7209
	8				4.4954
	10				5.2699
	15				7.206
		1.2			3.2916
		1.6			3.2111
		1.8			3.1726
		2			3.1351
			0.5		8.0226
			0.8		9.647
			1		10.391
			1.2		10.9719
				1	2.8055
				2	3.3337
				3	3.7368
				4	4.0624

Table – 2 demonstrates the numerical values of Nusselt number (Nu) for different values of Prandtl number (Pr), Radiation parameter (R), Heat source parameter (Q). From table 2, we notice that the Nusselt number increases with an increase in Prandtl number, Radiation parameter and Heat source parameter.

TABLE-2: VARIATIONS IN NUSSELT NUMBER

Pr	R	Q	Nu
0.11			1.4105
0.51			1.6266
0.71			1.7472
7			7.2439
	1		1.4874
	2		1.7472
	3		1.9657
	4		2.158
		1	1.7472
		2	2.2967
		3	2.7231
		4	3.0842

Table – 3 shows numerical values of Sherwood number (Sh) for the distinct values of Schmidt number (Sc), Chemical reaction parameter (Kr) and Soretnumber(So). It can be noticed from Table - 3 that the Sherwood number enhances with rising values of Schmidt number, and the Chemical reaction parameter whereas it decreases under the influence of Soret number(S_0).

TABLE-3: VARIATIONS IN SHERWOOD NUMBER

Sc	Kr	S_0	Sh
0.16			0.0288
0.22			0.0609
0.6			0.2966
0.8			0.3891
	0.4		0.0958
	0.5		0.1346
	0.8		0.2327
	1		0.2882
		0.5	0.1221
		1	-0.0609
		1.5	-0.2438
		2	-0.4267

IV.CONCLUSION

In this problem, is studied the multiple variables effect on MHD Jeffrey fluid flow past a vertical plate embedded in a porous medium. In the analysis of the flow the following conclusions are made:

1. Velocity increases with an increase in Grashof number and as well as modified Grashof number, porosity parameter and Jeffrey parameter of the porous medium while, it decreases in the existence of magnetic parameter.
2. Temperature decreases in the presence of Prandtl number, heat source parameter and radiation parameter.
3. Concentration decreases with an increase in the Schmidt number and chemical reaction parameter while it increases in the presence of the Soret parameter.
4. As a significant increase is seen in skin friction for Grashof number, modified Grashof number, porosity parameter and Jeffrey parameter while a decrease is seen in the presence of magnetic parameter.
5. The rate of heat transfer increases with Prandtl number, heat source parameter and radiation parameter.
6. The rate of mass transfer increases with the Schmidt number and Chemical reaction parameter while a decrease is seen in the presence of the Soret parameter.

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