TANGLE-TREE DUALITY: The Homology of a locally finite graph with ends

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Abstract--Graph Theory began with Leonhard Euler in his study of the Bridges of K¨onigsburg problem. Since Euler solved this very first problem in Graph Theory, the field has exploded, becoming one of the most important areas of applied mathematics we currently study. Generally speaking, Graph Theory is a branch of Combinatorics but it is closely connected to Applied Mathematics, Optimization Theory and Computer Science. At Penn State (for example) if you want to start a bar fight between Math and Computer Science (and possibly Electrical Engineering) you might claim that Graph Theory belongs (rightfully) in the Math Department. (This is only funny because there is a strong group of graph theorists in our Computer Science Department.) In reality, Graph Theory is cross-disciplinary between Math, Computer Science, Electrical Engineering and Operations Research. For any constant size sequence, we overcome the question of sub-graph isomorphism in planar graphs in linear time. Our findings are based on a technique used to partition the planar graph into small tree width sections and use dynamic programming in each section. Many flat graphic problems like connection, circumference, circle, isomorphism, or shortest paths can be solved using the same techniques.
Keywords—Tangle tree, finite graphs, Bridges.

I. INTRODUCTION

In this paper graph theoretic results concerning the degree sequence, vertex coloring, and the vertex independence number are used to derive theorems about finite groups. First, two elements X, y of the group G are connected by an edge whenever they commute: xy = yx. A well-known fact about coloring the vertices of a finite graph is shown to yield an upper bound to the order of the largest abelian.subgroup(s) of G, in terms of the cardinalities of the conjugacy classes of G. The same graph, and a lower bound to the vertex independence number in terms of the degree sequence, yields a sufficient condition on a non-abelian group G in order that G contain at least $\{[G^{1/3}]\}$ pairwise non-commuting elements, and hence cannot be covered by the union of fewer than $\{[G^{1/3}]\}$ abelian subgroups. Such groups are, for example, permutation groups of prime degree, Frobenius groups, the simple groups PSL(2, p), and the sporadic simple groups. Finally, turning to finite abelian groups G and an entirely different graph association, we use the vertex independence number to prove an extremal result concerning the cardinalities of the (disjoint) sets S U -S and S-S when S is a locally-maximal sum-free subset of G. Along the way we find a lower bound for all such S c G, of the form $|S| \geq$ Constant.

II.TERMINOLOGY AND BASIC FACTS

The isomorphic sub graph is an essential and very general type of exact correspondence of patterns. The sub graphic isomorphism generalizes other main graph problems, including Hamiltonian paths, cliques, correspondences, distribution, and the shortest. The sub graph isomorphism Sub graph isomorphic variations have likewise been used to model such diverse functional problems as comparison of molecular structure, integrated circuit checking, optimization of micro programmed controllers, prior art avoidance in genetically system development, study of Chinese ideographs, robot action preparation, semantic network recuperation and polyhedral arty facts. In the question of the sub graph isomorphism, provided a "template" K and a sequence "L, it is either appropriate to find an occurrence of H as a subsection of K or to list all occurrences. There may be an exponentially multiplication of occurrences for any choices of K and L, but the listing of all occurrences cannot be resolved in sub exponential periods. For this respect, we look at the unique case of K (and ultimately L) flat graphs, a restriction which is obviously present in many applications. We demonstrate that planar substrates can be resolved in linear time for every fixed sequence. The findings contain many other concerns, including access to the origin, isomorphic sub graphic effects and limited paths. A common decomposition method is implemented in our algorithm for classification of different NP issues on flat maps. The solution is to delete a set of small arboreal from the diagram, when we have a range with non-sub graphs with small arborescence covering each vertex.Each planar map needs to be presumed to be easy, and the number of boundaries is $P(j)$ at most. It is a straightforward matter of time because we do not need to consider the dependency of this figure. The only difficulties that pose a difference in this definition are isomorphic sub graphics, I and edge connectivity; for such issues it should be concluded that the graph has minimal edge multiplicity, so again $l = P(j)$.

III.LATEST OUTCOME

The following findings have been shown. Time dependency on me is not included. It's exponential generally (necessary except for $Q = QM$ because the isomorphism of the planar sub graph is $MQ - full$).

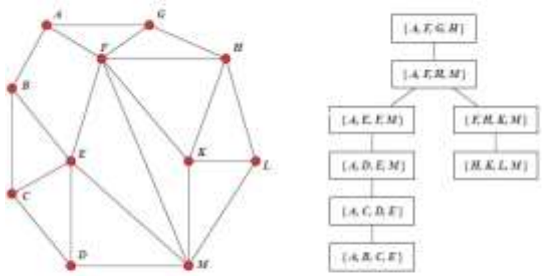
1. In the case of a planar graph H, we may check if a certain fixed pattern I is, or in time $P(m)$, count the number of events H as a subgraph I.
2. If the associated pattern is l, we can mention all occurred in time $P(o + l)$ as a sub graph of the planar graph H. If 3-linked, then l= m and all things can be described in $P(o)$ time.
3. The number of mediated subsections of a planar H isomorphous graph can be determined to be any fi xed sequence linked to G in time $P(o)$ in order to list them in time $P(O + L)$ if there are k occurrences.
4. In planar diagrams we may solve the h-and h-issues in time $P(o)$ for every constant i.
5. We can measure an exact girth in time off $P(o)$ for any planar graph H for which we are conscious of a constant circle boundary. The same restriction is whether the shortest separating cycle or the smallest non facial cycle in a specific plane is needed in a given diagram instead of the circuit.
6. We can measure vertex connectivity and edge connectivity H in time $P(o)$ for any planar graph H. We may check the k-edge connectivity of any fixed k in $P(o)$ period for planar multigraphs.
7. In time $P(m)$ we construct a linear routing data structure for any planar graph G and any constent m which can check for a pair of vertices if their distance is at its limit m and, wherever feasible, find a shortest routing distance in time $P(logm)$.

We may measure the exact diameter in time $P(o)$ for every planar map H for which we know that the diameter is constantly limits.

IV.WORK WHICH RELATED TO SUBJECT

The naïve exponential which is defined as $P(m^{|I|})$ and it is always weaker than the general subgraph isomorphism and the time taken $|I|^{P(|I|}n^{P(\sqrt{|I|})}$ which is much bigger than the given lower bound. Most articles examined isomorphism of flat sub graph with minimal patterns. Long ago it was understood that if model I was L_3 or L_4, at most P(m's instances could be a subgraph in a planar graph H. and such instances could be described in a linear time. It was an algorithm that measured accessibility to estimate maximal independent sets and checked inscrutability. Linear time and instance limits for L_3 and L_4 are based solely on the spartanfeatures of planar charts and related approaches are often common in the search problems and other full sub graphs.

The grapes that can be located at most $P(m)$ times may recently be described in a planar graph n-vertex as the sub-graph isomorphism: precisely these are the 3-connected planar maps. Nevertheless, our evidence does not contribute to an appropriate algorithm for the isomorphic three-connected planar subset. We use different strategies in this paper that do not rely on the strong communication order. Obviously all the shortest pairs of ports may be determined in time $P(on)$, after which the requests that we mention can be addressed in time $P(1)$; but certain faster algorithms are known to have estimated shorter planar route. Their details are often focused on Frederickson, the shortest route structure. Within less preparatory time than other established tests, our data structure reacts specifically to the shortest path questions, but can only identify paths of constant duration.

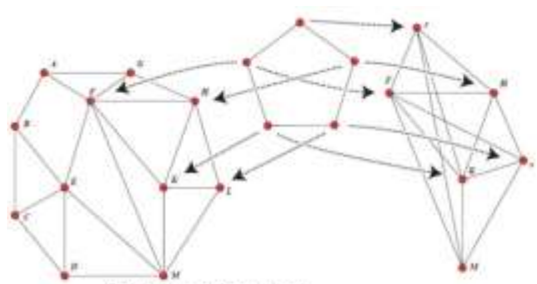


Planar Graph's Tree Decomposition

Isomorphism of restricted tree-width- Sub graph isomorphism checking in grapes with restricted width of the tree must be carried out as a subroutine. This can be done by a traditional technique of dynamic programming. A condition that we count or mention each subgraph isomorphs precisely one period complicates the exact description of the question we solve. For convenience, we define the limitations of this issue with a parameter that calculates both the text tree width and the pattern dimension.

Definition :A graph G tree decomposition is a T-in which each node of $M \epsilon S$ is labeled as $M(O) \subset W(H)$ that forms a contiguous S sub-with a number with tree nodes whose tree labels include some specific H vertex. Every border, in other words, links two vertices belonging to the same $M(O)$ label to at least a M node. The tree's breakup diameter is one smaller than the highest mark placed in S. H's treetop distance is the least range of any H-breakdown.

The sub-tree at node O is made up of O and the descendants. An induced sub graph of H is connected to each of these sub trees and has vertices in sub-tree node names.



Partial isomorph of a pentagon

Theorem : The M-rooted substratum supplies the association mediated substratum H with a tree decomposition.

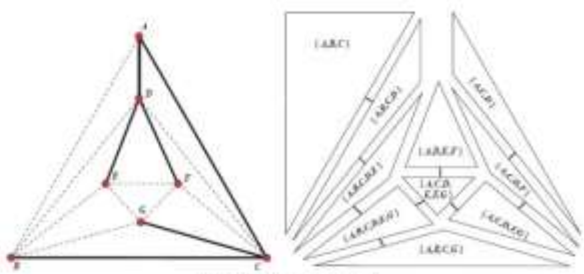
Proof- A tree breakup which does not instantly follow is just due to the need for each edge to connect two vertices on the label of a specific node. Because this is valid for G and T, any induced edge of sub graph (w, y) But if not, u belongs both to $M(p)$ and, if not, to $M(p)$ where M belongs to O. Therefore, through adjacency, at least one node in the sub tree also all belongs to the mark.

Lemma:-Suppose that graph H with m vertices and decomposition U of H with width w are given. Let s be a subset of H's vertices, and let H at most w vertices be a fixed diagram. All isomorphs of H in H may be identified in time with any vertex in S.

Proof: To compute Y_1 and Y_2 values for increasing partial isomorphic boundary, the above dynamic programming method. We calculate the pair set $(C;Y)$ in the top down of the tree in which B is a partial isomorphic limit and Y is either Y_1 or Y_2, such that the $Y(C)$ meaning contributes to the subgraph isomorphic nal counts. These pairs may be defined as those that have been marked as adding to the sum of $Y(C)$ and that have produced a non-change in this count in a higher pair in the list. Finally, we measure again from bottom up, naming the partial sub graph isomorphs for each contributing pair $(C;Y)$ in the $Y(C)$ meaning. The following is achieved by removing each addition of the $Y(C)$ description in the previous lemma and by removing any multiplication with a partly isomorphic one from a Cartesian combination of two pre-compute lists. It is possible to do this by mimicking a preliminary $Y(C)$ approximation described in the previous lemma. For this equation the number of steps is equal to the number of steps in the previous algorithm and the extra time for each iteration of a partial isomorphic pair. For each combination, the isomorphic component used in the output may be mounted, and each output is created from a binary combination panel that takes the execution of $P(z)$ time. $P(lz)$ is applied for the complete period.

Lemma: Let planar graph T have a root tree G where the longest path is long m. Then in time $P(mo)$ it is possible to consider a tree decomposition of T with width limit at 3m.

Proof: We presume that T is embedded with all sides of the circle, including the outside face. Form the trees with one node in each triangle, as well as an edge that links some 2 nodes, while the related triangles share an edge that is not in T. Label each node with a collection of vertices on the route that links each angle of the triangle with the centre. Then the endpoints of each edge are part of a marking collection (namely, the sets of two triangles that surround the tip) and the labelling of each vertex creates an adjacent substratum.



Triangulated planar graph

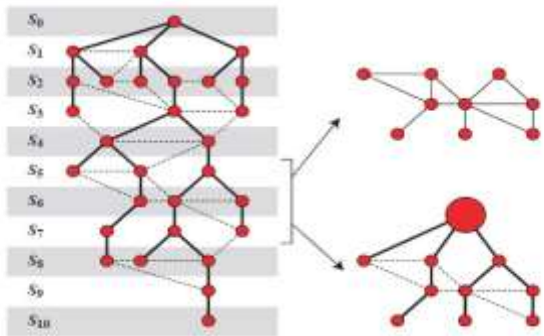
Owing to the exponential dependency of our overall algorithms on the tree-of covering segments, we concentrate our efforts to the as much as possible this distance, at the expense of that the total cover size by a factor of $P(z)$ by as little as possible.

Lemma : Let H be a flat graph and w be a specified parameter integer. So we may apply the following characteristics to a community of subgraphs H_i and to segment the vertices of H into sub set S_i:-

- The full number of w sub graphs of H_i is any vertex of H.
- Each subsection of H_i with a tree width may be decomposed to a limit of $3w - 1$.
- If H is a w- sub graph of H bound, and i is the lowest value $H \cap S_i$ does not have, H is a H_i subgrapher, but not a H_j a sub graph with $j > i$.
- $P(v^2m)$ Would be the average period for partitioning and tree decomposition measurements.

Proof: We select an arbitrary starting point w_o, and if the vertices are distant i from there, we let H_i be the plot of the vertex collection $\bigcup_{j=i} i + w - 1$.

Obviously, the S_i sets are a division of H, and each vertex is in H_i at its most which is shown in following diagram the sets S_i. if obviously create a break between the vertices of H and at most every vertex is w.



Planar graph with breadth first spanning tree

So the vertices for $i = 0$, H_i are at most $k - 1$ from whence, and the first spanning tree, using the top of the lemma, is able to detect the treet with a maximum width of $3(w - 1)$. In order to show that-H_i with $i > 0$ has little treetop space, generates a Ho auxiliary graph by breaking down all vertices less than I from where into one super vertex and removing it. Each vertical with at least $i + k$ width. Ho, i Dot-diagram H minor, and so is simple. Then, the collapse of a width that first extends H tree gives a twin $Ho\ i$ tree with a depth of w at the most, such that $Ho\ i$ has a tree decomposition with a width at 3W maximum, where any node of decomposition contains a supervertex collapsed on its name is created by extracting from Ho, i this supervertex so that we may shape a breakdown of H_i with a width of $3W - 1$ at most by taking the supervertex away from the breakdown.

V.ADDITIONAL UPGRADES

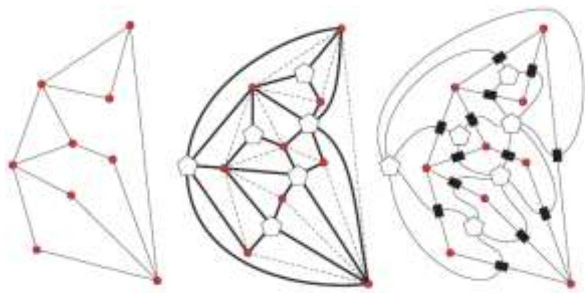
Lemma: Suppose planar diagram M is Hamiltonian or 3-linked, or linked and small, instead, $M - R$ has $P(|R|)$ connected components for every set of R of M vertices.

Proof:- This is easy with Hamiltonian diagrams and diagrams. For 3 linked graphs, without lack of generality, presume that there is no edge in M connecting two vertices in R; so the flat embedding of R, induced by a single embedding of M, has to be a different face for any aspect of $M - R$.

Theorem : In planar text graph H with n vertices in time $P(ml^2)$, you can count the isomorphic for any Vl-wheel.

Proof: They list the number of period cycles in the neighbours of w for every vertex w. The complete number of barrios in H is $P(m)$. the treewidth of every neighborhood is 2 by Lemma 3 at most. A single line, with maximum $l - 1$ vertices, which begins and terminates at two or three vertices in $m(O)$ and can or can not include the third vertex, is possible only in a partial isomorphism in any M node of this decomposition; hence it is only appropriate to keep line of $P(l)$ separate parametral isomorphism borders per node at each node, which is the period of $P(l)$.

Definition : This definition is designated by two mathematicians Keil and Brecht, The trouble with i is that the full clique can be approximated by adding a collection of vertices with the most borders necessary.



Embedded Planar Graph K

Theorem : For any K, in time $2^{P(k\ logk)}m$, we can overcome the flat k-clustering and the related k-clustering problems.

Proof: For any possible planar graph we will clearly check sub graph isomorphism in k vertices and return sub-graphs with the most edges. We describe now two connectivity apps which are linear without an exponential dependence on an individual parameter as opposed to previous applications. The vertex connectivity of K is the least number of vertices of which a disconnected subset is omitted. For any planar graph has a maximum grade vertex five, the connection vertex is five at most.

We now use the Nishizeki method to turn the question of the vertex relation into a short loop, similar to the cycles mentioned in the beginning of this segment. The latest K 'map, which has $o + g$ vertices: the n vertices of K originally and the current facial vertices associated with the faces in K, is built for any flat embedding of K. If the vertex and the face instances are in K, we place an edge in K' 'between an initial vertex and the nose. So k' is a two-party graph embedded in the plane and hence this is embedded in above diagram.

In this section we briefly run through any non-standard terminology we use. We also list without proof a few easy lemmas that we shall need, and use freely, later on. For graphs we use the terminology of [10], for topology that of Hatcher [26]. We reserve the word ‘boundary’ for homologous contexts and use ‘frontier’ for the closure of a set minus its interior. Our use of the words ‘path’, ‘cycle’ and ‘loop’, where these terminologies conflict, is as follows. The word path is used in both senses, according to context (such as ‘path in X’, where X was previously introduced as a graph or as a topological space). Note that while topological paths need not be injective, graph-theoretical paths are not allowed to repeat vertices or edges. The term cycle will be used in the topological sense only, for a (usually 1-dimensional) singular chain with zero boundary. When we do need to speak about graph-theoretic cycles (i.e., about finite connected graphs in which every vertex has exactly two incident edges) we shall instead refer to the edge sets of those graphs, which we shall call circuits. Our graphs may have multiple edges but no loops. This said, we shall from now on use the term loop topologically: for a topological path σ : [0, 1] → X with σ(0) = σ(1). This loop is based at the point σ(0). Given any path σ : [0, 1] → X, we write σ− : s 7→ σ(1 − s) for the inverse path. An arc in a topological space is a subspace homeomorphism to [0, 1].

Lemma ([25]). The image of a topological path with distinct endpoints x, y in a Hausdorff space X contains an arc in X between x and y.

Proof: Suppose that F(x,t) time-injective if for every x the map t 7→ F(x,t) is either constant or injective. Let σ be a topological path in X, with σ(a) = σ(b) =: z for some a < b, and let τ be the path obtained from σ by replacing σ π [a, b] with the constant map [a, b] → {z}. We say that a homotopy F from σ to τ retracts σ π [a, b] to z in Y (⊆ X) if F is relative to [0, a] ∪ [b, 1], time injective, and maps [a, b] × [0, 1] to σ ([a, b]) ⊆ Y .

All homotopies between paths that we consider are relative to the first and last point of their domain, usually {0, 1}. We shall often construct homotopies between paths segment by segment. The following lemma enables us to combine certain homotopies defined separately on infinitely many segments.

Lemma ([18]). A path in |G| traverses each edge only finitely often.

Proof :A loop that is injective on [0, 1) is a circle in |G|. (In most of our references, the term circle is used for the image of such a loop.) The set of all edges traversed by a circle is a circuit. It is easy to show that the image of a circle is uniquely determined by its circuit C, being the closure of U^{C} in |G|.

Let → E = → E (G) denote the set of all integer-valued functions ϕ on the set → E of all oriented edges of G that satisfy ϕ(←e) = −ϕ(→e) for all →e ∈ → E. This is an abelian group under pointwise addition. A family (ϕi | i ∈ I) of elements of → E is thin if for every →e ∈ → E we have ϕi(→e) 6= 0 for only finitely many i. Then ϕ = P i∈I ϕi is a well-defined element of → E : it maps each →e ∈ → E to the (finite) sum of those ϕi(→e) that are non-zero. We shall call a function ϕ ∈ → E obtained in this way a thin sum of those ϕi. We can now define our oriented version of the topological cycle space of G.

The topological cycle space C(G) can be characterized as the set of those subsets of E that meet every finite cut of G in an even number of edges [12, 10]. The characterization has an oriented analogue:

Lemma : Let G = (V, E) be a graph and suppose that t is a non-trivial tour (closed trail) in G. Then t contains a cycle.

Proof. The fact that t is closed implies that it contains at least one pair of repeated vertices. Therefore a closed sub-walk of t must exist since t is itself has these repeated vertices. Let c be a minimal (length) closed sub-walk of t. We will show that c must be a cycle. By way of contradiction, suppose that c is not a cycle. Then since it is closed it must contain a repeated vertex (that is not its first vertex). If we applied our observation from Remark 3.24 we could produce a smaller closed walk c ′, contradicting our assumption that c was minimal. Thus c must have been a cycle. This completes the proof.

Theorem: Let G = (V, E) be a graph and suppose that t is a non-trivial tour (closed trail). Then t is composed of edge disjoint cycles.

Proof. We will proceed by induction. In the base case, assume that t is a one edge closed tour, then G is a non-simple graph that contains a self-loop and this is a single edge in t and thus t is a (non-simple) cycle2 . Now suppose the theorem holds for all closed trails of length N or less. We will show the result holds for a tour of length N + 1. Applying Lemma, we know there is at least one cycle c in t. If we reduce tour t by c to obtain t 0 , then t is still a tour and has length at most N. We can now apply the induction hypothesis to see that this new tour t 0 is composed of disjoint cycles. When taken with c, it is clear that t is now composed of disjoint cycles. The theorem is illustrated in Figure 3.5. This completes the proof.

Theorem : Let G = (V, E) be a connected graph and let e ∈ E. Then G' = G − {e} is connected if and only if e lies on a cycle in G.

Proof. (⇐) Recall a graph G is connected if and only if for every pair of vertices v_1 and v_{n+1} there is a walk w from v_1 to v_1 with:

$w = (v_1, e_1, v_2, \ldots, v_n, e_n, v_{n+1})$.

Let G' = G − {e}. Suppose that e lies on a cycle c in G and choose two vertices v_1 and v_{n+1}in G. If e is not on any walk w connecting v1 to vn+1 in G then the removal of e does not affect the reach ability of v_1 and v_{n+1} in G' . Therefore assume that e is in the walk w. The fact that e is in a cycle of G implies we have vertices $u_1, \ldots$, um and edges $f_1, \ldots, f_m$ so that: $c = (u_1, f_1, \ldots, u_m, f_m, u_1)$ is a cycle and e is among the $f_1, \ldots$, fm. Without loss of generality, assume that e = fm and that $e = \{u_m, u_1\}$. (Otherwise, we can re-order the cycle to make this true.) Then in G' we will have the path: c ' = $(u_1, f_1, \ldots, u_m)$ The fact that e is in the walk w implies there are vertices v_i and v

$_{i+1}$ so that e = {v_i, v_{i+1}} (with $v_i = u_1$ and $v_{i+1} = u_m$). In deleting e from G we remove the sub-walk (v_i, e, v_{i+1}) from w. But we can create a new walk with structure:

w ‘ = (v_1, e_1, . . . , v_i , f_1, u_2, . . . , u_{m-1}, f_{m-1}, u_m, . . . , e_n, v_{n+1}).

This is illustrated in Figure:

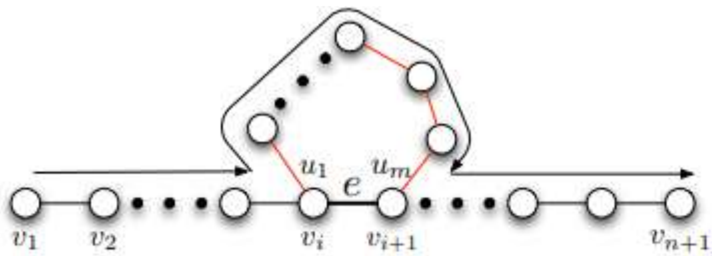


Figure: If e lies on a cycle, then we can repair path w by going the long way around the cycle to reach v_{n+1} from v_1.

(⇒) Suppose G’ = G − {e} is connected. Now let e = {v_1, v_{n+1}}. Since G’ is connected, there is a walk from v_1, to, v_{n+1}. Applying Remark, we can reduce this walk to a path p with: p = (v_1, $e_{1,\dots,}$, v_n, e_n, v_{n+1}) Since p is a path, there are no repeated vertices in p. We can construct a cycle c containing e in G as: p = (v_1 , e_1, . . . , v_n, e_n, v_{n+1}, e, v_1) since e = {v_1, v_{n+1}} = {v_{n+1}, v_1}. Thus, e lies on a cycle in G. This completes the proof.

Corollary: Let G = (V, E) be a connected graph and let e ∈ E. The edge e is a cut edge if and only if e does not lie on a cycle in G.

Remark: The next result is taken from Extremal Graph Theory, the study of extremes or bounds in properties of graphs. There are a number of results in Extremal Graph Theory that are of interest.

Remark: There are many situations in which we’d like to measure the importance of a vertex in a graph. The problem of measuring this quantity is usually called determining a vertex’s centrality.

Lemma: Every consistent T^* -avoiding orientation O of S_k avoids T , as long as |G|≥k.

Proof. Suppose O has a subset σ ∈ T . We show that as long as this set is not an inclusion-minimal nested set in T , we can either delete one of its elements, or replace it by a smaller separation in O, so that the resulting set σ ‘ ⊆ O is still in T but is smaller or contains fewer pairs of crossing separations. Iterating this process, we eventually arrive at a minimal nested set in T that is still a subset of O. By its minimality, this set is an antichain (compare the definition of T), and all consistent nested antichains are stars.9 Our subset of O will thus lie in T^* , contradicting our assumption that O avoids T^* .

If σ has two comparable elements, we delete the smaller one and retain a subset of O in T . We now assume that σ is an antichain, but that it contains two crossing separations, → r = (A,B) and →s = (C,D) say. As these and their inverses lie in $S\sim_k$, sub-modularity implies that one of the separations (A ∩ D,B ∪C) ≤ (A,B) and (B ∩ C,A ∪ D) ≤ (C,D) also lies in $S\sim_k$. Let us assume the former; the other case is analogous.

Theorem : Suppose G = (V, E) and G0 = (V ’ , E ’) are graphs with G ∼= G ’ with f : V → V ’ the graph isomorphism between the graphs. Further suppose that the degree sequence of G is d and the degree sequence of G’ is d ‘. Then:

(1) |V | = |V ’| and |E| = |E ‘ |,

(2) For all v ∈ V, deg(v) = deg(f(v))

(3) d = d ’ ,

(4) For all v ∈ V, ecc(v) = ecc(f(v))

(5) ω(G) = ω(G ’) (recall ω(G) is the clique number of G),

(6) α(G) = α(G ’) (recall α(G) is the independence number of G),

(7) c(G) = c(G’) (recall c(G) is the number of components of G),

(8) diam(G) = diam(G ’),

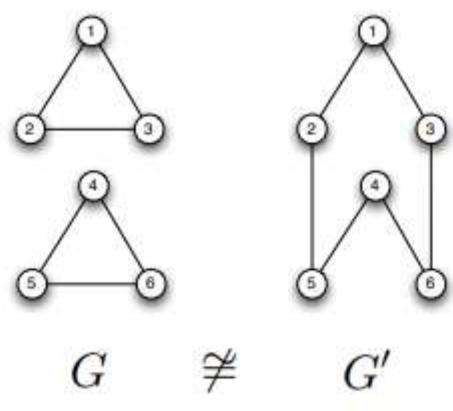
(9) rad(G) = rad(G ’)

(10) The girth of G is equal to the girth of G ’ ,

(11) The circumference of G is equal to the circumference of G ’.

Remark : Isomorphism is really just a way of renaming vertices; we assume that the vertices in graph G are named from the set V , while the vertices in the set G ’. are named from the set V ‘ . If the graphs are identical except for the names we give the vertices (and thus the names that appear in the edges), then the graphs are isomorphic and all structural properties are preserved as a result of this.

Example: Given two graphs G and G ’. , we can see through example that the degree sequence does not uniquely specify the graph G and thus if G and G0 have degree sequences d and d 0 it is necessary that d = d 0 when G ∼= G ’.but not sufficient to establish isomorphism. To see this, consider the graphs shown in Figure It’s clear that d = (2, 2, 2, 2, 2, 2) = d ’.



Two graphs that have identical degree sequences, but are not isomorphic, but these graphs cannot be isomorphic, since they have different numbers of components. The same is true with the other graph properties. The equality between a property of G and that same property for G0 is a necessary criterion for the isomorphism of G and G ’, but not sufficient.

Lemma: For every integer k ≥ 3, [10] a graph G of order at least k has branch-width < k if and only if G has an S_k-tree over T^* .

Theorem: (Tangle-tree duality theorem for graphs, extended). The following assertions are equivalent for all finite graphs $G \neq \emptyset$ and $0< k \leq |G|$:

(i) G has a tangle of order k.

(ii) G has a T -tangle of S_k.

(iii) G has a T^* -tangle of S_k.

(iv) G has no S_k-tree over T^* .

(v) G has branch-width at least k, or k = 1 and G has no edge, or k = 2 and G is a disjoint union of stars and isolated vertices and has at least one edge.

Proof. If k = 1, then all statements are true. If k = 2, they are all true if G has an edge, and all false if not. Assume now that $k \geq 3$.

(i)↔(ii) follows from the definition of a tangle at the start of this section, and our observation that they are consistent.

(ii)→ (iii) is trivial; the converse is Lemma.

(iii)↔ (iv) is an application of Theorem.

(iv)↔ (v) is Lemma 4.3. The exceptions in (v) for $k \leq 2$ are due to a quirk in the notion of branch width, which results from its emphasis on separating individual edges. The branch-width of all nontrivial trees other than stars is 2, but it is 1 for stars $K_{1,n}$. For a clean duality theorem (even one just in the context of [20]) it should be 2 also for stars: every graph with at least one edge has a tangle of order 2, because we can orient all separations in S2 towards a fixed edge. Similarly, the branch-width of a disjoint union of edges is 0, but its tangle number is 2.

Lemma : Every consistent F^* -avoiding orientation of S_k avoids F, as long as $|V| \geq k$.

Theorem :(Tangle-tree duality theorem for set separations). Given a universe U~ of separations of a set V with a sub-modular order function, and $k \leq |V|$, the following assertions are equivalent:

(i) U~ has a tangle of order k.

(ii) U~ has an F^* -tangle of S_k.

(iii) U~ has no $S\sim_k$-tree over F^*.

Proof: Applying Theorem proofs with the appropriate order functions yields duality theorems for all known width parameters based on set separations. For example, let V be the vertex set of a graph G, with bipartitions as separations. Counting the edges across a bipartition defines an order function whose F-tangles are known as the edge-tangles of G, so Theorem proofs yields a duality theorem for these. See Liu [16] for more on edge-tangles, as well as their applications to immersion problems.

The duals to edge-tangles of order k are S_k-trees over F^*. These were introduced by Seymour and Thomas [22] as carvings. The least k such that G has a carving is its carving-width. We thus have a duality theorem between edge-tangles and carving-width.

Taking as the order of a vertex bipartition the rank of the adjacency matrix of the bipartite graph that this partition induces (which is sub-modular [18]) gives rise to a width parameter called rank-width. In our terminology, G has rank-width < k if and only if it admits an S_k-tree over F *. The corresponding F-tangles of S_k, then, are necessary and sufficient witnesses for having rank-width ≥k, and we have a duality theorem for rank-width.

If V is the vertex set of a hyper graph or the ground set of a matroid, the F-tangles coincide, just as for graphs, with the hyper graph tangles of [20] or the matroid tangles of [13]. As in the proof of Lemma 4.3, a hyper graph or matroid has branch-width < k if and only if it has an S_k-tree over F *. Theorem proofs thus yields the original duality theorems of [20] and [13] in this case.

Our tangle-tree duality theorem for set separations can also be applied in contexts quite different from graphs and matroids. As soon as a set comes with a natural type of set separation – for example, bipartitions – and a (sub-modular) order function on these, it is natural to think of the tangles in this separation universe as clusters in that set. Theorem proofs then applies to these clusters: if there is no cluster of some given order, then this is witnessed by a nested set of separations which cut the given set, recursively, into small pieces.

The interpretation is that the separations to be oriented have small enough order that they cannot cut right through a cluster. So if there exists a cluster, it can be thought of as orienting all these separations towards it. If not, the nested subset of the separations returned by the theorem divides the ground set into pieces too small to accommodate a cluster. This tree set of separations, therefore, will be an easily checkable witness for the non-existence of a cluster.

Even if we base our cluster analysis just on bipartitions, we still need to define an order function to make this work. This will depend both on the type of data that our set represents and on the envisaged type of clustering. In [12] there are some examples of how this might be done for a set of pixels of an image, where the clusters to be captured are the natural regions of this image such as a nose, or a cheek, in a portrait of the Mona Lisa. The corresponding duality theorem then reads as follows.

Corollary ([12]). For every picture π on a canvas and every integer k > 0, either π has a non-trivial region of coherence at least k, or there exists a laminar set of lines of order < k all whose splitting stars are void 3-stars or single pixels. For no picture do both of these happen at once.

Theorem : Let r ≥ 3 be given. There is a positive constant such that if $p \leq n^{-2/r+}\in$ then, for some $\pi \approx p$ (r 2) , we may couple $G = G_{n,p}$ and the random r-uniform hypergraph $H = H_{n,\pi;r}$ such that w.h.p. to every edge e of H there is a corresponding copy of Kr in G with V (Kr) = e.

Proof: We will also need the following theorem from Dudek, Frieze, Loh and Speiss [2], which removed some divisibility constraints from [1], [5]. A loose Hamilton cycle C in an r-uniform hypergraph H = (V, E) of order n is a collection of edges of H such that for some cyclic ordering of V , every edge consists of r consecutive vertices, and for every pair of consecutive edges E_{i-1}, E_i in C (in the natural ordering of the edges), we have $|E_{i-1} \cap E_i| = 1$.

VI.TANGLE DUALITY FOR TREE-WIDTH IN GRAPHS

We now apply our abstract duality theorem to obtain a new duality theorem for tree-width in graphs. Its witnesses for large tree-width will be orientations of S_k, like tangles, and thus different from brambles (or ‘screens’), the dual objects in the classical tree-width duality theorem of Seymour and Thomas [21].

This latter theorem, which ours easily implies, says that a finite graph either has a tree-decomposition of width less than k−1 or a bramble of orders at least k, but not both. The original proof of this theorem is as mysterious as the result is beautiful. The shortest known proof is given in [5] (where we refer the reader also for definitions), but it is hardly less mysterious. A more natural, if slightly longer, proof due to Mazoit is presented in [6]. The proof via our abstract duality theorem, as outlined below, is perhaps not shorter all told, but it seems to be the simplest available so far.

Finding, Conclusion and Accessible Assumptions:-

For any period $P(M)$ pattern we have seen how to overcome planar subgraphic isomorphism. In identical time frames, we have discussed several relevant problems. Several generalizations remain available for the problem:

- ✓ We showed that also for disconnected patterns in time $P(m)$ we could overcome planar subset isomorphism. Could we mention all disconnected sequence occurrences in $P(m+l)$ time?
- ✓ Keil and Brecht describe an algorithm that separates the smallest edge set into two vertices of different dimensions while their algorithm has a minimal duration, and their algorithm a cubic period is restricted. Could we use our resources to find a performance faster?
- ✓ It seems feasible to increase the reliability of the pattern from $2^{P(klogk)}$ to $2^{P(v)}$ however; the newly discovered randomized coloring technique could only boost the decision of sub-graph isomorphism. May we boost the numbering and classification of the subgraph isomorphism question in a similar way?

REFERENCES

[1] O. Amini, F. Mazoit, N. Nisse and S. Thomasse´: Submodular partition functions, Discrete Appl. Math. 309 (2009), 6000–6008.

[2] D. Bienstock, N. Robertson, P. Seymour and R. Thomas: Quickly excluding a forest, J. Combin. Theory Ser. B 52 (1991), 274–283.

[3] N. Bowler: Presentation at Hamburg workshop on graphs and matroids, Spiekeroog 2014.

[4] J. Carmesin, R. Diestel, F. Hundertmark and M. Stein: Connectivity and tree structure in finite graphs, Combinatorica 34 (2014), 1–35.

[5] R. Diestel: Graph Theory, Springer, 4th edition, 2010.

[6] R. Diestel: Graph Theory (5th edition), Springer-Verlag, 2017, Electronic edition available at http://diestel-graph-theory.com/.

[7] R. Diestel: Abstract separation systems, Order 35 (2018), 157–170.

[8] R. Diestel, P. Eberenz and J. Erde: Duality theorem for blocks and tangles in graphs, SIAM J. Discrete Math. 31 (2017), 1514–1528.

[9] R. Diestel and S. Oum: Unifying duality theorems for width parameters, II. General duality, arXiv:1406.3798, 2014.

[10] R. Diestel and S. Oum: Tangle-tree duality in abstract separation systems, arXiv:1701.02509, 2017. [11] R. Diestel and S. Oum: Tangle-tree duality in graphs, matroids and beyond (extended version), arXiv:1701.02651, 2017.

[12] R. Diestel and G. Whittle: Tangles and the Mona Lisa, arXiv:1603.06652.

[13] J. Geelen, B. Gerards, N. Robertson and G. Whittle: Obstructions to branchdecomposition of matroids, J. Combin. Theory (Series B) 96 (2006), 560–570.

[14] P. Hlinenˇ y´ and G. Whittle: Matroid tree-width, European J. Combin. 27 (2006), 1117–1128.

[15] P. Hlinenˇ y´ and G. Whittle: Addendum to matroid tree-width, European J. Combin. 30 (2009), 1036–1044.

[16] C.-H. Liu: Packing and covering immersions in 4-edge-connected graphs, arXiv:1505.00867, 2015. [17] L. Lyaudet, F. Mazoit and S. Thomasse´: Partitions versus sets: a case of duality, European J. Combin. 31 (2010), 681–687.

[18] S. Oum and P. Seymour: Approximating clique-width and branch-width, J. Combin. Theory Ser. B 96 (2006), 514–528.

[19] J. Oxley: Matroid Theory, Oxford University Press, 1992.

[20] N. Robertson and P. Seymour: Graph minors. X. Obstructions to treedecomposition, J. Combin. Theory (Series B) 52 (1991), 153–190.

[21] P. Seymour and R. Thomas: Graph searching and a min-max theorem for treewidth, J. Combin. Theory (Series B) 58 (1993), 22–33.

[22] P. Seymour and R. Thomas: Call routing and the ratcatcher, Combinatorica 14 (1994), 217–241.

[23] A. Georgakopoulos and P. Sprussel, ¨ Geodesic topological cycles in locally finite graphs, preprint 2006.

[24] R. Halin, Ub¨ er unendliche Wege in Graphen, Math. Annalen 157 (1964), 125–137.

[25] D.W. Hall and G.L. Spencer, Elementary topology, John Wiley, New York 1955.

[26]M. M. Sys lo. The subgraph isomorphism problem for outerplanar graphs. Theoretical Computer Science 17:91{97, 1982.}

[27]N. Robertson and P. D. Seymour. Graph Structure Theory: Proc. Joint Summer Res. Conf. Graph Minors. Contemporary Mathematics 147. Amer. Math. Soc., 1991.

[28] C. H. Papadimitriou and M. Yannakakis. The clique problem for planar graphs. Information

Processing Letters 13:131{133, 1981.}

[29] A. Lingas and M. M. Sys lo. A polynomial-time algorithm for subgraph isomorphism of two-connected series-parallel graphs. Proc. 15th Int. Colloq. Automata, Languages and Programming, pp. 394{409. Springer-Verlag, Lecture Notes in Computer Science 317, 1988.

[30] S. Y. T. Lang and A. K. C. Wong. A sensor model registration technique for mobile robot localization. Proc. Int. Symp. Intelligent Control, pp 298{305. Inst. Electrical & Electronics Engineers, 1991.}

[31] G. Kortsarz and D. Peleg. On choosing a dense subgraph. Proc. 34th Symp. Foundations of Computer Science, pp. 692{703. Inst. Electrical & Electronics Engineers, 1993.}

[32] P. N. Klein and S. Sairam. Fully dynamic approximation schemes for shortest path problems in planar graphs. Algorithmica 22(3):235{249, November 1998.}

[33] J. M. Keil and T. B. Brecht. The complexity of clustering in planar graphs. Combinatorial Mathematics and Combinatorial Computing 9:155{159,1991.}

[34] A. Kanevsky, R. Tamassia, G. Di Battista, and J. Chen. On-line maintenance of the four-connected components of a graph. Proc. 32nd IEEESymp. Foundations of Computer Science, pp. 793{801, 1991.}

[35] G. N. Frederickson and R. Janardan. Ecient message routing in planar networks. SIAM J. Computing 18:843{857, 1989.}

[36] G. N. Frederickson. Fast algorithms for shortest paths in planar graphs, with applications. SIAM J. Computing 16:1004{1022, 1987}

[37] D. Eppstein. Connectivity, graph minors, and subgraph multiplicity. J. Graph Theory 17:409{416, 1993.}

[38] D. Eppstein. Diameter and treewidth in minor-closed graph families. To appear in Algorithmica, math.CO/9907126.