PEROFRMANCE ANALYSIS OF K-PHASE FUZZY QUEUEING SYSTEM

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Abstract

In any situation the decision maker did not like breaking to any system. Consider the error optimality value and simulate the corresponding system and find the best optimal interval value using α - cut approach. The basis of the α - cut representation and the extension principle of a pair of mathematical program are formulated and derived the performance measure in the family of crisp queue. Here the queue parameters are considered as a triangular fuzzy number.

Keywords – Triangular Fuzzy Number, α - cut membership function, NLP,

I. INTRODUCTION

Generally, the decision maker take a decision based on improve the quality and profit. But the category depends on the human resources, quality, and employer's skills to control the service station. But the service provided due to some interval. Clearly, requirement of the arrival and service are getting in randomly the decision maker decision is good that is entire of the system is controllable and profitable. In case any one of the parameters are loss the function decision maker has faced so many problem because irregularity of the arrival and service for that way entire system has collapsed and suffered in economically, so, we never expected the exact optimal value. Queuing theory is most helpful to simulate the problem. A.K.Erlang has very first introduced the concept of queueing theory and stated many results. Paricularly, Kuo-Hsiung Wang et al., presented a recursive method, using the supplementary variable technique in remaining service time and to analyzed the steady state probability distribution of the number of customer in the finite system.Mishra.S.S and Dinesh KumarYadav discussed the Cost and Profit analysis of the k-phase erlang distribution in single service queuing system M / E_k / 1 with Removable Service Station. Latter, Many researchers has derived different algorithms on how to control the error value of the queuing system. But due to some insufficient human resources or some natural calamities the queues parameters are not certain. So, we are never obtained the exact optimal value. In this case fuzzy set theory is most helpful to analyze the exact optimal value.

II.MODEL DESCRIPTION

Let us consider the FM/FE_k/1 queueing model with removable service station in which idle fraction of server's time is optimized by shifting the server to another service station in the turned off state of the system. We may also consider the arrival follows the Poisson and service follows the Erlang distribution with parameter k and μ. The k-type Erlang distribution made up of K independent and identical exponential stages each with mean $1/k\mu$. In this paper we follow the First Come First service Discipline and they are interested in the optimal management policy of the FM/FE_k/1queueing system. We develop the membership function of the total expected cost function per unit time. Then we will try to find the optimum value of the management parameter N, denoted by N^*.The total expected cost function per unit time is given by $G(N) = H\frac{N}{2} + \frac{(S+R)x(y-x)}{Ny}$ where x, y, h, s, r are the fuzzy variables. Corresponding to

$\overline{\lambda}$ →arrival rate,Ch→holding cost, Cs→start-up cost and C_r→removable cost. N is the optimal management policy given by $N^* = \left(\frac{2(S+R)x(y-x)}{Hy}\right)^{\frac{1}{2}}$.

III.SOLUTION PROCEDURE

Here, the parameters of removable queueing systems are considered as a fuzzy numbers and it's denoted by $\overline{\lambda}, \overline{\mu}, \overline{C_h}, \overline{C_s} and \overline{C_r}$ .In this model parameters are defined as follows: $\overline{\lambda} = \{(x, \mu_{\overline{\lambda}}(x)) / x \in X\}$, $\overline{\mu} = \{(y, \mu_{\overline{\lambda}}(y)) / y \in Y\}$, $\overline{C_h} = \{(h, \mu_{\overline{c_h}}(h) / h \in H)\}$, $\overline{C_s} = \{(s, \mu_{\overline{c_s}}(s) / s \in S)\}$, $\overline{C_r} = \{(r, \mu_{\overline{c_r}}(r) / r \in R)\}$ $\overline{C_h} = \{(h, \mu_{\overline{C_h}}(h)) / h \in H\}$, where X ,Y,S,H and R are the crisp rate, and $\mu_{\overline{\lambda}}(x)$, $\mu_{\overline{\mu}}(y)$, $\mu_{\overline{c_h}}(h)$, $\mu_{\overline{c_s}}(s)$ and $\mu_{\overline{c_r}}(r)$ are the corresponding membership functions. The performance of the system is mentioned by P(x,y,r,s,h). If $\overline{\lambda}, \overline{\mu}, \overline{c}, \overline{r}$ and $\overline{h}$ are fuzzy numbers then P($\overline{\lambda}, \overline{\mu}, \overline{c}, \overline{r}, \overline{h}$) is also fuzzy number. On the basis of Zadeh's extension principle The performance measure functions is defined as

$$\mu_{\overline{p}(\overline{\lambda}, \overline{\mu}, \overline{c}, \overline{s}, \overline{h})}(z) = \sup \min \left\{\mu_{\overline{\lambda}}(x), \mu_{\overline{\mu}}(y), \mu_{\overline{c_h}}(h), \mu_{\overline{c_s}}(s), \mu_{\overline{c_r}}(r) / z = p(x, y, h, r, s)\right\} ------(I)$$

where $z = P(x, y, r, s, h) = H\left(\frac{N}{2}\right) + \frac{(S+R)x(y-x)}{Ny}$. We extended in this system using α cut approach to develop the Mathematical form of Non-Linear Programming.

IV.PROPOSED ALGORITHM OF NLP

It is helpful to analyze the fuzzy membership function of the parameters and to construct the membership function $\mu(\overline{\lambda}, \overline{\mu}, \overline{c_h}, \overline{c_r}, \overline{c_s})$ is formation on the basis derivation of α - cut and removable station parameters as follows: $\lambda(\alpha) = [X_\alpha^L, X_\alpha^U] = \left[\min_{x \in X}\{x / \mu_{\overline{\lambda}}(x) \geq \alpha\}, \max_{x \in X}\{x / \mu_{\overline{\lambda}}(x) \geq \alpha\}\right]$, $\mu(\alpha) = [Y_\alpha^L, Y_\alpha^U] = \left[\min_{y \in Y}\{y / \mu_{\overline{\mu}}(y) \geq \alpha\}, \max_{y \in Y}\{y / \mu_{\overline{\mu}}(y) \geq \alpha\}\right]$, $C_h(\alpha) = [w_\alpha^L, w_\alpha^U] = \left[\min_{h \in H}\{h / \mu_{\overline{C_h}}(h) \geq \alpha\}, \max_{h \in H}\{h / \mu_{\overline{C_h}}(h) \geq \alpha\}\right]$, $C_s(\alpha) = [s_\alpha^L, s_\alpha^U] = \left[\min_{s \in S}\{s / \mu_{\overline{C_s}}(s) \geq \alpha\}, \max_{s \in S}\{s / \mu_{\overline{C_s}}(s) \geq \alpha\}\right]$ $C_r(\alpha) = [r_\alpha^L, r_\alpha^U] = \left[\min_{r \in R}\{r / \mu_{\overline{C_r}}(r) \geq \alpha\}, \max_{r \in R}\{r / \mu_{\overline{C_r}}(r) \geq \alpha\}\right]$.

The parameters of the removable system can be represented as a different level of intervals [14, 19]. Subsequently $FM / FE_k / 1$ can be reduced the family of crisp $M / E_k / 1$ queues with different of α - level sets. For this set of relationship between the ordinary set and fuzzy sets [5]. There are some more methods solving these problems [20]. Moreover, analyze how to change the optimal solution as Y_α^l , Y_α^u , S_α^l , S_α^u , R_α^l , R_α^u . X_α^l , X_α^u H_α^l and H_α^u Where $\alpha \in [0,1]$; they fall into the category of NLP [21]. The level of α sets can be obtained as the convexity of the fuzzy numbers as follows: $X_\alpha^L = \min \mu_{\overline{\lambda}}^{-1}(x)$, $X_\alpha^U = \max \mu_{\overline{\lambda}}^{-1}(x)$, $Y_\alpha^L = \min \mu_{\overline{\mu}}^{-1}(y)$, $Y_\alpha^U = \max \mu_{\overline{\mu}}^{-1}(y)$, $H_\alpha^L = \min \mu_{\overline{C_h}}^{-1}(h)$, $H_\alpha^U = \max \mu_{\overline{C_h}}^{-1}(h)$ $S_\alpha^L = \min \mu_{\overline{C_s}}^{-1}(s)$, $S_\alpha^U = \max \mu_{\overline{C_s}}^{-1}(s)$, $R_\alpha^L = \min \mu_{\overline{C_r}}^{-1}(r)$, $R_\alpha^U = \max \mu_{\overline{C_r}}^{-1}(r)$. Using Zadeh's Extension Principle, we obtained the performance of the parameter are as follows

$$\mu_{\overline{p}(\overline{\lambda}, \overline{\mu}, \overline{C_h}, \overline{C_s}, \overline{C_r})}(Z)^L = \sup_{\substack{x \in X, y \in Y, h \in H, \\ s \in S, r \in R}} \min\left\{\left(\mu_{\overline{\lambda}}(x), \mu_{\overline{\mu}}(y), \mu_{\overline{C_h}}(h), \mu_{\overline{C_s}}(S), \mu_{\overline{C_r}}(r)\right)\right\},$$

$$\mu_{\overline{p}(\overline{\lambda}, \overline{\mu}, \overline{C_h}, \overline{C_s}, \overline{C_r})}(Z)^U = \sup_{\substack{x \in X, y \in Y, h \in H, \\ s \in S, r \in R}} \max\left\{\left(\mu_{\overline{\lambda}}(x), \mu_{\overline{\mu}}(y), \mu_{\overline{C_h}}(h), \mu_{\overline{C_s}}(S), \mu_{\overline{C_r}}(r)\right)\right\}$$

At least one of the following cases must hold such that Z=P(x,y,h,s,r) satisfied as follows:

$\mu_{\overline{p}(x,y,h,r,s)}(z) = \alpha$,

(i) $\{\mu_{\bar{\lambda}}(x)=\alpha, \mu_{\bar{\mu}}(y)\geq\alpha, \mu_{\bar{C_h}}(H)\geq\alpha, \mu_{\bar{C_s}}(S)\geq\alpha \text{ and } \mu_{\bar{C_r}}(R)\geq\alpha\}$

(ii) $\{\mu_{\bar{\lambda}}(x)\geq\alpha, \mu_{\bar{\mu}}(y)=\alpha, \mu_{\bar{C_h}}(H)\geq\alpha, \mu_{\bar{C_s}}(S)\geq\alpha \text{ and } \mu_{\bar{C_r}}(R)\geq\alpha\}$

(iii) $\{\mu_{\bar{\lambda}}(x)\geq\alpha, \mu_{\bar{\mu}}(y)\geq\alpha, \mu_{\bar{C_h}}(H)=\alpha, \mu_{\bar{C_s}}(S)\geq\alpha \text{ and } \mu_{\bar{C_r}}(R)\geq\alpha\}$

(iv) $\{\mu_{\bar{\lambda}}(x)\geq\alpha, \mu_{\bar{\mu}}(y)\geq\alpha, \mu_{\bar{C_h}}(H)\geq\alpha, \mu_{\bar{C_s}}(S)=\alpha \text{ and } \mu_{\bar{C_r}}(R)\geq\alpha\}$

(v) $\{\mu_{\bar{\lambda}}(x)\geq\alpha, \mu_{\bar{\mu}}(y)\geq\alpha, \mu_{\bar{C_h}}(H)\geq\alpha, \mu_{\bar{C_s}}(S)\geq\alpha \text{ and } \mu_{\bar{C_r}}(R)=\alpha\}$

This can be accomplished by the parameter NLP technique. The performance calculated as $\bar{P}(x,y,h,r,s)=H\frac{N}{2}+\frac{(S+R)x(y-x)}{Ny}$ and derived the upper and lower bound of $\mu_{\bar{p}(x,y,h,r,s)}(Z)$ are as follows: $C_s(\alpha)=[s_\alpha^L, s_\alpha^U]=\mu_{\bar{p}}(z)$

$\lfloor\mu_{\bar{p}}(z)\rfloor_\alpha^{L_1}=\min_{x,y,h,r,s\in R}\{P(x,y,h,r,s)\}$ Such that $X_\alpha^L\leq x\leq X_\alpha^U, y\in\mu(\alpha), h\in C_h(\alpha), s\in C_s(\alpha) \text{ and } r\in C_r(\alpha)$

$\lfloor\mu_{\bar{p}}(z)\rfloor_\alpha^{U_1}=\max_{x,y,h,r,s\in R}\{P(x,y,h,r,s)\}$ Such that $X_\alpha^L\leq x\leq X_\alpha^U, y\in\mu(\alpha), h\in C_h(\alpha), s\in C_s(\alpha) \text{ and } r\in C_r(\alpha)$

Similarly, we derived $C_r(\alpha)=[r_\alpha^L, r_\alpha^U]=\mu_{\bar{p}}(z)$ $C_h(\alpha)=[h_\alpha^L, h_\alpha^U]=\mu_{\bar{p}}(z)$. The membership function $\mu_{\bar{G}(N)}$ is serves to interval of $[G(N)_\alpha^L, G(N)_\alpha^U]$ of α-cuts of can be written as a pair of mathematical programming as: $G(N)_\alpha^L=\min_{x,y,h,r,s}\left\{H\frac{N}{2}+\frac{(S+R)x(y-x)}{Ny}\right\}$

Such that $X_\alpha^L\leq x\leq X_\alpha^U, Y_\alpha^L\leq y\leq Y_\alpha^U$ $H_\alpha^L\leq h\leq H_\alpha^U$ $S_\alpha^L\leq s\leq S_\alpha^U$ and $R_\alpha^L\leq r\leq R_\alpha^U$.

Using G(N) to make a differentiation with respect to the parameter we easily obtained the extreme value of the function which formulate $\mu_{\bar{p}}^U=\min_{N_0>0}G(N)$ and the error optimum can be calculated the interval $\lfloor\mu_{\bar{p}}^L, \mu_{\bar{p}}^U\rfloor$ which is represented as the α-cuts of $\mu_p(Z)$. In other words $G(N)_\alpha^L$ is increasing with respect to α and $G(N)_\alpha^U$ is decreasing with respect toα. If both $G(N)_\alpha^L$ and $G(N)_\alpha^U$ are invertible with respect toα, a left shape function $L(G)=[G(N)_\alpha^L]^{-1}$ can be obtained from which the membership function is constructed

$$\mu_{\bar{G}(N)}=\begin{cases}L(Z) & ;G(N)_{\alpha=0}^L\leq z\leq G(N)_{\alpha=1}^L\\ 1 & ;G(N)_{\alpha=1}^L\leq z\leq G(N)_{\alpha=1}^U\\ R(Z) & ;G(N)_{\alpha=1}^U\leq z\leq G(N)_{\alpha=1}^L\end{cases}$$

But $\mu_p(Z)$ cannot be solved numerically because we get data's are different levels of α can be together to approximate the shapes of L(Z) and R(Z). Since the optimal beginning is described by a membership function, the value conserves completely all of the fuzziness parameters. So, using Yager's ranking index method easily obtained the optimal value and formulated as $O(N^*)=\int_0^1\frac{(N^*)_\alpha^L+(N^*)_\alpha^U}{2}dx$. If $O(N^*)$ is not an integer, the best positive integer value of N is one of the integers surrounding $O(N^*)$. The corresponding optimum cost $O(G(N))=\int_0^1\frac{G(N_\alpha)^L+G(N_\alpha)^U}{2}dx$.

V. NUMERICAL EXAMPLE

Consider a M/Ek/1 queuing system of N policy with removable service station. Also we may consider the arrival and service rate are λ and μ and the corresponding holding cost, setup cost and removable cost per unit as C_h, C_s and C_r respectively. Let us assume that all the parameters are fuzzy number, $\bar{\lambda}$ = (2 3 4); $\bar{\mu}$ = (7 8 9); $\bar{C_h}$ =(0.4 0.5 0.6); $\bar{C_s}$ =(20 30 40); $\bar{C_r}$ =(0.1 0.2 0.3)Theα-cut value of arrival rate, service rate, Holding cost, set-up cost and removable cost are

$\overline{\lambda} = [x_\alpha^L, x_\alpha^U] = (\alpha + 2,\ 4 - \alpha)$ $\overline{\mu} = [y_\alpha^L, y_\alpha^U] = (\alpha + 7,\ 9 - \alpha)$ $\overline{C_h} = [w_\alpha^L, w_\alpha^U] = (0.1\alpha + 0.4,\ 0.6 - 0.1\alpha)$;
$\overline{C_s} = [v_\alpha^L, v_\alpha^U] = (10\alpha + 20,\ 40 - 10\alpha)$ $\overline{C_r} = [u_\alpha^L, u_\alpha^U] = (0.1\alpha + 0.1,\ 0.3 - 0.1\alpha)$.

The optimal management policy N is given by $N = \left[\frac{2x(u+v)(y-x)}{wy}\right]^{\frac{1}{2}}$, The upper and lower bound value of this policy is given by , $N_\alpha^L = \left[\frac{2(10.1\alpha + 20.1)(\alpha+2)(\alpha+7-\alpha-2)}{(0.1\alpha+0.4)(\alpha+7)}\right]^{\frac{1}{2}} = \left[\frac{10(10.1\alpha+20.1)(\alpha+2)}{(0.1\alpha+0.4)(\alpha+7)}\right]^{\frac{1}{2}}$

$N_\alpha^U = \left[\frac{2(40.3-10.1\alpha)(4-\alpha)(9-\alpha-4+\alpha)}{(0.6-0.1\alpha)(9-\alpha)}\right]^{\frac{1}{2}} = \left[\frac{10(40.3-10.1\alpha)(4-\alpha)}{(0.6-0.1\alpha)(9-\alpha)}\right]^{\frac{1}{2}}$.

The total expected cost function per unit time of per customer is given by $G(N) = \left[\frac{2w(u+v)(y-x)}{y}\right]^{\frac{1}{2}}$ where x, y, h, r and s are the fuzzy variable corresponding to $\overline{\lambda}$, $\overline{\mu}$, $\overline{C_h}$, $\overline{C_s}$ and $\overline{C_r}$ respectively. The upper and lower bound value of total expected cost function is

$G(N)_\alpha^L = \left[\frac{2(0.1\alpha+0.4)(\alpha+2)(10.1\alpha+20.1)(\alpha+7-\alpha-2)}{(\alpha+7)}\right]^{\frac{1}{2}} = \left[\frac{10(0.1\alpha+0.4)(\alpha+2)(10.1\alpha+20.1)}{(\alpha+7)}\right]^{\frac{1}{2}}$,

$G(N)_\alpha^U = \left[\frac{2(0.6-0.1\alpha)(40.3-10.1\alpha)(4-\alpha)(9-\alpha-4+\alpha)}{(9-\alpha)}\right]^{\frac{1}{2}} = \left[\frac{10(0.6-0.1\alpha)(40.3-10.1\alpha)(4-\alpha)}{(9-\alpha)}\right]^{\frac{1}{2}}$

Table

α	N_α^L	N_α^U	N	$G(N)_\alpha^L$	$G(N)_\alpha^u$	G(N)
0	11.9821	17.2777	14.6299	4.7929	10.3667	7.5798
0.1	12.3405	17.0825	14.7115	5.0596	10.0787	7.56915
0.2	12.6857	16.8822	14.78395	5.328	9.7915	7.55975
0.3	13.0184	16.6757	14.84705	5.598	9.5051	7.55155
0.4	13.3393	16.4635	14.9014	5.8693	9.2196	7.54445
0.5	13.649	16.2451	14.94705	6.1421	8.9348	7.53845
0.6	13.948	16.0203	14.98415	6.4162	8.6509	7.53355
0.7	14.2375	15.7887	15.0131	6.6916	8.368	7.5298
0.8	14.5172	15.55	15.0336	6.9682	8.086	7.5271
0.9	14.7883	15.3038	15.04605	7.246	7.805	7.5255
1	15.0499	15.0499	15.0499	7.525	7.525	7.525

VI.CONCLUSION

In this paper, Analyzed the system performance of the k-phase fuzzy queuing system with the help of proposed algorithm and also analyzed the optimal cost. G(N) is the line of regression on interval of N and G(N) is zero because of the covariance's zero. So, the process of N and G(N) are independent. Consequently, we obtained exact optimal value. Our proposed method is most helpful to the design of optimization studies.

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