ALGEBRAIC OPERATIONS OF T-FUZZY IDEALS IN TERNARY Γ-SEMIRING

[1] **R.PadmaPriya**, [2] **Dr.D.Sivakumar**, [3] **Dr.S.AnjalMose**

[1,3] Assistant Professor, St.Joseph's College of Arts & Science (Autonomous), Cuddalore-607001

[2] Professor, Annamalai University,Annamalainagar-608002.

ABSTRACT: In this paper, we introduced the notion of T-fuzzy ideals, T-fuzzy left(resp., right, lateral) ideal in ternary Γ-semiring. Also we investigated algebraic operations on T-fuzzy ideals in ternary Γ-semiring and to prove subsequently these operations give rise to different structures on T-fuzzy ideals in ternary Γ-semiring. Some characterization of ternary Γ-semiring has been obtained in terms of fuzzy subsets.

KEYWORDS: T-fuzzy ideals, T-fuzzy left ideal, T-fuzzy right ideal, T-fuzzy lateral ideal, Ternary Γ-semiring, t-norm, Composition of ternary Γ-semiring.

1.INTRODUCTION

The theory of fuzzy sets was first proposed by Zadeh[4]. Triangular norm was introduced by Schweizer and skla[1]. In fuzzy set theory t-norm is extensively used to model the logical connective conjunction (AND), Which have application in several fields of mathematics and artifical intelligence. Zimmerman [2] have introduced fuzzy set theory and its application. Dudek and Jun[17] studied fuzzy subquasigroups over a t-norm. Srinivas and Nagaiah [14] studied T-fuzzy ideals of gamma near rings. The concepts of fuzzy ideals in algebraic structures have introduced by Chinnadurai[16] and in 2014, Chinnadurai et.al.[15] have studied fuzzy lateral ideals in ternary near-rings. Akram[6] studied T-Fuzzy ideals in near rings. SajaniLavanya et.al.[7,9] introduced the notion of ternary Γ-semirings. Murali Krishna Rao et.al [8] studied L-fuzzy ideals in Γ-semirings. In the year 2017, Revathi et.al.[11] introduced the compositions of fuzzy TΓ-ideals in ternary Γ-semiring. Marapureddy Murali Krishna Rao [10] have introduced T-fuzzy ideals in ordered Γ-semirings. Kavikumar et.al. [3] studied fuzzy ideals and fuzzy quasi-ideals of ternary semirings. The concepts of L-fuzzy ideals in semirings were studied by Jun,Neggers and Kim[18]. Ronnason chinram et.al. [12] have introduced the L-fuzzy ternary subsemirings and L-fuzzy ideals in ternary semirings. The theory of ternary rings was introduced by Lister[5].In [13], Kar studied quasi-ideals and bi-ideals in ternary semirings. The main purpose of this paper is to introduced the notion of T-fuzzy ideal, T-fuzzy left ideal, T-fuzzy right ideal, T- fuzzy lateral ideal in ternary Γ-Semiring and to investigate the algebraic operations and relations between them.

2.PRELIMINARIES

Definition 2.1 ([12]). A non empty set $\tilde{S}$ together with two associative binary operations called addition and multiplication (denoted by + and · respectively) is called a **semiring** if

i) ($\tilde{S}$, +) is a commutative semi-group
ii) ($\tilde{S}$, .) is a semi-group
iii) Multiplicative distributes from the left and from the right over addition.
(i.e) a(b+c) = ab+ac
and (a+b)c =ac+bc for all $a, b, c \in \tilde{S}$.

Definition 2.2 ([15]). A non empty set $\tilde{S}$ together with a binary operation and ternary operation called addition + and ternary multiplication, respectively is said to be a **ternary semiring** if $\tilde{S}$ is an additive commutative semigroup satisfying the following conditions.

(i) (abc)de = a(bcd)e = ab(cde)
(ii) (a+b)cd = acd + bcd,
(iii) a(b+c)d = abd +acd and
(iv) ab(c +d) = abc + abd for all $a, b, c, d, e \in \tilde{S}$

Definition 2.3 ([3]). Let (Ã, +) and (Γ,+) be commutative semigroup. If there exists a mapping $\tilde{A} \times \Gamma \times \tilde{A} \times \Gamma \times \tilde{A} \rightarrow \tilde{A}$ which maps (a,α,b, β,c)→[a$\alpha b \beta c$] satisfying the conditions for all $a, b, c, d, e \in \tilde{A}$ and $\alpha, \beta, \gamma, \delta \in \Gamma$

i) [(a$\alpha a \beta c)\gamma d \delta y e$ = [aα(b$\beta c \gamma d)\delta e$]=[a$\alpha b \beta (c \gamma d \delta e)$]
ii) [(a+b)$\alpha c \beta d$] = [a$\alpha c \beta d$+b$\alpha c \beta d$]
iii) [a$\alpha(b+c)\beta d$] = [a$\alpha b \beta d$+a$\alpha c \beta d$]
iv) [a$\alpha b \beta (c+d)$] = [a$\alpha b \beta c$+a$\alpha b \beta d$]
Then Ã is called a **ternary Γ- semiring**

A Ternary Γ- semi-ring Ã is said to have zero element if there exists an element 0∈ μ such that 0+x=x=x+0 and 0αaβb= aα0bv=a$\alpha b \beta$0=0, ∀ a,b ∈ Ã and α ,β ∈ Γ.

Example 2.4([10]) Let Ã be the additive semi-group of all m×n matrices over the set of non-negative rational numbers and Γ be the additive semigroup of all m×n matrices over the set of non-negative integers and ternary operation is defined as $\tilde{A} \times \Gamma \times \tilde{A} \times \Gamma \times \tilde{A} \rightarrow \tilde{A}$ by (a,α,b,β, c)→ a$\alpha b \beta c$ using matrix multiplication for all $a, b, c \in \tilde{A}$ and α ,β ∈ Γ . Then Ã is a ternary Γ –semiring.

Definition 2.5 ([10]). A non empty subset $\tilde{S}$ of ternary Γ – semiring Ã is called a **ternary Γ – subsemiring** of $\tilde{A}$ if

i) ($\tilde{S}$,+) is a subsemigroup of (Ã,+) and

ii) $a\alpha b\beta c \in \tilde{S}$; for all $a, b, c \in \tilde{S}$ and $\alpha, \beta \in \Gamma$.

Definition 2.6 ([12]). An additive subsemigroup $\tilde{S}$ of Ã is called a **left (resp., right and lateral) ideal** of ternary Γ-semiring Ã if $a\alpha b\beta s$(resp., $s\alpha a\beta b$, $a\alpha s\beta b$) $\in \tilde{S}$ for all $a, b \in$ Ã, α,β∈Γand s∈ $\tilde{S}$. If $\tilde{S}$ is both left and right ideal of ternary Γ-semiring Ã then $\tilde{S}$ is called a two sided ideal of Ã. If $\tilde{S}$ is a left, right and lateral ideal of ternary Γ-semiring Ã then $\tilde{S}$ is called **an ideal** of Ã.

Definition 2.7 ([10]). A mapping T: [0,1] ×[0,1]→[0,1] is called a **triangular norm** (t-norm) if it satisfying the following conditions, for each p,q,r,s ∈[0,1].

i) T(0,p)=0 ; T(p,1)=p
ii) T(p,q)=T(q,p) (commutative)
iii)T(p,T(q,r))=T(T(p,q),r) (Associative)
iv) T(p,q)≤ T(r,s) if p≤r and q≤s (Monotonicity)

Definition 2.8 ([2]). A fuzzy subset of a set X is a function $\mu : X \to [0,1]$. This function is also called a membership function. A membership function is a generalization of a characteristic function or an indicator function of a subset defined for [0,1].

Definition 2.9 ([12]). Let $\tilde{A}$ *and* $\tilde{B}$ be ternary Γ-semiring. A mapping $\phi : \tilde{A} \to \tilde{B}$ is called a **homomorphism** if

(i) $\phi(a+b) = \phi(a) + \phi(b)$
(ii) $\phi(a\alpha b\beta c) = \phi(a)\phi(b)\phi(c)$ for all $a, b, c \in$ Ã , α,β∈ Γ

Definition 2.10 ([12]). Given any two sets $\tilde{A}$ *and* $\tilde{B}$, let $\nu \in F(\tilde{A})$ and $\phi : \tilde{A} \to \tilde{B}$ be a function. Define $\mu \in F(\tilde{B})$ by for $\nu \in \tilde{B}$

$$\mu(b) = \begin{cases} \sup\limits_{a \in \phi^{-1}(b)} \nu(a) & if\ \phi^{-1}(b) \neq 0 \\ 0 & otherwise \end{cases}$$

We call μ the image of ν under ϕ which is denoted by $\phi(\nu)$. Conversely for $\mu \in F(\phi(a))$, define $\nu \in F(\tilde{A})$ by $\nu(a) = \mu(\phi(a))$ *for all* $a \in \tilde{A}$ and we call ν the preimage of μ under ϕ which is denoted by $\phi^{-1}(\mu)$.

Definition 2.11([8]). Let μ be a fuzzy subset of a ternary Γ –semiring Ã. Then the set μ_t ={ a,b ϵ Ã / $\mu(a)$ ≥t} , tϵ[0,1] is called the **level subset** of Ã with respect to μ.

Definition 2.12([12]). A fuzzy subset μ of ternary Γ- semiring Ã is said to be **normal fuzzy subset** if μ(0)=1.

Let Ã be a ternary Γ- semi-ring Ã and $\mu \in F(\tilde{A})$. Define a T-fuzzy subset μ^{+} of Ã by

$$\mu^{+}(a) = \mu(a) + 1 - \mu(0), \forall a \in \tilde{A}$$

Definition 2.13([10]). Let $\tilde{S}$ be a non-empty subset of Ã. The characteristic function of $\tilde{S}$ is a fuzzy subset of Ã, defined by

$$\chi_{\tilde{s}}(a) = \{1, \quad if\ a \in \tilde{S}$$
$$0, \quad if\ a \notin \tilde{S}$$

3. T-FUZZY IDEALS

Definition 3.1 A fuzzy subset μ of a fuzzy subsemigroup of Ã is called a **fuzzy ternary** Γ**- subsemiring** of Ã if

$i)\mu$ (a+b)≥min{ $\mu(a), \mu(b)$ }

$ii)\mu$(aαbβc) ≥ min{μ (a), μ (b), μ (c)} for all a,b,c ∈ Ã and $\alpha, \beta \in \Gamma$.

Definition 3.2 A T-fuzzy subset μ of a ternary Γ semiring Ã is called a **T-fuzzy ternary** Γ **–subsemiring** of Ã if

$i)\mu$ (a+b)≥T{ $\mu(a), \mu(b)$ }

$ii)\mu$(aαbβc) ≥ $T\{T\{\mu$ (a),$\mu(b)\}$,μ (c)$\}$ for all a,b,c ∈ Ã and $\alpha, \beta \in \Gamma$.

Definition 3.3 A non empty fuzzy subset μ of a ternary Γ- semiring Ã is called a fuzzy left (resp., right and lateral) ideal of Ã if

$i)\mu$ (a+b)≥min{ $\mu(a), \mu(b)$ }

$ii)\mu$(aαbβc) ≥ μ(c) [μ(a$\alpha b \beta c$)≥ $\mu(a)$ and μ(a$\alpha b \beta c$)≥ $\mu(b)$] for all a,b,c ∈ Ã and $\alpha, \beta \in \Gamma$.

If μ is a fuzzy left, right and lateral ideal of ternary Γ-semiring then μ is called a **fuzzy ideals** of Ã.

Definition 3.4 A fuzzy subset μ of ternary Γ- semiring Ã is called a T-fuzzy left (resp.,right and lateral) ideal of Ã, if it satisfies the following conditions for all a,b∈ Ã , $\alpha, \beta \in \Gamma$.

$i)$ μ (a+b)≥ T{ $\mu(a), \mu(b)$ }

$ii)$ μ(aαbβc) ≥ μ(c) [μ(a$\alpha b \beta c$)≥ $\mu(a)$ and μ(a$\alpha b \beta c$)≥ $\mu(b)$]

If μ is a T-fuzzy left, right and lateral ideal of ternary Γ- semiring then μ is called a **T- fuzzy ideals** of Ã.

Theorem 3.5 Let μ be a T- fuzzy left (resp.,right, lateral) ideal of a ternary Γ- semiring Ã. Then the level set μ_t $(t \leq \mu(0))$ is the left (resp.,right, lateral) ideal of ternary Γ- semiring Ã.

Proof: Let a,bϵ μ_t, then $\mu(a) \geq t,\ and\ \mu(b) \geq t$

then $\mu(a+b) \geq T\{\mu(a), \mu(b)\} \geq t$ that implies a+b ϵ μ_t

On the other hand, if a,b ϵÃ, $x\epsilon\mu_t\ and\ \alpha, \beta\epsilon\ \Gamma$ then

$\mu(a\alpha b\beta x) \geq \mu(x) \geq t\ \ [\mu(x\alpha a\beta b) \geq \mu(x) \geq t, \mu(a\alpha x\beta b) \geq \mu(x) \geq t]$

So that $a\alpha b\beta x\ \epsilon\ \mu_t[x\alpha a\beta b\epsilon\ \mu_t, a\alpha x\beta b\epsilon\ \mu_t]$

Hence $\mu_t$is a left (resp.,right, lateral) ideal of ternary Γ-semiring Ã.

Theorem 3.6 Every fuzzy ideal of ternary Γ-semiring Ã is a T-fuzzy ideal of Ã.

Proof: Suppose μ is a fuzzy ideal of ternary Γ-semiring Ã and if a,b ϵÃ $and\ \alpha, \beta\epsilon\ \Gamma$ then

$$\mu(a+b) \geq \min\{\mu(a), \mu(b)\} \geq T\{\mu(a), \mu(b)\}$$
$$\mu(a\alpha b\beta c) \geq \mu(c), \mu(a\alpha b\beta c) \geq \mu(a), \mu(a\alpha b\beta c) \geq \mu(b)$$

Hence μ is a T-fuzzy ideal of Ã.

Definition 3.7 Let Ã ba a ternary Γ-semiring and μ,λ,δ be T-fuzzy ideals, then the intersection of μ,λ, μ+λ sum and the ternary product μΓλΓδ and composition $\mu_o\lambda_o\delta$ of μ,λ,δ are defined as follows

$$(\mu \cap \lambda)(x) = T\{\mu(x), \lambda(x)\}\ for\ all\ x \in \tilde{A}$$

$$(\mu + \lambda)(x) = \{\sup_{x=a+b}\{T[\mu(a), \lambda(b)]\ for\ all\ a,b \in \tilde{A}\}$$

0 otherwise

$$(\mu\Gamma\lambda\Gamma\delta)(x) = \{\sup_{x=a\alpha b\beta c}\{T[T[\mu(a), \lambda(b)], \delta(c)]\ for\ all\ a,b,c \in \tilde{A}; \alpha, \beta \in \Gamma\}$$

0 otherwise

$$(\mu \circ \lambda \circ \delta)(x) = \{\sup_{x=\sum_{i=1}^{n} a_i\alpha_i b_i\beta_i c}\ \{T[T[\mu(a_i), \lambda(b_i)], \delta(c_i)]\ for\ all\ a_i, b_i, c_i \in \tilde{A}; \alpha_i, \beta_i \in \Gamma\}$$

0 otherwise

Theorem 3.8 If μ and λ are two T-fuzzy left(resp.,right,lateral)ideals of ternary Γ-semiring, then $\mu \cap \lambda$ is also a T-fuzzy left(resp.,right,lateral)ideals of Ã.

Proof: Let μ and λ be two T-fuzzy left ideal of Ã and if a,b ϵÃ and α, βϵ Γ
Consider,

$$(\mu \cap \lambda)(a+b) = T[\mu(a+b), \lambda(a+b)]$$
$$\geq T[T(\mu(a), \mu(b)), T(\lambda(a), \lambda(b))]$$
$$= T[T(\mu(a), \lambda(a)), T(\mu(b), \lambda(b))]$$
$$= T[(\mu \cap \lambda)(a), (\mu \cap \lambda)(b)] \quad \textit{and}$$

$$(\mu \cap \lambda)(a\alpha b\beta c) = T[\mu(a\alpha b\beta c), \lambda(a\alpha b\beta c)]$$

$$\geq T[\mu(c), \lambda(c)]$$ (Since μ and λ are T-fuzzy left ideal of Ã)

$$= (\mu \cap \lambda)(c)$$

Thus $\mu \cap \lambda$ is a T-fuzzy left ideal of Ã.

Similarly we can prove for T-fuzzy right ideal, T-fuzzy lateral ideal of Ã.

Theorem 3.9 Let μ,λ are two T-fuzzy left(resp.,right, lateral) ideals of ternary Γ-semiring Ã then μ+λ is also a T-fuzzy left(resp.,right, lateral) ideals of Ã.

Proof: Let μ,λ are two T-fuzzy left(resp.,right, lateral) ideals of ternary Γ-semiring Ã and a,bЄ Ã ;α,βЄ Γ

Consider,

$$(\mu + \lambda)(a + b) = \sup_{a+b=p+q} \{T[\mu(p), \lambda(q)] : p, q \in \tilde{A}\}$$

$$= \sup_{a=u+v, b=s+t} \{T[\mu(u+s), \lambda(v+t)]; u, v, s, t \in \tilde{A}\}$$

$$\geq \sup_{a=u+v, b=s+t} \{T[T[\mu(u), \mu(s)], T[\lambda(v), \lambda(t)]]; u, v, s, t \in \tilde{A}\}$$

$$= \sup_{a=u+v, b=s+t} \{T[T[\mu(u), \lambda(v)], T[\mu(s), \lambda(t)]]; u, v, s, t \in \tilde{A}\}$$

$$= T\{\sup_{a=u+v,} T[\mu(u), \lambda(v)], \sup_{b=s+t} T[\mu(s), \lambda(t)]; u, v, s, t \in \tilde{A}\}$$

$$= T\{(\mu + \lambda)(a), (\mu + \lambda)(b)\}$$

And $(\mu+\lambda)(a\alpha b\beta c)=\sup\limits_{a\alpha b\beta c=p+q}\{T[\mu(p),\lambda(q)]: p,q\in \tilde{A}\}$

$$=\sup_{c=u+v}\{T[\mu(a\alpha b\beta u),\lambda(a\alpha b\beta v)]: u,v\in \tilde{A}\}$$

[since $a\alpha b\beta c=a\alpha b\beta(u+v)=a\alpha b\beta u+a\alpha b\beta v$]

$$\geq \sup_{c=u+v}\{T[\mu(u),\lambda(v)]\}$$

$$=(\mu+\lambda)(c)$$

Hence $(\mu+\lambda)$ is a T-fuzzy left ideal of Ã.

Similarly we can prove for T-fuzzy right ideal, T-fuzzy lateral ideal of Ã.

Corollary 3.10 Let μ, λ are two fuzzy ideal of ternary Γ-semiring Ã then μ+λ is also a fuzzy ideal of Ã.

Proof: By using the theorem 3.9, we can prove this corollary easily.

Proposition 3.11 Let Ã be a ternary Γ-semiring and $\mu\in F(\tilde{A})$. The following statements are true

(i) μ^{+} is a normal T-fuzzy subset of Ã containing μ.
(ii) $(\mu^{+})^{+}=\mu^{+}$
(iii) μ is normal iff $\mu^{+}=\mu$

Proof:

(i) We Know that, $\mu^{+}(a)=\mu(a)+1-\mu(0), \forall a\in\tilde{A}$

$$\mu^{+}(0)=\mu(0)+1-\mu(0)$$

Therefore $\mu^{+}(0)=1$

(i.e) $\mu(a)\leq\mu^{+}(a)$ This implies $\mu\subseteq\mu^{+}$

(ii) By (i), we have

$$(\mu^{+})^{+}(a)=\mu^{+}(a)+1-\mu^{+}(0), \forall a\in\tilde{A}$$

$$=\mu^{+}(a)+1-1 \qquad \text{[By (i)]}$$

$$=\mu^{+}(a)$$

Therefore $(\mu^{+})^{+} = \mu^{+}$

(iii)Assume that μ is normal. Then

$$\mu^{+}(a) = \mu(a) + 1 - \mu(0),\ \forall a \in \widetilde{A}$$

$$= \mu(a) + 1 - 1 \qquad \text{[since } \mu \text{ is normal]}$$

$$= \mu(a)$$

The converse is obvious by (i).

This completing the proof

Corollary 3.12 Let Ã be a ternary Γ-semiring and $\mu \in F(\widetilde{A})$ and $a \in \widetilde{A}$. If $\mu^{+}(a) = 0$ then $\mu(a) = 0$.

Proof: By Proposition 3.11 (i), we have $\mu(a) \leq \mu^{+}(a)$ this implies $\mu(a) = 0$

Proposition 3.13 Let Ã be a ternary Γ-semiring and $\mu \in F(\widetilde{A})$. The following statements are true

(i) If μ is a T-fuzzy ternary Γ − subsemiring of Ã, then μ^{+} is a normal T-fuzzy ternary Γ − subsemiring of Ã containing μ.

(ii) If μ is a T-fuzzy left(resp., right, lateral) ideal of Ã, then μ^{+} is a normal T-fuzzy left(resp., right, lateral) ideal of Ã containing μ.

(iii) If μ is a T-fuzzy ideal of Ã, then μ^{+} is a normal T-fuzzy ideal of Ã containing μ.

Proof: Let a,b,c ∈ Ã and $\alpha, \beta \in \Gamma$, then

$$\begin{aligned} \mu^{+}(a+b) &= \mu(a+b) + 1 - \mu(0) \\ &\geq T\{\mu(a), \mu(b)\} + 1 - \mu(0) \\ &= T\{\mu(a) + 1 - \mu(0), \mu(b) + 1 - \mu(0)\} \\ &= T\{\mu^{+}(a), \mu^{+}(b)\} \end{aligned}$$

$$\begin{aligned} \mu^{+}(a\alpha b\beta c) &= \mu(a\alpha b\beta c) + 1 - \mu(0) \\ &\geq T\{T\{\mu(a), \mu(b)\}, \mu(c)\} + 1 - \mu(0) \\ &= T\{T\{\mu(a), \mu(b)\} + 1 - \mu(0), \mu(c) + 1 - \mu(0)\} \\ &= T\{T\{\mu(a) + 1 - \mu(0), \mu(b) + 1 - \mu(0)\}, \mu(c)\} + 1 - \mu(0)\} \\ &= T\{T\{\mu^{+}(a), \mu^{+}(b)\}, \mu^{+}(c)\} \end{aligned}$$

Hence μ^{+} is a T-fuzzy ternary Γ −subsemiring of Ã. By Proposition 3.6 (i), μ^{+} is a normal T-fuzzy ternary Γ −subsemiring of Ã containing μ .

The proofs of (ii) and (iii) are similar to the proof of (i).

This completing the proof.

Propostion 3.14 Let Ã be a ternary Γ-semiring, $\phi:\widetilde{A}\to\widetilde{A}$ an onto homomorphism and $\mu\in F(\widetilde{A})$.

Define $\mu^{\phi}\in F(\widetilde{A}) by\, \mu^{\phi}(a)=\mu(\phi(a))\,\forall a\in\widetilde{A}$. The following statements are true

(i) If μ is a T-fuzzy ternary Γ − subsemiring of Ã, then μ^{ϕ} is a T-fuzzy ternary Γ − subsemiring of Ã.

(ii) If μ is a T-fuzzy left(resp., right, lateral) ideal of Ã, then μ^{ϕ} is a T-fuzzy left(resp., right, lateral) ideal of Ã.

(iii) If μ is a T-fuzzy ideal of Ã, then μ^{ϕ} is a T-fuzzy ideal of Ã.

Proof: Let a,b,c ∈ Ã and $\alpha,\beta\in\Gamma$, then

$$\begin{aligned}\mu^{\phi}(a+b)&=\mu(\phi(a+b))\\&=\mu(\phi(a)+\phi(b))\\&\geq T\{\mu(\phi(a)),\mu(\phi(b))\}\\&=T\{\mu^{\phi}(a),\mu^{\phi}(b)\}\end{aligned}$$

$$\begin{aligned}\mu^{\phi}(a\alpha b\beta c)&=\mu(\phi(a\alpha b\beta c))\\&=\mu(\phi(a)\phi(b)\phi(c))\\&\geq T\{T\{\mu(\phi(a)),\mu(\phi(b))\},\mu(\phi(c))\}\\&=T\{T\{\mu^{\phi}(a),\mu^{\phi}(b)\},\mu^{\phi}(c)\}\end{aligned}$$

Therefore μ^{ϕ} is a T-fuzzy ternary Γ − subsemiring of Ã.

The proofs of (ii) and (iii) are similar to the proof of (i).

This completing the proof.

Propostion 3.15 Let Ã and $\widetilde{B}$ be a ternary Γ-semiring and $\phi:\widetilde{A}\to\widetilde{\mathrm{B}}$ be an onto homomorphism. The following statements are true

(i) If μ is a T-fuzzy ternary Γ − subsemiring of $\widetilde{B}$, then the preimage of μ under ϕ is a T-fuzzy ternary Γ − subsemiring of Ã.

(ii) If μ is a T-fuzzy left(resp., right, lateral) ideal of $\widetilde{B}$, then the preimage of μ under ϕ is a T-fuzzy left(resp., right, lateral) ideal of Ã.

(iii) If μ is a T-fuzzy ideal of $\widetilde{B}$, then the preimage of μ under ϕ is a T-fuzzy ideal of Ã.

Proof: Let $\mu \in F(\widetilde{B})$ be a T-fuzzy ternary $\Gamma-$ subsemiring and ν be the preimage of μ under ϕ then for any a,b,c ∈ Ã and $\alpha, \beta \in \Gamma$, then

$$\begin{aligned}\nu^{\phi}(a+b) &= \mu(\phi(a+b)) \\ &= \mu(\phi(a),\phi(b)) \\ &\geq T\{\mu(\phi(a)),\mu(\phi(b))\} \\ &= T\{\nu(a),\nu(b)\}\end{aligned}$$

$$\begin{aligned}\nu^{\phi}(a\alpha b\beta c) &= \mu(\phi(a\alpha b\beta c)) \\ &= \mu(\phi(a)\phi(b)\phi(c)) \\ &\geq T\{T\{\mu(\phi(a)),\mu(\phi(b))\},\mu(\phi(c))\} \\ &= T\{T\{\nu(a),\nu(b)\},\nu(c)\}\end{aligned}$$

This shows that ν is a T-fuzzy ternary $\Gamma-$ subsemiring of Ã.

The proofs of (ii) and (iii) are similar to the proof of (i).

Theorem 3.16 In a ternary Γ-semiring Ã the following are equivalent

(i) μ is a T-fuzzy left(resp.,right,lateral) ideals of Ã.

(ii) $\chi_{\tilde{A}} + \mu \subseteq \mu$ *and* $\chi_{\tilde{A}}\Gamma\chi_{\tilde{A}}\Gamma\mu \subseteq \mu(\mu\Gamma\chi_{\tilde{A}}\Gamma\chi_{\tilde{A}} \subseteq \mu, \chi_{\tilde{A}}\Gamma\mu\Gamma\chi_{\tilde{A}} \subseteq \mu)$ where $\chi_{\tilde{A}}$ is the characteristic function of Ã.

Proof: Let μ be a T-fuzzy left ideal of ternary Γ-semiring Ã. Let x∈ Ã. Suppose there exist a,b,c∈ Ã and α,β∈Γ such that x=aαbβc, then we have

$$(\chi_{\tilde{A}} + \mu)(a+b) = \sup_{a+b=p+q}\{T[\chi_{\tilde{A}}(p),\mu(q)],\ for\ all\ p,q \in \widetilde{A}\}$$

$$\leq \sup_{a=u+v,b=s+t}\{T[\chi_{\tilde{A}}(u+s),\mu(v+t)],\ for\ all\ u,v,s,t \in \widetilde{A}\}$$

$$\leq \sup_{a=u+v,b=s+t}\{T[T[\chi_{\tilde{A}}(u),\chi_{\tilde{A}}(s)],T[\mu(v),\mu(t)]],\ for\ all\ u,v,s,t \in \widetilde{A}\}$$

$$= \sup_{a=u+v,b=s+t}\{T[T[\chi_{\tilde{A}}(u),\mu(v)],T[\chi_{\tilde{A}}(s),,\mu(t)]],\ for\ all\ u,v,s,t \in \widetilde{A}\}$$

$$= \sup_{a=u+v,b=s+t}\{T[T[\chi_{\tilde{A}}(u),\mu(v)],T[\chi_{\tilde{A}}(s),\mu(t)]],\ for\ all\ u,v,s,t \in \widetilde{A}\}$$

$$= T\{\sup_{a=u+v}\{T[\chi_{\tilde{A}}(u), \mu(v)]\}, \sup_{b=s+t}\{T[\chi_{\tilde{A}}(s), \mu(t)]\},\ for\ all\ u,v,s,t \in \tilde{A}\}$$

$$= T[(\chi_{\tilde{A}} + \mu)(a), \chi_{\tilde{A}} + \mu)(b)]$$

$$= T[T[\chi_{\tilde{A}}(a), \mu(a)], T[\chi_{\tilde{A}}(b), \mu(b)]]$$

$$= T[T[1, \mu(a)], T[1, \mu(b)]]$$

since $\chi_{\tilde{A}}$ is the characteristic function of $\tilde{A}$

$$= T[\mu(a), \mu(b)]$$

Therefore $(\chi_{\tilde{A}} + \mu)(a+b) \leq T[\mu(a), \mu(b)]$

(i.e) $\chi_{\tilde{A}} + \mu \subseteq \mu$ and

$$(\chi_{\tilde{A}}\Gamma\chi_{\tilde{A}}\Gamma\mu)(x) = \sup_{x=a\alpha b\beta c}\{T[T[\chi_{\tilde{A}}(a), \chi_{\tilde{A}}(b)], \mu(c)]\}$$

$$= \sup_{x=a\alpha b\beta c}\{T[T[1,1], \mu(c)]\}$$

$$= \sup_{x=a\alpha b\beta c}\{T[1, \mu(c)\}$$

$$= \sup_{x=a\alpha b}\{\mu(c)\}$$

$$= \mu(c)$$

Now since μ is a T-fuzzy left ideal of Ã then

$$\mu(a\alpha b\beta c) \geq \mu(c)\ ; \forall a,b,c \in \tilde{A}\ and\ \alpha, \beta \in \Gamma$$

So in particular $\mu(x) \geq \mu(c)$; $\forall\ x = a\alpha b\beta c$

Hence $\sup_{x=a\alpha b\beta c} \mu(c) \leq \mu(x)$

Thus $\mu(x) \geq (\chi_{\tilde{A}}\Gamma\chi_{\tilde{A}}\Gamma\mu)(x)$

If there do not exist a,b,c∈ Ã and α,β∈ Γ such that x=aαbβc then $(\chi_{\tilde{A}}\Gamma\chi_{\tilde{A}}\Gamma\mu)(x) = 0 \leq \mu(x)$

Therefore $\chi_{\tilde{A}}\Gamma\chi_{\tilde{A}}\Gamma\mu \subseteq \mu$

Conversely,

Suppose that $\chi_{\tilde{A}}\Gamma\chi_{\tilde{A}}\Gamma\mu \subseteq \mu$. Let a,b,c∈ Ã and α,β∈ Γ such that x=aαbβc, then

$\mu(a\alpha b\beta c) = \mu(x) \geq (\chi_{\tilde{A}}\Gamma\chi_{\tilde{A}}\Gamma\mu)(x)$

Now,

$$(\chi_{\tilde{A}}\Gamma\chi_{\tilde{A}}\Gamma\mu)(x) = \sup_{x=a\alpha b\beta c}\{T[T[\chi_{\tilde{A}}(a), \chi_{\tilde{A}}(b)], \mu(c)]\}$$

$$= \sup_{x=a\alpha b\beta c}\{T[T[1,1], \mu(c)]\}$$

$$= \sup_{x=a\alpha b\beta c}\{T[1, \mu(c)]\}$$

$$= \sup_{x=a\alpha b\beta c}\{\mu(c)\}$$

$$= \mu\ (c)$$

Hence $\mu(a\alpha b\beta c) \geq \mu(c), \forall a,b,c \in \tilde{A}\ and \alpha, \beta \in \Gamma.$ Therefore μ is a T-fuzzy left ideal of ternary Γ-semiring Ã.

Similarly, we can prove for T-fuzzy right ideal and T-fuzzy lateral ideal of ternary Γ-semiring Ã.

Corollary 3.17 In a ternary Γ-semi-ring Ã the following are equivalent

(i) μ is a fuzzy left(resp.,right,lateral) ideals of Ã.

(ii) $\chi_{\tilde{A}} + \mu \subseteq \mu\, and\, \chi_{\tilde{A}}\Gamma\chi_{\tilde{A}}\Gamma\mu \subseteq \mu, (\mu\Gamma\chi_{\tilde{A}}\Gamma\chi_{\tilde{A}} \subseteq \mu, \chi_{\tilde{A}}\Gamma\mu\Gamma\chi_{\tilde{A}} \subseteq \mu)$ where $\chi_{\tilde{A}}$ is the characteristic function of Ã.

Proof: By using the theorem 3.16, we can find the proof of this corollary easily.

Theorem 3.18 Let μ, λ, δ are T-fuzzy left(resp.,right, lateral) ideal of ternary Γ-semiring Ã then $\mu \circ \lambda \circ \delta$ is also a T-fuzzy left(resp.,right, lateral) ideal of Ã.

Proof: Let μ, λ, δ are T-fuzzy left(resp.,right, lateral) ideal of ternary Γ-semiring Ã and a,b,c∈ Ã ; α,β∈ Γ

Consider,

$$(\mu \circ \lambda \circ \delta)(a+b) = \sup_{a+b=\sum_{i=1}^{n} a_i\alpha_i b_i\beta_i c_i} \left\{ \underset{1\leq i\leq n}{T} [T[\mu(a_i), \lambda(b_i)], \delta(c_i)] : a_i, b_i, c_i \in \tilde{A}; \alpha_i, \beta_i \in \Gamma, n \in Z^+ \right\}$$

$$\geq \sup_{a=\sum_{i=1}^{n} a_i\alpha_i b_i\beta_i c_i,\, b=\sum_{k=1}^{l} p_k\alpha_k q_k\beta_k r_k} \{ \underset{1\leq j\leq m, 1\leq k\leq l}{T} [T[T[\mu(a_j),\lambda(b_j)],\delta(c_j)], T[T[\mu(p_k),\lambda(q_k)],\delta(r_k)]]$$

$$for\ all\ a_j, b_j, c_j, p_k, q_k, r_k \in \widetilde{A}; \alpha_j, \beta_j, \alpha_k, \beta_k \in \Gamma; m, l \in Z^+\}$$

$$= T[\sup_{a=\sum_{i=1}^{n} a_i\alpha_i b_i\beta_i c_i,} \{ \underset{1\leq j\leq m}{T} [T[T[\mu(a_j),\lambda(b_j)],\delta(c_j)]],\ for\ all\ a_j, b_j, c_j \in \widetilde{A}; \alpha_j, \beta_j \in \Gamma; m \in Z^+\},$$

$$\sup_{b=\sum_{k=1}^{l} p_k\alpha_k q_k\beta_k r_k} \{ \underset{1\leq k\leq l}{T} [T[T[\mu(p_k),\lambda(q_k)],\delta(r_k)]],\ for\ all\ p_k, q_k, r_k \in \widetilde{A};, \alpha_k, \beta_k \in \Gamma; l \in Z^+\}]$$

$$= T[(\mu\circ\lambda\circ\delta)(a), (\mu\circ\lambda\circ\delta)(b)]$$

Therefore $(\mu\circ\lambda\circ\delta)(a+b) \geq T[(\mu\circ\lambda\circ\delta)(a), (\mu\circ\lambda\circ\delta)(b)]$

Also,

$$(\mu\circ\lambda\circ\delta)(a\alpha b\beta c) = \sup_{a\alpha b\beta c=\sum_{i=1}^{n} x_i\alpha_i y_i\beta_i z_i} \{ \underset{1\leq i\leq n}{T} [T[T[\mu(x_i),\lambda(y_i)],\delta(z_i)]],\ for\, all\, x_i, y_i, z_i \in \widetilde{A}; \alpha_i, \beta_i \in \Gamma; n \in Z^+\}$$

$$\geq \sup_{c=\sum r_j\alpha_j s_j\beta_j t_j} \{ \underset{1\leq j\leq m}{T} [T[T[\mu(a\alpha b\beta r_j),\lambda(s_j)],\delta(t_j)]],\ for\, all\, r_j, s_j, t_j \in \widetilde{A}; \alpha_j, \beta_j \in \Gamma; n \in Z^+\}$$

$$\geq \sup_{c=\sum r_j\alpha_j s_j\beta_j t_j} \{ \underset{1\leq j\leq m}{T} [T[T[\mu(r_j),\lambda(s_j)],\delta(t_j)]],\ for\, all\, r_j, s_j, t_j \in \widetilde{A}; \alpha_j, \beta_j \in \Gamma; n \in Z^+\}$$

$$= (\mu\circ\lambda\circ\delta)(c)$$

Therefore $(\mu\circ\lambda\circ\delta)(a\alpha b\beta c) = (\mu\circ\lambda\circ\delta)(c)$

Hence $\mu\circ\lambda\circ\delta$ is T-fuzzy left ideal of Ã. Similarly, we can prove for T-fuzzy right ideal and T-fuzzy lateral ideal of Ã.

Corollary 3.19 Let μ,λ,δ are fuzzy ideal of ternary Γ-semiring Ã then $\mu\circ\lambda\circ\delta$ is also a fuzzy ideal of Ã.

Proof: By using the theorem 3.13, we can prove this corollary easily.

Theorem 3.20 Let μ,λ,δ are T-fuzzy left(resp.,right, lateral) ideals of ternary Γ-semiring Ã then

(i) μ,λ is commutative

(ii) μ,λ,δ is associative

(iii) μ is idempotent

Proof: Let μ, λ, δ are T-fuzzy left(resp.,right, lateral) ideals of ternary Γ-semiring Ã then

(i) To prove $\mu + \lambda = \lambda + \mu$

$$(\mu + \lambda)(a) = \sup_{a=x+y}\{T[\mu(x), \lambda(y)] : x, y \in \tilde{A}\}$$

$$= \sup_{a=x+y}\{T[\lambda(y), \mu(x)] : x, y \in \tilde{A}\}$$

$$= (\lambda + \mu)(a)$$

Therefore $\mu + \lambda = \lambda + \mu$.

(i.e) μ, λ is commutative.

(ii) To prove $[(\mu + \lambda) + \delta] = [\mu + (\lambda + \delta)]$

Consider,

$$((\mu + \lambda) + \delta)(a) = \sup_{a=x+y}\{T[(\mu + \lambda)(x), \delta(y)],\ for\,all\, x, y \in \tilde{A}\}$$

$$= \sup_{a=x+y}\{T[\sup_{x=u+v}\{T[\mu(u), \lambda(v)],\ for\,all\, u, v \in \tilde{A}\}, \delta(y)],\ for\,all\, x, y \in \tilde{A}\}$$

$$= \sup_{a=x+y}\{\sup_{x=u+v}\{T[T[\mu(u), \lambda(v)], \delta(y)],\ for\,all\, u, v, x, y \in \tilde{A}\}\}$$

$$= \sup_{a=u+v+y}\{T[T[\mu(u), \lambda(v)], \delta(y)],\ for\,all\, u, v, x, y \in \tilde{A}\} \cdots\cdots\cdots (1)$$

Also consider,

$$[\mu + (\lambda + \delta)](a) = \sup_{a=u+x}\{T[\mu(u), (\lambda + \delta)(x)] : u, x \in \tilde{A}\}$$

$$= \sup_{a=u+x}\{T[\mu(u), \sup_{x=v+y}\{T[\lambda(v), \delta)(y)] : v, y \in \tilde{A}\}] : u, x \in \tilde{A}\}$$

$$= \sup_{a=u+v+y}\{T[\mu(u), T[\lambda(v), \delta)(y)]] : u, v, x, y \in \tilde{A}\} \quad \ldots\ldots[2]$$

From $[1]\, and\, [2]$

$$[(\mu + \lambda) + \delta](a) = [\mu + (\lambda + \delta)](a)$$

Therefore, $(\mu+\lambda)+\delta=\mu+(\lambda+\delta)$

Hence μ,λ,δ are associative.

(iii) To prove $\mu+\mu=\mu$

Let $a\in\tilde{\mathrm{A}}$

$$(\mu+\mu)(a)=\sup_{a=x+y}\{T[\mu(x),\mu(y)]:x,y\in\tilde{\mathrm{A}}\}$$

$$=\sup_{a=x+y}\{\mu(x+y);x,y\in\tilde{\mathrm{A}}\}$$

$$=\mu(a)$$

Hence, $\mu+\mu\subseteq\mu\ldots\ldots[1]$

Again, $\mu(a)=T[\lambda(0),\mu(a)]$

$$\leq\sup_{a=x+y}\{T[\mu(x),\mu(y)]:x,y\in\tilde{\mathrm{A}}\}$$

$$=(\mu+\mu)(a)$$

Therefore, $\mu\subseteq\mu+\mu\ldots\ldots[2]$

From $[1] and [2]$

$$\mu+\mu=\mu$$

Hence μ is idempotent.

Defintion 3.21 Let μ *and* λ be fuzzy subsets of Ã. Then the cartesian product of μ *and* λ is defined by

$$(\mu\times\lambda)(a,b)=T[\mu(a),\lambda(b)],\ for\, all\, a,b\in\tilde{\mathrm{A}}$$

Theorem 3.22 Let μ *and* λ be T-fuzzy left ideal of ternary Γ-semiring Ã. Then $\mu\times\lambda$ is also T-fuzzy left ideal of ternary Γ-semiring Ã×Ã.

Proof: Let μ *and* λ be T-fuzzy left ideal of ternary Γ-semiring Ã and $(x,r),(y,s).(z,t)\in$ Ã×Ã; $\alpha,\beta\in\Gamma$, then

(i) $(\mu\times\lambda)((x,r)+(y,s))$ = $(\mu\times\lambda)((x+y),(r+s))$

$$= T(\mu(x+y), \lambda(r+s))$$

$$\geq T(T(\mu(x), \mu(y)), T(\lambda(r), \lambda(s)))$$

$$= T(T(\mu(x), \lambda(r)), T(\mu(y), \lambda(s)))$$

$$= T((\mu \times \lambda)(x,r), (\mu \times \lambda)(y,s))$$

(ii) $\mu \times \lambda((x,r)\alpha(y,s)\beta(z,t)) = \mu \times \lambda((x\alpha y\beta z, r\alpha s\beta t))$

$$= T(\mu(x\alpha y\beta z), \lambda(r\alpha s\beta t))$$

$$\geq T(\mu(z), \lambda(t)) \quad (Since\ \mu, \lambda\ is\, a\, T - fuzzy left ideal of\ \tilde{A})$$

$$= \mu \times \lambda(z,t)$$

Thus $\mu \times \lambda$ is a T-fuzzy left ideal of ternary Γ-semiring Ã×Ã.

4. CONCLUSION

In this paper, we introduced the notion of T-fuzzy ideals, T-fuzzy left(resp., right, lateral) ideal in ternary Γ-semiring. Also we investigated algebraic operations on T-fuzzy ideals in ternary Γ-semiring and we have proved subsequently these operations give rise to different structures on T-fuzzy ideals in ternary Γ-semiring. A characterization of ternary Γ-semiring has been obtained in terms of fuzzy subsets.

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