FREE CONVECTION AND HEAT TRASNFER OF A COUETTE FLOW AN INFINITE POROUS PLATE IN THE PRESENCE RADIATION EFFECT

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Abstract - **This paper theoretically analyzes radiation effect on three dimensional Couette flow of a viscous incompressible electrically conducting heat generating fluid between two infinite horizontal parallel porous flat plates in presence of a transverse magnetic field. The stationary plate and the plate in uniform motion are, respectively, subjected to a transverse sinusoidal injection and uniform suction of the fluid. The governing equations of the flow field are solved by using perturbation method and the expressions for the velocity profiles, the temperature profiles, skin friction and heat flux in terms of Nusselt number are obtained. The effects of the flow parameters analyzed with the help of graphs and tables for various parameters involving in the flow problem.**

Keywords: **Hydromagnetic, Radiation, Couette flow, heat transfer, three dimensions**

I. INTRODUCTION

The problem of hydromagnetic Couette flow with heat transfer has been a subject of interest of many researchers because of its possible applications in many branches of science and technology. Channel flows have several engineering and geophysical applications, such as, in the field of chemical engineering for filtration and purification processes; in the field of agricultural engineering to study the underground water resources; in petroleum industry to study the movement of natural gas, oil and water through the oil channels and reservoirs. In view of these applications a series of investigations have been made by different scholars where the medium is either bounded by horizontal or vertical surfaces. Gersten and Gross [10] studied the flow and heat transfer along a plane wall with periodic suction. Gulab and Mishra [11] analyzed the unsteady MHD flow of a conducting fluid through a porous medium. Kaviany [15] explained the laminar flow through a porous channel bounded by isothermal parallel plates. Vajravelu and Hadjinicolaou [26] have investigated the heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. Attia and Kotb [2] explained the MHD flow between two parallel plates with heat transfer. The unsteady hydromagnetic natural convection in a fluid saturated porous channel was studied by Chamkha [5]. Attia [1] analyzed the transient MHD flow and heat transfer between two parallel plates with temperature dependent viscosity. Krishna et. al. [16] presented the hydromagnetic oscillatory flow of a second order Rivlin-Erickson fluid in a channel. Sharma and Yadav [21] analyzed the heat transfer through three dimensional Couette flow between a stationary porous plate bounded by porous medium and a moving porous plate. Sharma et. al. [22] explained the steady laminar flow and heat transfer of a non-Newtonian fluid through a straight horizontal porous channel in the presence of heat source. Recently, Jain et. al. [14] discussed the three dimensional couette flow with transpiration cooling through a porous medium in the slip flow regime, Srinathuni Lavanya and Chenna Kesavaiah [27] studied heat transfer to MHD free convection flow of a viscoelastic dusty gas through a porous medium with chemical reaction, Chenna Kesavaiah and Sudhakaraiah [28] considered effects of heat and mass flux to MHD flow in vertical surface with radiation absorption, Chenna Kesavaiah and Satyanarayana [29] analyzed MHD and Diffusion Thermo effects on flow accelerated vertical plate with chemical reaction, Chenna Kesavaiah et.al. [30] Reviewed radiation and thermo - diffusion effects on mixed convective heat and mass transfer flow of a viscous dissipated fluid over a vertical surface in the presence of chemical reaction with heat source, Karunakar Reddy et.al. [31] Presented MHD heat and mass transfer flow of a viscoelastic fluid past an impulsively started infinite vertical plate with chemical reaction, Ch Kesavaiah et.al. [32] Shown radiation and mass transfer effects on moving vertical plate with variable temperature and viscous dissipation, Chenna Kesavaiah et.al. [35] Analyzed chemical reaction effects on MHD flow over vertical surface through porous medium, Chenna Kesavaiah [36] studied radiative flow of MHD Jeffery fluid over a stretching vertical surface in a porous medium.

Radiation is a form of energy that is emitted or transmitted in the form of rays, electromagnetic waves, and/or particles. In some cases, radiation can be seen (visible light) or felt (infrared radiation), while other forms like x-rays and gamma rays are not visible and can only be observed directly or indirectly with special equipment. Although radiation can have negative effects both on biological and mechanical systems, it can also be carefully used to learn more about each of those systems. The motion of electrically charged particles produces electromagnetic waves. These waves are also called “electromagnetic radiation” because they radiate from the electrically charged particles. They travel through empty space as well as through air and other substances. In our daily lives we are exposed to electromagnetic radiation through the use of microwaves, cell phones, and diagnostic medical applications such as x-rays. In addition to human created technologies that emit electromagnetic radiation such as radio transmitters, light bulbs, heaters, and gamma ray sterilizers (tools that kill microbes in fresh or packaged food), there are many naturally occurring sources of electromagnetic and ionizing radiation. These include radioactive elements in the Earth's crust, radiation trapped in the Earth's magnetic field, stars, and other astrophysical objects like quasars or galactic centers. Radiative convective flow occurs in several industrial and environmental situations. The applications are found mostly in cooling chambers, fossil fuel combustion, energy processes, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat transfer is found to have an important role in manufacturing sectors for the design of highly precision equipment. Generally, nuclear power plants, gas turbines and propulsion devises for air craft, medical applications, missiles and space vehicles are few such examples. Stokes [25] was the first to examine the problem of viscous incompressible fluid past an impulsively started infinite horizontal moving plate in its own plane. Subsequently, Brinkman [4] analyzed the viscous force imparted by a flowing fluid in a dense swarm of particles. Later, an analytical solution for a viscous flow past an impulsively started semi infinite horizontal plate was presented by Stewartson [24]. Thereafter, Berman [3] analyzed the case of two dimensional steady state flow of an incompressible fluid with parallel rigid porous walls, with the flow being influenced by uniform suction or injection. The flow between two vertical plates which are electrically non-conducting and under the assumption that the wall temperature varies linearly in the direction of the flow and existence of heat source in the vertical channel was presented in detail by Mori [19], Macy [17] examined the flow in the renal tubules as viscous flow through circular tube of uniform cross section with permeable boundary by prescribing their radial velocity at the wall as exponentially decreasing function as axial distance. Subsequently, Hall [12] studied similar problem by using finite differences method of a mixed explicit and implicit time for a convergence and stability of the solution. While, Chang et. al. [7] investigated the effects of radioactive heat transfer on free convection regimes in an enclosure with specialized applications in geophysics and geothermal reservoirs. The influence of viscous heat dissipating effect in natural convective flows was presented by Mahajan et. al. [18]. It was established that the heat transfer rates are reduced by an increase in the dissipation parameter. Later, Soundalgekar and Thaker [23] examined the thermal radiation effects of an optionally thin gray gas bounded by a stationary vertical plate. A higher order numerical approximation for the mass transfer effect on the steady flow past an accelerated vertical porous plate was analyzed by Das et. al. [9], while Hossain et. al. [13] analyzed the radiation effects on a mixed convection along a vertical plate with a uniform surface temperature by applying Rossland's approximation. The effects of thermal radiation and convective flow past a moving infinite vertical plate was presented by Raptis and Perdikis [20]. Subsequently, Chandrakala and Antony Raj [6] studied the effects of thermal radiation on the flow past a semi infinite vertical isothermal plate with uniform heat flux in the presence of transversely applied magnetic field, Chenna Kesavaiah and Venkateswarlu [33] studied chemical reaction and radiation absorption effects on convective flows past a porous vertical wavy channel with travelling thermal waves, Srinathuni Lavanya et.al. [34] Considered radiation effect on unsteady free convective MHD flow of a viscoelastic fluid past a tilted porous plate with heat source.

The proposed study considers the radiation effects on three dimensional Couette flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous flat plates in presence of a transverse magnetic field with heat transfer. The stationary plate and the plate in uniform motion are, respectively, subjected to a transverse sinusoidal injection and uniform suction of the fluid. The governing equations of the flow field are solved by using series expansion method and the expressions for the velocity field, the temperature field, skin friction and heat flux in terms of Nusselt number are obtained. The effect of the flow parameters on the velocity field, temperature field, skin friction and Nusselt number have been studied and analyzed with the help of figures and tables. It is observed that the magnetic parameter (M) has a retarding effect on the main velocity (u) and an accelerating effect on the cross velocity $\left(w_1\right)$ of the flow field. The suction parameter (Re) has a retarding effect on the main velocity as well as on the temperature field. The Prandtl number (Pr) reduces the temperature of the flow field and increases the rate of heat transfer at the wall (Nu). The effect of suction parameter is to reduce the x-component of skin friction and to enhance the magnitude of z- component of the skin friction at the wall. The problem is very much significant in view of its several engineering, geophysical and industrial applications.

In view of the above we analyzes radiation effect on three dimensional Couette flow of a viscous incompressible electrically conducting heat generating fluid between two infinite horizontal parallel porous flat plates in presence of a transverse magnetic field. The stationary plate and the plate in uniform motion are, respectively, subjected to a transverse sinusoidal injection and uniform suction of the fluid. The governing equations of the flow field are solved by using perturbation method and the expressions for the velocity profiles, the temperature profiles, skin friction and heat flux in terms of Nusselt number are obtained.

II. FORMULATION OF THE PROBLEM

Consider the three dimensional couette flow of a viscous incompressible electrically radiating heat transfer conducting fluid bounded between two infinite horizontal parallel porous plates in presence of a uniform transverse magnetic field B_0. The physical model and geometry of the problem is shown in **figure (1)**. A coordinate system is chosen with its origin at the lower stationary plate lying horizontally in $x^* - z^*$ plane and the upper plate at a distance *l* from it is subjected to a uniform velocity *U*.

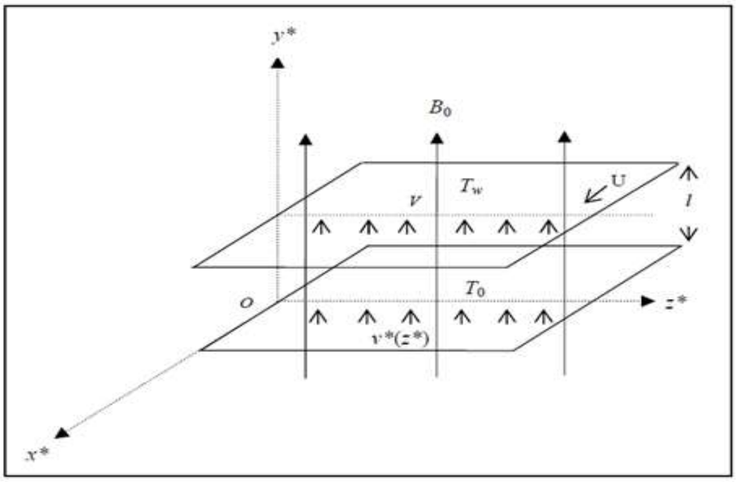

Fig. 1. Physical sketch and geometry of the problem

The $y^* -$axis is taken normal to the planes of the plates. The lower and the upper plates are assumed to be at constant temperatures T_0 and T_w, respectively, with $T_w > T_0$. The upper plate is subjected to a constant suction velocity *V* whereas the lower plate to a transverse sinusoidal injection velocity of the form:

$$v^*\left(z^*\right) = V\left(1 + \varepsilon \cos \pi z^* / l\right) \tag{1}$$

where $\varepsilon\left(<<1\right)$ is a very small positive constant quantity, *l* is taken equal to the wavelength of the injection velocity.

Due to this kind of injection velocity the flow remains three dimensional. All the physical quantities involved are independent of x* for this fully developed laminar flow. Denoting the velocity components u*, v*, w* in x*, y*, z* directions, respectively and the temperature by T*, the problem is governed by the following equations:

$$\frac{\partial v^*}{\partial y^*}+\frac{\partial w^*}{\partial z^*}=0 \tag{2}$$

$$v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}=\nu\left(\frac{\partial^2 u^*}{\partial y^{*^2}}+\frac{\partial^2 u^*}{\partial z^{*^2}}\right)+\frac{\sigma B_0^2}{\rho}u^* \tag{3}$$

$$v^*\frac{\partial v^*}{\partial y^*}+w^*\frac{\partial v^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\nu\left(\frac{\partial^2 v^*}{\partial y^{*^2}}+\frac{\partial^2 v^*}{\partial z^{*^2}}\right) \tag{4}$$

$$v^*\frac{\partial w^*}{\partial y^*}+w^*\frac{\partial w^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\nu\left(\frac{\partial^2 w^*}{\partial y^{*^2}}+\frac{\partial^2 w^*}{\partial z^{*^2}}\right)+\frac{\sigma B_0^2}{\rho}w^* \tag{5}$$

$$v^*\frac{\partial T^*}{\partial y^*}+w^*\frac{\partial T^*}{\partial z^*}=\alpha\left[\frac{\partial^2 T^*}{\partial y^{*^2}}+\frac{\partial^2 T^*}{\partial y^{*^2}}\right]-\frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*}-\frac{Q_0}{\rho C_p}\left(T^*-T_d\right) \tag{6}$$

where ρ is the density, σ is the electrical conductivity, p^* is the pressure, ν is coefficient of the kinematic viscosity and α is the thermal diffusivity

The radiative heat flux is given by Cogly et. al. [8]

$$\frac{\partial q_r^*}{\partial y^*}=4\alpha^2(T^*-T_d)$$

where $\alpha^2=\int_0^\infty \delta\lambda\frac{\partial B}{\partial T^*}$, $B-$ is Planck's function

The boundary conditions for the problem are

$$\begin{aligned}&u^*=0, w^*=0, v^*=V\left(1+\varepsilon\cos\pi z^*/l\right), T^*=T_0^* \quad at\ y^*=0\\ &u^*=U, v^*=V, w^*=0, T^*=T^*_w \qquad at\ y^*=l\end{aligned} \tag{7}$$

Introducing the following non-dimensional quantities,

$$y=\frac{y^*}{l}, z=\frac{z^*}{l}, v=\frac{v^*}{V}, w=\frac{w^*}{V}, u=\frac{u^*}{U}, p=\frac{p^*}{\rho V^2} T=\frac{T^*-T_0^*}{T_w^*-T_0^*} \tag{8}$$

equations (2) - (6) reduce to the following forms

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{9}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{M^2}{\mathrm{Re}}u \tag{10}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{11}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\frac{M^2}{\mathrm{Re}}w \tag{12}$$

$$v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left[\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial y^2}\right]-\frac{4\alpha^2}{\rho C_p}T-\frac{Q_0}{\rho C_p}T \tag{13}$$

Introducing the non-dimensional quantities

$$\mathrm{Re}=\frac{Vl}{\nu},\ M^2=\frac{B_0^{\,2}}{\rho}\frac{l^2}{\nu},\ \mathrm{Pr}=\frac{\nu}{\alpha},\ R=\frac{4\alpha l}{\rho C_p},\ Q_0=\frac{\phi\nu\rho C_p}{l} \tag{14}$$

where Reynolds number, Magnetic parameter, Prandtl number, radiation parameter and heat source parameter

The corresponding boundary conditions become

$$\begin{aligned}&u=0, w=0, v=1+\varepsilon\cos\pi z, T=0 \quad at\ \ y=0\\ &u=1, v=1, w=0, T=1 \qquad\qquad at\ \ y=1\end{aligned} \tag{15}$$

III. METHOD OF SOLUTION

In order to solve the problem, we assume the solutions of the following form because the amplitude $0<\varepsilon<<1$ of the permeability variation is very small:

$$\begin{aligned}u(y,z)&=u_0(y)+\varepsilon u_1(y,z)+\varepsilon^2 u_2(y,z)+\ldots\ldots\ldots\\ v(y,z)&=v_0(y)+\varepsilon v_1(y,z)+\varepsilon^2 v_2(y,z)+\ldots\ldots\ldots\\ w(y,z)&=w_0(y)+\varepsilon w_1(y,z)+\varepsilon^2 w_2(y,z)+\ldots\ldots\ldots\\ p(y,z)&=p_0(y)+\varepsilon p_1(y,z)+\varepsilon^2 p_2(y,z)+\ldots\ldots\ldots\\ T(y,z)&=T_0(y)+\varepsilon T_1(y,z)+\varepsilon^2 T_2(y,z)+\ldots\ldots\ldots\end{aligned} \tag{16}$$

when ε =0, the problem reduces to the two dimensional free convective MHD flow which is governed by the following equations:

$$\frac{dv_0}{dy}=0 \tag{17}$$

$$\frac{d^2u_0}{dy^2}-v_0\,\mathrm{Re}\frac{du_0}{dy}-M^2u_0 \tag{18}$$

$$\frac{d^2T_0}{dy^2}-v_0\,\mathrm{Re}\,\mathrm{Pr}\frac{dT_0}{dy}-\mathrm{Re}\left(R+\mathrm{Pr}\,\phi\right)T_0 \tag{19}$$

The corresponding boundary conditions become

$$\begin{aligned}&u_0=0, v_0=1, T_0=0 \quad at\ \ y=0\\ &u_0=1, v_0=1, T_0=1 \quad at\ \ y=1\end{aligned} \tag{20}$$

The solutions of these equations for this two dimensional problem are

$$u_0(y)=\frac{e^{\lambda_1 y}-e^{\lambda_2 y}}{e^{\lambda_1}-e^{\lambda_2}} \tag{21}$$

$$T_0(y)=\left(\frac{1}{e^{\beta_1}-e^{\beta_2}}\right)e^{\beta_1 y}+\left(\frac{1}{e^{\beta_1}+e^{\beta_2}}\right)e^{\beta_2 y} \tag{22}$$

with $v_0=1, w_0=0, p_0=$ constant (23)

when $\varepsilon \neq 0$, substituting (16) into Equations (9) - (13) and comparing the coefficients of like powers of ε, neglecting those of ε^2, we get the following first order equations with the help of equation (23):

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0 \tag{24}$$

$$v_1\frac{\partial u_0}{\partial y}+\frac{\partial u_1}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right)+\frac{M^2}{\mathrm{Re}}u_1 \tag{25}$$

$$\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right) \tag{26}$$

$$\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right)+\frac{M^2}{\mathrm{Re}}w_1 \tag{27}$$

$$v_1\frac{\partial T_0}{\partial y}+\frac{\partial T_1}{\partial y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left[\frac{\partial^2 T_1}{\partial y^2}+\frac{\partial^2 T_1}{\partial z^2}\right]-RT_1-\phi T_1 \tag{28}$$

The corresponding boundary conditions become

$$\begin{aligned} &u_1=0, v_1=\cos\pi z, w_1=0, T_1=0 \quad at\ y=0 \\ &u_0=0, v_0=0, \quad w_1=0,\ T_0=0 \quad at\ y=1 \end{aligned} \tag{29}$$

equations (24) - (28) are the linear partial differential equations which describe the MHD three-dimensional flow through a porous medium. For solution we shall first consider three Equations (24), (26) and (27) being independent of the main flow component u_1 and the temperature field T_1.

we assume v_1, w_1, and p_1 of the following form:

$$v_1(y,z)=v_{11}(y)\cos\pi z \tag{30}$$

$$w_1(y,z)=\frac{1}{\pi}v_{11}'(y)\sin\pi z \tag{31}$$

$$P_1(y,z)=P_{11}(y)\cos\pi z \tag{32}$$

where the prime in $v'_{11}(y)$ y denotes the differentiation with respect to y. Expressions for $v_1(y,z)$ and $w_1(y,z)$ have been chosen so that the equation of continuity (24) is satisfied.

Substituting these expressions (30) - (32) into (26) and (27) and solving under corresponding transformed boundary conditions, we get the solutions of v_1, w_1 and p_1 as:

$$v_1(y,z) = \frac{1}{A}\left[A_1 e^{m_1 y} + A_2 e^{m_2 y} + A_3 e^{\pi y} + A_4 e^{-\pi y}\right]\cos \pi z \tag{33}$$

$$w_1(y,z) = \frac{1}{\pi A}\left[A_1 m_1 e^{m_1 y} + A_2 m_2 e^{m_2 y} + A_3 \pi e^{\pi y} - A_4 \pi e^{-\pi y}\right]\sin \pi z \tag{34}$$

$$P_1(y,z) = \frac{1}{\pi \operatorname{Re} A}\left[A_3\left(\pi \operatorname{Re} + M^2\right)e^{\pi y} + A_4\left(\pi \operatorname{Re} - M^2\right)e^{-\pi y}\right]\cos \pi z \tag{35}$$

To solve Equations (25) and (28) for u_1 and T_1, we assume

$$u_1(y,z) = u_{11}(y)\cos \pi z \tag{36}$$

$$T_1(y,z) = T_{11}(y)\cos \pi z \tag{37}$$

substituting the values of u_1 and T_1 from Equations (36) and (37) into Equations (25) and (28), we get

$$u''_{11} - \operatorname{Re} u'_{11} - \left(\pi^2 + M^2\right)u_{11} = \operatorname{Re} v_{11} u_0 \tag{38}$$

$$T''_{11} - \operatorname{Re}\Pr T'_{11} - \operatorname{Re}\Pr\left(\pi^2 + \frac{R}{\Pr} + \phi\right)T_{11} = \operatorname{Re}\Pr v_{11} T_0 \tag{39}$$

where the primes denote the differentiation with respect to y

The corresponding boundary conditions are

$$\begin{aligned} u_{11} &= 0, T_{11} = 0 \quad at \ y = 0 \\ u_{11} &= 0, \ T_{11} = 0 \quad at \ y = 1 \end{aligned} \tag{40}$$

solving Equations (38) and (39) under the boundary conditions (40) and using Equations (36) and (37), we get

$$\begin{aligned} u_1(y,z) = \Big[& B_1 e^{-m_3 y} - B_2 e^{m_4 y} + B_3 e^{2m_1 y} + B_4 e^{(m_1+m_2)y} - B_5 e^{2m_2 y} + B_6 e^{(m_1+\pi)y} \\ & - B_7 e^{(m_1-\pi)y} - B_8 e^{(m_1-\pi)y} + B_9 e^{(m_2-\pi)y}\Big]\cos \pi z \end{aligned} \tag{41}$$

$$\begin{aligned} T_1(y,z) = \Big[& D_1 e^{-m_5 y} + D_2 e^{m_6 y} + D_3 e^{(m_1+\operatorname{Re}\Pr)y} + D_4 e^{(m_2+\operatorname{Re}\Pr)y} + D_5 e^{(m_2+\operatorname{Re}\Pr)y} \\ & + D_6 e^{(\pi+\operatorname{Re}\Pr)y}\Big]\cos \pi z \end{aligned} \tag{42}$$

substituting the values of $u_0, u_1, T_0,$ and T_1 from Equations (21), (41), (22) and (42) in Equation (16), the solutions for velocity and temperature are given by

$$u(y,z)=\frac{e^{\lambda_1 y}-e^{\lambda_2 y}}{e^{\lambda_1}-e^{\lambda_2}}+\varepsilon\Big[B_1e^{-m_3 y}-B_2e^{m_4 y}+B_3e^{2m_1 y}+B_4e^{(m_1+m_2)y}-B_5e^{2m_2 y}$$
$$+B_6e^{(m_1+\pi)y}-B_7e^{(m_1-\pi)y}-B_8e^{(m_1-\pi)y}+B_9e^{(m_2-\pi)y}\Big]\cos\pi z \tag{43}$$

$$T(y,z)=\left(\frac{1}{e^{\beta_1}-e^{\beta_2}}\right)e^{\beta_1 y}+\left(\frac{1}{e^{\beta_2}+e^{\beta_1}}\right)e^{\beta_2 y}+\varepsilon\Big[D_1e^{-m_5 y}+D_2e^{m_6 y}+D_3e^{(m_1+\mathrm{RePr})y}$$
$$+D_4e^{(m_2+\mathrm{RePr})y}+D_5e^{(m_2+\mathrm{RePr})y}+D_6e^{(\pi+\mathrm{RePr})y}\Big]\cos\pi z \tag{44}$$

Skin Friction

The *x*- and *z*-components of skin friction at the wall are given by

$$\tau_x=\left(\frac{du_0}{dy}\right)_{y=0}+\varepsilon\left(\frac{du_1}{dy}\right)_{y=0} \text{ and } \tau_z=\varepsilon\left(\frac{dw_1}{dy}\right)_{y=0} \tag{45}$$

Rate of Heat Transfer

The rate of heat transfer i.e. heat flux at the wall in terms of Nusselt number (Nu) is given by

$$Nu=\left(\frac{dT_0}{dy}\right)_{y=0}+\varepsilon\left(\frac{dT_1}{dy}\right)_{y=0} \tag{46}$$

IV. RESULTS AND DISCUSSION

The hydromagnetic three dimensional couette flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous flat plates with heat transfer has been analyzed. The governing equations of the flow field are solved by using perturbation method and the expressions for the velocity profiles, temperature profiles, skin friction and heat flux in terms of Nusselt number are obtained. The effect of the flow parameters on the velocity field and temperature field have been studied and discussed with the help of velocity profiles shown in **figures (2) – (4)** and temperature profiles shown in **figures (5) – (8)** and the effects of the flow parameters on the skin friction and heat flux have been discussed with the help of **tables (1) and (2)** respectively.

Main velocity profile:

The major change in the main velocity (u) of the flow field is due to the variation of magnetic parameter (M) and suction / injection parameter (Re). The magnetic parameter affects the main velocity of the flow field to a greater extent than the suction / injection parameter. The effects of these parameters have been presented in **figures (2) and (3)** respectively. In **figure (2),** we present the variation in the main velocity of the flow field due to the change of the suction / injection parameter (Re) keeping other parameters of the flow field constant. It is observed that the suction / injection parameter retards the velocity of the flow field at all points. As the suction / injection of the fluid through the plate increases, the plate is cooled down and in consequence of which the viscosity of the flowing fluid increases. Therefore, there is a gradual decrease in velocity of the fluid as Re increases. Further, the velocity increases slowly from zero to its maximum value as we proceed from the inlet section. But in absence of suction / injection (Re=0), there is a rapid increase in velocity and the velocity is proportional to the distance from the inlet section. **Figure (3)** depicts the effect of the magnetic parameter (M) on the main velocity of the flow field. The curve with M=0 corresponds to the flow in absence of magnetic field. The main velocity is observed to increase slowly from zero to its maximum value as we proceed from the inlet section. But in absence of magnetic field (M=0), there is a uniform variation in the velocity of the flow field. Comparing the curves of **figure (3)**, it is observed that the magnetic parameter has a retarding effect on the main velocity of the flow field due to the action of Lorentz force on the flow field. Further, comparing the curves of **figures (2) and (3)** it is observed that the magnetic parameter has a very dominant effect on the main velocity field over the suction / injection parameter.

Cross flow velocity profile:

The variation in the magnitude of the cross flow velocity $\left(w_1\right)$ of the flow field is shown in **figure (4)** for three different values of the magnetic parameter (M = 3, 5, 7, 9). The magnetic parameter has an accelerating effect on the cross velocity of the flow field near the lower plate. It is further observed that the cross velocity at first increases sharply to a peak value and then decreases to zero.

Temperature profile:

The temperature of the flow field is affected by the variation of Prandtl number, suction / injection parameter, radiation parameter and heat source parameter. These variations are shown in **figures (5) – (8)** respectively. The suction / injection parameter affects the temperature field to a greater extent than the Prandtl number. In **figure (5),** we discuss the effect of Prandtl number (Pr) on the temperature of the flow field. This figure is a plot of temperature against the non-dimensional distance for three different values of Pr (= 0.71, 1, 2, 3). A comparison of the curves of the said figure shows that the Prandtl number decreases the temperature at all points of the flow field. With the increase of Prandtl number, the thermal conduction in the flow field is lowered and the viscosity of the flowing fluid becomes higher. Consequently, the molecular motion of the fluid elements is lowered down and therefore, the flow field suffers a decrease in temperature at all points as we increase Pr. The effect of suction / injection parameter (Re) on the temperature of the flow field is shown in **figure (6)**. The temperature of the flow field is found to decrease in presence of growing suction/injection. The temperature profile becomes very much linear in absence of suction/injection (Re = 0). In presence of higher suction / injection more amount of fluid is pushed into the flow field through the plate due to which the flow field suffers a decrease in temperature of the flow field at all points. The effect of radiation parameter (R) on the temperature of the flow field is shown in **figure (7)**. The temperature of the flow field is found to decrease in presence of growing radiation parameter. The effect of heat source parameter (ϕ) on the temperature of the flow field is shown in **figure (8),** form this figure we observed that the temperature of the flow field is found to decrease in presence of increase the heat source parameter.

Skin friction:

The skin friction at the wall for different values of suction / injection parameter (Re) has been entered in **table (1)**. The suction / injection parameter reduces the skin friction at the wall in x-direction while it enhances the magnitude of z - component of the skin friction at the wall.

Table (1): Skin friction

Re	τ_x	τ_z
0	0.9102	-0.0873
0.01	0.9021	-0.0837
0.20	0.8453	-0.7225
0.50	0.7153	1.3541

Rate of heat transfer

The rate of heat transfer at the wall in terms of Nusselt number (Nu) for different values of the Prandtl number (Pr) is presented in **table (2)**. The Prandtl number (Pr) is found to enhance the rate of heat transfer at the wall. It is interesting to observe that for lower value of $\Pr(\leq 1)$, the rate of heat transfer assumes negative values while for higher values $\Pr(\geq 1)$, it takes positive values.

Table (2): Nusselt number

Pr	*Nu*
0.71	-1.6030
1.0	-0.7586
2.0	1.5345
5.0	0.1856

V. CONCLUSION

The present analysis brings out the following interesting results of physical interest on the velocity and temperature of the flow field:

1. The magnetic parameter (M) retards the main velocity (u) at all points of the flow field due to the magnetic pull of the Lorentz force acting on the flow field and accelerates the cross velocity (w_1) of the flow field.
2. The suction / injection parameter (Re) decelerates the main velocity of the flow field while no appreciable effect is observed for cross velocity of the flow field.
3. The Prandtl number (Pr) reduces the temperature of the flow field at all points.
4. The suction / injection parameter (Re) diminishes the temperature of the flow field at all points.
5. The suction / injection parameter reduces the *x*-component of skin friction and enhances the magnitude of *z* component of the skin friction at the wall.
6. The rate of heat transfer at the wall (Nu) increases with the increase of the Prandtl number (Pr) of the flow field.

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APPENDIX

$$\lambda_1 = \frac{\mathrm{Re}+\sqrt{\mathrm{Re}^2+4M^2}}{2}, \lambda_2 = \frac{\mathrm{Re}-\sqrt{\mathrm{Re}^2+4M^2}}{2} \quad \beta_1 = \frac{\mathrm{Re}\,\mathrm{Pr}+\sqrt{\mathrm{Re}^2\,\mathrm{Pr}^2+4\,\mathrm{Re}\left(R+\mathrm{Pr}\,\phi\right)}}{2}$$

$$\beta_2 = \frac{\mathrm{Re}\,\mathrm{Pr}-\sqrt{\mathrm{Re}^2\,\mathrm{Pr}^2+4\,\mathrm{Re}\left(R+\mathrm{Pr}\,\phi\right)}}{2}, \; m_1 = \frac{\mathrm{Re}}{2}+\sqrt{\frac{\mathrm{Re}^2}{4}+\left(\pi^2+M^2\right)}$$

$$m_2 = \frac{\mathrm{Re}}{2}-\sqrt{\frac{\mathrm{Re}^2}{4}+\left(\pi^2+M^2\right)}, \; m_3 = -\frac{\mathrm{Re}}{2}+\sqrt{\frac{\mathrm{Re}^2}{4}+\left(\pi^2+M^2\right)}, m_4 = \frac{\mathrm{Re}}{2}-\sqrt{\frac{\mathrm{Re}^2}{4}+\left(\pi^2+M^2\right)}$$

$$m_5 = \frac{\mathrm{Re}\,\mathrm{Pr}}{2}-\sqrt{\frac{\mathrm{Re}^2}{4}+\pi^2}, \; m_6 = \frac{\mathrm{Re}\,\mathrm{Pr}}{2}-\sqrt{\frac{\mathrm{Re}^2\,\mathrm{Pr}^2}{4}+\pi^2}$$

$$A=\left(\pi-m_1\right)\left(\pi+m_2\right)\left[e^{m_2-\pi}+e^{m_1+\pi}\right]+\left(\pi+m_1\right)\left(m_2-\pi\right)\left[e^{m_2+\pi}+e^{m_1-\pi}\right]-2\pi\left(m_2-m_1\right)\left[e^{m_1+m_2}+1\right]$$

$$A_1=-2\pi m_2+\pi\left(m_2+\pi\right)e^{\pi+m_2}-\pi\left(\pi-m_2\right)e^{m_2-\pi}, \; A_2=2\pi m_1-\pi\left(m_1+\pi\right)e^{m_1-\pi}-\pi\left(m_1-\pi\right)e^{m_1+\pi}$$

$$A_3=-m_1\left(m_2+\pi\right)e^{m_2-\pi}+m_2\left(m_1+\pi\right)e^{m_1-\pi}-\pi\left(m_2-m_1\right)e^{m_1+m_2}$$

$$A_4=m_1\left(m_2-\pi\right)e^{m_2+\pi}-m_2\left(m_1-\pi\right)e^{m_1+\pi}-\pi\left(m_2-m_1\right)e^{m_1+m_2}, \; A_5=B_3+B_4-B_5+B_6-B_7-B_8+B_9$$

$$A_6 = B_3 e^{2m_1} + B_4 e^{m_1+m_2} - B_5 e^{2m_1} + \pi B_6 e^{(m_1+\pi)} - B_7 e^{(m_1-\pi)} + B_8 e^{(m_1+\pi)} - B_9 e^{(m_1-\pi)}$$

$$A_7 = D_3 + D_4 + D_5 + D_6 + D_7,\ A_8 = D_3 e^{m_1+\mathrm{Re}\,\mathrm{Pr}} + D_4 e^{m_2+\mathrm{Re}\,\mathrm{Pr}} + D_3 e^{m_1+\mathrm{Re}\,\mathrm{Pr}} + D_6 e^{\pi+\mathrm{Re}\,\mathrm{Pr}} + D_7 e^{-\pi+\mathrm{Re}\,\mathrm{Pr}}$$

$$B_1 = \left(\frac{A_5 - A_6 e^{m_3}}{1-e^{m_3+m_4}} - A_5\right), B_2 = \left(\frac{A_5 - A_6 e^{m_3}}{1-e^{m_3+m_4}}\right), B_3 = \frac{\mathrm{Re}\,A_1}{Am_1\left(e^{\lambda_1} - e^{\lambda_2}\right)\left(2m_1 + m_3\right)}$$

$$B_4 = \frac{\mathrm{Re}\left(A_2 - A_1\right)}{Am_1\left(e^{\lambda_1} - e^{\lambda_2}\right)\left(m_1 + m_2 + m_3\right)}, B_5 = \frac{\mathrm{Re}\,A_2}{A\left(e^{\lambda_1} - e^{\lambda_2}\right)\left(2m_2 + m_3\right)\left(2m_2 - m_4\right)}$$

$$B_6 = \frac{\mathrm{Re}\,A_3}{\pi A\left(e^{\lambda_1} - e^{\lambda_2}\right)\left(m_1 + \pi + m_3\right)}, B_7 = \frac{\mathrm{Re}\,A_4}{\pi A\left(e^{\lambda_1} - e^{\lambda_2}\right)\left(m_1 - \pi + m_3\right)}$$

$$B_8 = \frac{\mathrm{Re}\,A_3}{A\left(e^{\lambda_1} - e^{\lambda_2}\right)\left(m_1 + \pi + m_3\right)\left(m_1 + \pi - m_4\right)}, B_9 = \frac{\mathrm{Re}\,A_4}{\pi A\left(e^{\lambda_1} - e^{\lambda_2}\right)\left(m_1 - \pi + m_3\right)\left(m_1 - \pi - m_4\right)}$$

$$D_1 = -\left(\frac{A_7 - A_8 e^{-m_5}}{e^{m_6-m_5} - 1} + A_7\right), D_2 = \left(\frac{A_7 - A_8 e^{-m_5}}{e^{m_6-m_5} - 1}\right)$$

$$D_3 = \frac{\mathrm{Re}^2\,A_1\,\mathrm{Pr}^2}{A\left(e^{\mathrm{Re}\,\mathrm{Pr}} - 1\right)\left(m_1 + \mathrm{Re}\,\mathrm{Pr} - m_5\right)\left(m_1 + \mathrm{Re}\,\mathrm{Pr} - m_6\right)}$$

$$D_4 = \frac{\mathrm{Re}^2\,A_2\,\mathrm{Pr}^2}{A\left(e^{\mathrm{Re}\,\mathrm{Pr}} - 1\right)\left(m_2 + \mathrm{Re}\,\mathrm{Pr} - m_5\right)\left(m_2 + \mathrm{Re}\,\mathrm{Pr} - m_6\right)}$$

$$D_5 = \frac{\mathrm{Re}^2\,A_3\,\mathrm{Pr}^2}{A\left(e^{\mathrm{Re}\,\mathrm{Pr}} - 1\right)\left(m_2 + \mathrm{Re}\,\mathrm{Pr} - m_5\right)\left(m_2 + \mathrm{Re}\,\mathrm{Pr} - m_6\right)}$$

$$D_6 = \frac{\mathrm{Re}^2\,A_3\,\mathrm{Pr}^2}{A\left(e^{\mathrm{Re}\,\mathrm{Pr}} - 1\right)\left(\pi + \mathrm{Re}\,\mathrm{Pr} - m_5\right)\left(\pi + \mathrm{Re}\,\mathrm{Pr} - m_6\right)}$$

$$D_7 = \frac{\mathrm{Re}^2\,A_4\,\mathrm{Pr}^2}{A\left(e^{\mathrm{Re}\,\mathrm{Pr}} - 1\right)\left(\mathrm{Re}\,\mathrm{Pr} - \pi - m_5\right)\left(\mathrm{Re}\,\mathrm{Pr} - \pi - m_6\right)}$$

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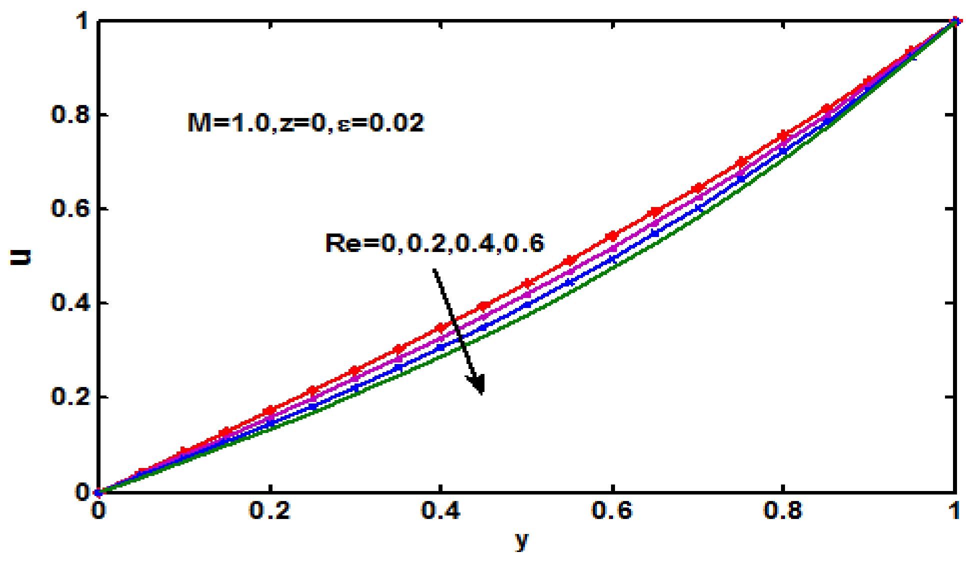

Fig. 2. Velocity profiles for different values of Re

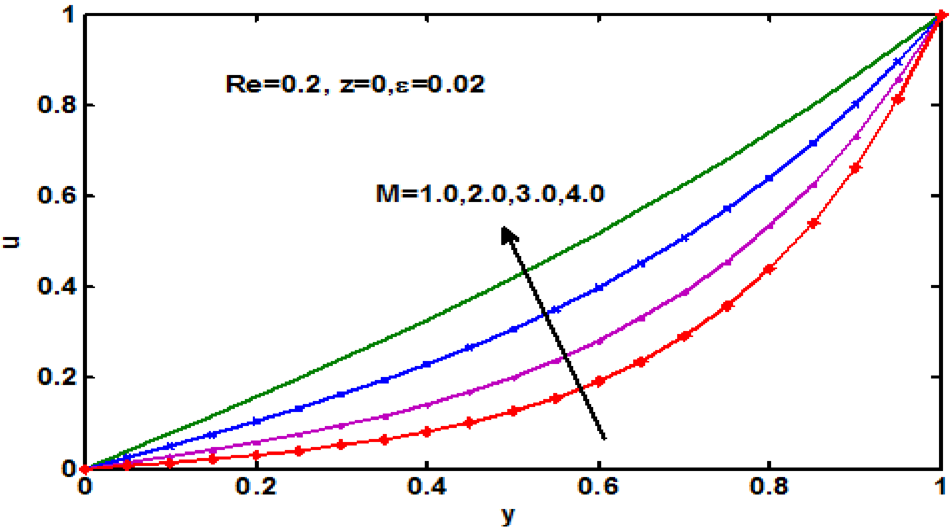

Fig. 3. Velocity profiles for different values of M

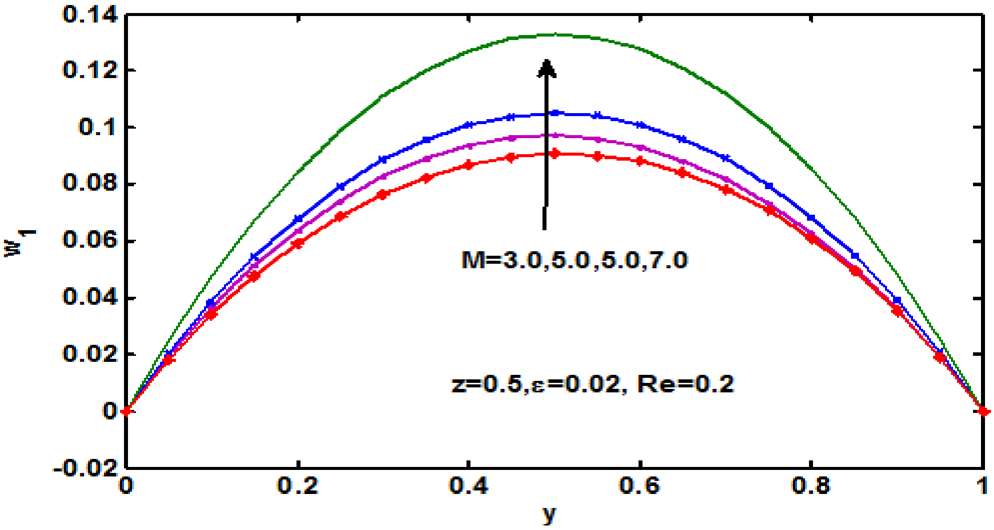

Fig. 4. Cross flow velocity profiles for different values of M

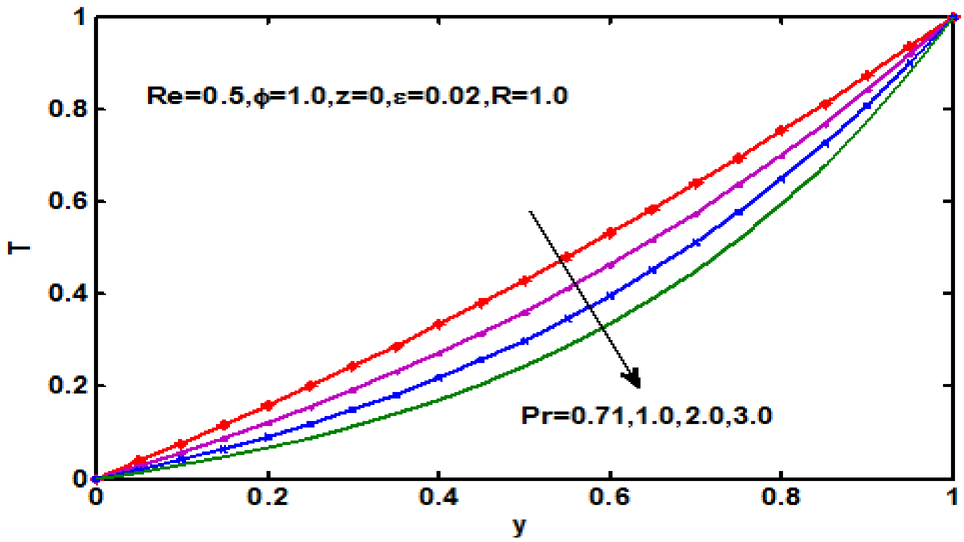

Fig. 5. Temperature profiles for different values of Pr

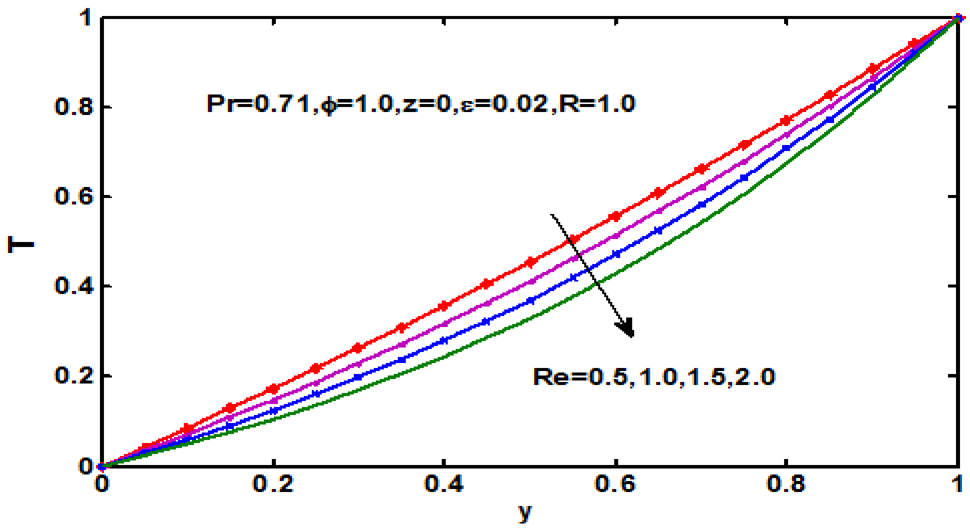

Fig. 6. Temperature profiles for different values of Re

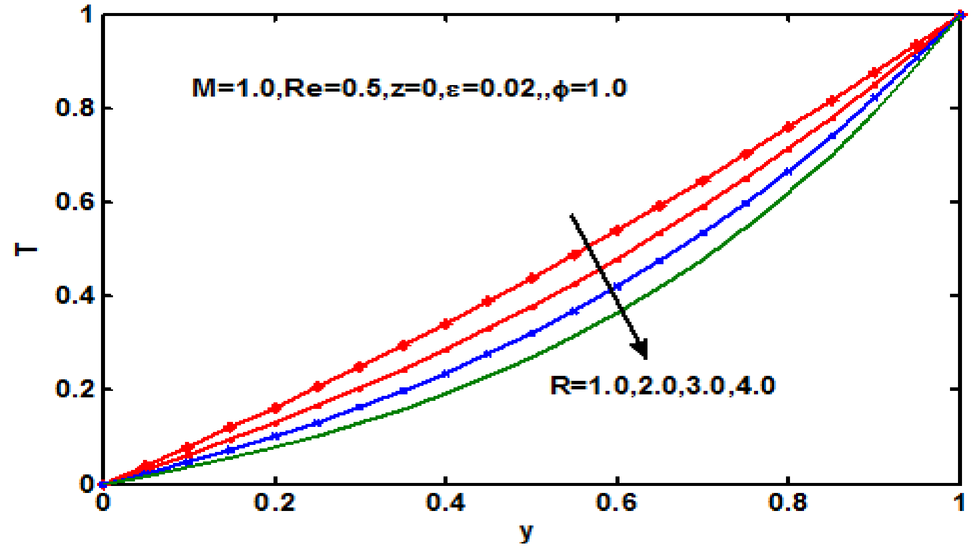

Fig. 7. Temperature profiles for different values of R

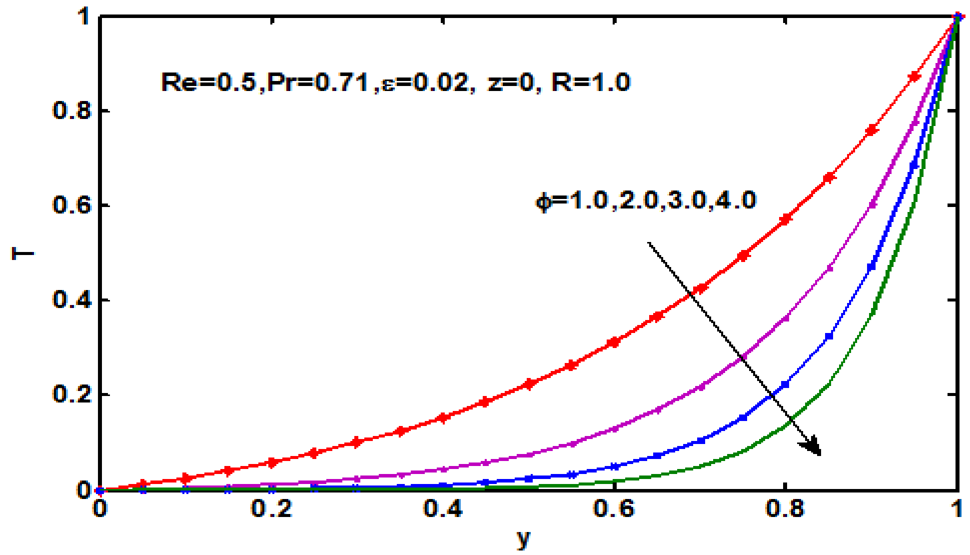

Fig. 8. Temperature profiles for different values of ϕ