Bayesian Estimation of Inverse Ailamujia Distribution Using Different Loss Functions

Ahmad Aijaz[1], S. Qurat ul Ain[2], Afaq Ahmad[3], Rajnee Tripathi[4]

Department of Mathematics, Bhagwant University, Ajmer, Rajasthan, India[1,2,4]

Department of Mathematical Sciences, Islamic University of Science & Technology, Awantipora, Kashmir[3]

Abstract:

This paper deals with the Bayesian estimation of the parameter of inverse Ailamujia distribution. The extended Jeffery's prior and gamma prior is used to obtain Bayesian distribution. The estimators are obtained under squared error loss function, Entropy loss function, precautionary loss function and Linex loss function. Eventually a real data set is considered to compare the performance of these estimators under different loss functions.

Key words: Bayesian analysis, priors, maximum likelihood estimator, different loss functions.

Mathematics subject classification: 60-XX, 62-XX, 11-KXX

1. Introduction:

In the literature of statistics Ailamujia distribution established by Lv et al (2002) is a new distribution with several applications in different fields of engineering. They have expounded its various distributional properties which includes moments, moment generating function, mode, median, order statistics. They have derived and discussed various reliability functions. The probability density function and cumulative distribution function of Ailamujia distribution are respectively given as

$$f(x,\alpha) = 4\alpha^2 \, x \, e^{-2\alpha x} \, ; x > 0 \, , \alpha > 0$$

$$F(x,\alpha) = 1 - (1 + 2\alpha x)e^{-2\alpha x} \, , \alpha > 0, y > 0$$

In recent past decade authors have proposed several extensions of Ailamujia distribution. Pan et al (2009) has worked on Ailamujia distribution for interval estimation and hypothesis testing based on small sample size. Long (2015) has obtained its Bayesian estimation under type II censoring on the basis of conjugate prior, Jeffrey's prior and no informative prior distribution. Yu et al (2015) proposed a new method by applying Ailamujia distribution to solve the problem in the production and distribution of battle field injury in campaign macrocosm. Recently Ahmad et al (2020) developed the inverse analogue of Ailamujia distribution and examine its usefulness through two real life time data sets.

Suppose Y is a random variable follows inverse Ailamujia distribution. Then its probability density function (p.d.f), is given by

$$f(y,\alpha) = 4\alpha^2 \frac{1}{y^3} e^{-\frac{2\alpha}{y}} \quad , y > 0 \,, \alpha > 0 \qquad (1.1)$$

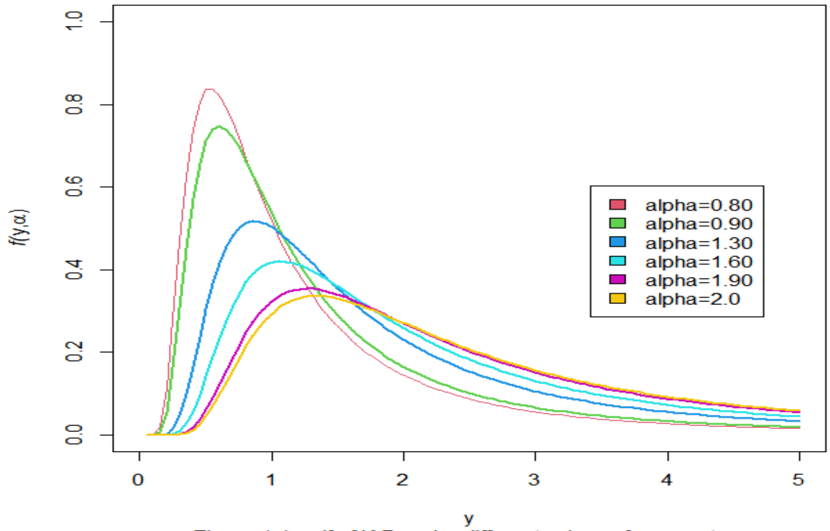

Figure 1.1: pdf of IAD under different values of parameters

Figure (1.1) illustrates some possible shapes of p.d.f for varying parameters

The corresponding cumulative distribution function (c.d.f), is given by

$$F(Y) = \frac{(2\alpha + y)}{y} e^{-\frac{2\alpha}{y}} \quad , y > 0 \,, \alpha > 0 \qquad (1.2)$$

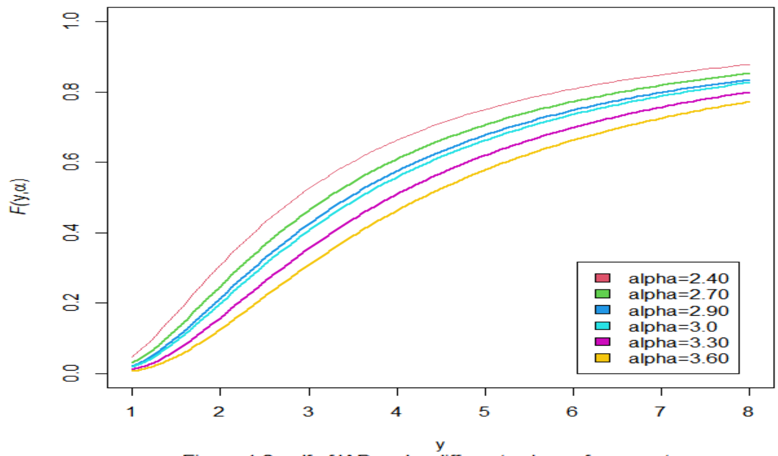

Figure 1.2: cdf of IAD under different values of parameters

Figure (1.2) illustrates some possible shapes of c.d.f for varying parameters

2: Maximum Likelihood Estimation

Let $Y_1, Y_2 \dots Y_n$ be random samples from the inverse Ailamujia distribution. Then the likelihood function of inverse Ailamujia distribution is given as

$$l = \prod_{i=1}^{n} f(y_i, \alpha)$$

$$= \prod_{i=1}^{n} 4\alpha^2 \frac{1}{y_i^3}\, e^{-\frac{2\alpha}{y_i}} = (4\alpha^2)^n \prod_{i=1}^{n} \frac{1}{y_i^3}\;\; e^{-2\alpha \sum_{i=1}^{n} \frac{1}{y_i}}$$

Taking log we get log likelihood function as

$$\log l = \; 2n \log 2\alpha - 3\sum_{i=1}^{n} \log y_i - 2\alpha \sum_{i=1}^{n} \frac{1}{y_i}$$

Differentiating w.r.t, we get

$$\frac{\partial \log l}{\partial \alpha} = 2n\frac{1}{2\alpha} - 2\sum_{i=1}^{n} \frac{1}{y_i}$$

Now equating $\frac{\partial \log l}{\partial \alpha} = 0$, we get

$$\hat{\alpha} = \frac{n}{2S}$$

Where $S = \sum_{i=1}^{n} y_i^{-1}$

3: Bayesian Estimation Of Inverse Ailamujia Distribution

Bayesian estimation procedure is a remarkable way to estimate the parameters of the distribution model. This estimation provides a posterior distribution of an existing life time distribution by considering prior information. From Bayesian point of view there can’t be put the lid on selecting prior(s) by considering one’s prior(s) is more suitable than others. In case of meager interpretative information about the unknown parameter it is preferable to select non informative prior. However, if one has sufficient information about the parameter(s) it is better to select informative prior. The aim of present study is to obtain a Bayesian estimation of parameter α of inverse Ailamujia distribution by using extended Jeffrey’s and gamma prior. In recent past years several research papers have been published in this direction. Afaq et al(2018) estimation of parameters of two parameter exponentiated gamma distribution. Mudasir et al (2017) studied the Bayesian estimation of weighted Erlang distribution. Raqab and Madi(2009) studied Bayesian estimation for exponentiated Rayleigh distribution. Fatima Bi and Afaq Ahmad (2019) studied Bayesian estimation of the parameter of Ailamujia distribution. In this paper our goal is to find the Bayesian estimators of the parameters of inverse Ailamujia distribution using extended Jeffery's prior and gamma prior under different loss functions.

3.1: Bayesian Estimation Of Inverse Ailamujia Distribution Under The Assumption Of Extended Jeffery's Prior

We assume the prior distribution of α to be extended Jeffrey's prior i.e $g(\alpha) \propto \frac{1}{\alpha^{2c}}$

Under the assumption of extended Jeffrey's prior. The posterior distribution of α can be obtained as

$$\pi(\alpha|y) \propto l(\alpha|y)g(\alpha)$$

$$\Rightarrow \pi(\alpha|y) \propto \left(4^n \prod_i^n \frac{1}{y_i^3}\right) \alpha^{2n} \, e^{-2\alpha \sum_i^n \frac{1}{y_i}} \frac{1}{\alpha^{2c}}$$

$$\Rightarrow \pi(\alpha|y) = k \, \alpha^{2(n-c)} e^{-2\alpha \sum_i^n \frac{1}{y_i}}$$

Where k is independent of α and

$$k^{-1} = \int_0^\infty \alpha^{2(n-c)} e^{-2\alpha \sum_i^n \frac{1}{y_i}} \, d\alpha$$

$$k^{-1} = \frac{\Gamma(2n - 2c + 1\,)}{\left(2 \sum_i^\infty \frac{1}{y_i}\right)^{2n-c+1}}$$

So that $$k = \frac{\left(2 \sum_i^\infty \frac{1}{y_i}\right)^{2(n-c)+1}}{\Gamma(2n-2c+1\,)} = \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-2c+1\,)}$$

Where $S = \sum_i^\infty \frac{1}{y_i}$

Hence the posterior distribution of α is given as

$$\pi(\alpha|y) = \frac{(2S)^{2(n-c)+1}}{\Gamma(2n - 2c + 1\,)} \, \alpha^{2(n-c)} \, e^{-2S\alpha}$$

Where $S = \sum_i^\infty \frac{1}{y_i}$

3.1.1: Estimation Under Squared Error Loss Function (SELF)

The squared error loss function is defined as $l(\hat{\alpha}, \alpha) = c_1(\hat{\alpha} - \alpha)^2$ for some constantant c_1 the risk function is given as

$$R(\hat{\alpha}, \alpha) = E[I(\hat{\alpha}, \alpha)]$$

$$= \int_0^\infty c_1 (\hat{\alpha} - \alpha)^2 \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-2c+1)} \alpha^{2(n-c)} e^{-2S\alpha} d\alpha$$

$$= c_1 \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-2c+1)} \left[\hat{\alpha} \int_0^\infty \alpha^{2(n-c)} e^{-2S\alpha} d\alpha + \int_0^\infty \alpha^{2(n-c)+2} e^{-2S\alpha} d\alpha - 2\hat{\alpha} \int_0^\infty \alpha^{2(n-c)+1} e^{-2S\alpha} d\alpha \right]$$

After solving the integral, we obtain

$$= c_1 \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-c+1)} \left[\frac{\hat{\alpha}\Gamma(2n-2c+1)}{(2S)^{2(n-c)+1}} + \frac{(2n-2c+2)(2n-2c+1)\Gamma(2n-2c+1)}{(2S)^{2(n-c)+3}} - \frac{(2n-2c+1)\Gamma(2n-2c+1)}{(2S)^{2(n-c)+2}} \right]$$

$$R(\hat{\alpha}, \alpha) = c_1 \left[\hat{\alpha}^2 + \frac{(2n-2c+2)(2n-2c+1)}{(2S)^2} - \frac{\hat{\alpha}(2n-2c+1)}{(2S)} \right]$$

Now solving $\frac{\partial R(\hat{\alpha}, \alpha)}{\partial \hat{\alpha}} = 0$, we get

$$\hat{\alpha}_s = \frac{(2n-2c+1)}{4S}$$

Where $s = \sum_i^\infty \frac{1}{y_i}$

3.1.2: Estimation Under Entropy Loss Function

The entropy loss function is defined as $L(\delta) = b[\delta - \log(\delta) - 1]; b > 0, \delta = \frac{\hat{\alpha}}{\alpha}$ the risk functions given as

$$R(\hat{\alpha}, \alpha) = \int_0^\infty b[\delta - \log(\delta) - 1] \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-c+1)} \alpha^{2(n-c)} e^{-2S\alpha} d\alpha$$

$$R(\hat{\alpha}, \alpha) = b \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-c+1)} \int_0^\infty \left[\frac{\hat{\alpha}}{\alpha} - \log \hat{\alpha} + \log \alpha - 1 \right] \alpha^{2(n-c)} e^{-2S\alpha} d\alpha$$

$$= b \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-2c+1)} \left[\hat{\alpha} \int_0^\infty \alpha^{2(n-c)-1} e^{-2S\alpha} d\alpha - \log \hat{\alpha} \int_0^\infty \alpha^{2(n-c)} e^{-2S\alpha} d\alpha + \int_0^\infty (\log \alpha) \alpha^{2(n-c)} e^{-2S\alpha} d\alpha - \int_0^\infty \alpha^{2(n-c)} e^{-2S\alpha} d\alpha \right]$$

After solving the integral, we obtain

$$= b\frac{(2S)^{2(n-c)+1}}{\Gamma(2n-2c+1\,)}\left[\hat{\alpha}\frac{\Gamma(2n-2c)}{(2S)^{2(n-c)}} - \log\hat{\alpha}\frac{\Gamma(2n-2c+1)}{(2S)^{2(n-c)+1}} + \frac{\Gamma'(2n-2c+1)}{(2S)^{2(n-c)+1}} - \frac{\Gamma(2n-2c+1)}{(2S)^{2(n-c)+1}}\right]$$

$$= b\left[\frac{\hat{\alpha}(S)}{(n-c)} - \log\hat{\alpha} + \frac{\Gamma'(2n-2c+1\,)}{\Gamma(2n-2c+1)} - 1\right]$$

Now solving $\frac{\partial R(\hat{\alpha},\alpha)}{\partial\hat{\alpha}} = 0$, we get

$$\hat{\alpha}_e = \frac{n-c}{S}$$

Where $s = \sum_i^{\infty}\frac{1}{y_i}$

3.1.3: Estimation Under Precautionary Loss Function

The precautionary loss function is defined as $(\hat{\alpha},\alpha) = \frac{(\hat{\alpha}-\alpha)^2}{\hat{\alpha}}$, the risk function is given as

$$R(\hat{\alpha},\alpha) = \int_0^{\infty}\frac{(\hat{\alpha}-\alpha)^2}{\hat{\alpha}}\frac{(2S)^{2(n-c)+1}}{\Gamma(2n-2c+1\,)}\alpha^{2(n-c)}\,e^{-2S\alpha}d\alpha$$

$$R(\hat{\alpha},\alpha) = \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-c+1\,)}\int_0^{\infty}\frac{(\hat{\alpha}-\alpha)^2}{\hat{\alpha}}\alpha^{2(n-c)}\,e^{-2S\alpha}d\alpha$$

$$= \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-2c+1\,)}\left[\hat{\alpha}\int_0^{\infty}\alpha^{2(n-c)}e^{-2S\alpha}d\alpha + \frac{1}{\hat{\alpha}}\int_0^{\infty}\alpha^{2(n-c)+2}e^{-2S\alpha}d\alpha - 2\int_0^{\infty}\alpha^{2(n-c)+1}e^{-2S\alpha}d\alpha\right]$$

After solving the integral, we obtain

$$= \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-2c+1\,)}\left[\hat{\alpha}\frac{\Gamma(2n-2c+1)}{(2S)^{2(n-c)+1}} + \frac{1}{\hat{\alpha}}\frac{\Gamma(2n-2c+3)}{(2S)^{2(n-c)+3}} - 2\frac{\Gamma(2n-2c+2)}{(2S)^{2(n-c)+2}}\right]$$

$$= \left[\hat{\alpha} + \frac{(2n-2c+2)(2n-2c+1)}{\hat{\alpha}\,(2S)^2} - \frac{2(2n-2c+1)}{(2S)}\right]$$

Now solving $\frac{\partial R(\hat{\alpha},\alpha)}{\partial\hat{\alpha}} = 0$, we get

$$\hat{\alpha}_p = \frac{[(n-c+1)(2n-2c+1)]^{\frac{1}{2}}}{(S)}$$

Where $S = \sum_{i}^{\infty} \frac{1}{y_i}$

3.1.4: Estimation Under Linex Loss Function

The linex loss function is defined as $L(\hat{\alpha}, \alpha) = exp\{b_1(\hat{\alpha} - \alpha)\} - b_1(\hat{\alpha} - \alpha) - 1$, the risk function is given as

$$l(\hat{\alpha}, \alpha) = \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-2c+1)} \int_0^{\infty} \{e^{\left(b_1(\hat{\alpha}-\alpha)\right)} - b_1(\hat{\alpha} - \alpha) - 1\} \alpha^{2(n-c)} \, e^{-2S\alpha} d\alpha$$

$$= \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-c+1)} \left[e^{b_1\hat{\alpha}} \int_0^{\infty} \alpha^{2(n-c)} e^{-\alpha(b_1+2S)} d\alpha - b_1\hat{\alpha} \int_0^{\infty} \alpha^{2(n-c)} e^{-2S\alpha} d\alpha + b_1 \int_0^{\infty} \alpha^{2(n-c)+1} e^{-2S\alpha} d\alpha - \int_0^{\infty} \alpha^{2(n-c)} e^{-2S\alpha} d\alpha \right]$$

$$= \frac{(2S)^{2(n-c)+1}}{\Gamma(2n-2c+1)} \left[e^{b_1\hat{\alpha}} \frac{\Gamma(2n-2c+1)}{(b_1+2S)^{2(n-c)+1}} - b_1\hat{\alpha} \frac{\Gamma(2n-2c+1)}{(2S)^{2(n-c)+1}} + b_1 \frac{\Gamma(2n-2c+2)}{(2S)^{2(n-c)+2}} - \frac{\Gamma(2n-2c+1)}{(2S)^{2(n-c)+1}} \right]$$

$$= \left[e^{b_1\hat{\alpha}} \left(\frac{2S}{b_1+2S} \right)^{2(n-c)+1} - b_1\hat{\alpha} + b_1 \frac{(2n-2c+1)}{(2S)} - 1 \right]$$

Now solving $\frac{\partial l(\hat{\alpha},\alpha)}{\partial \hat{\alpha}} = 0$, we get

$$\hat{\alpha}_l = \frac{1}{b_1} \log \left(\frac{b_1+2S}{2S} \right)^{2(n-c)+1}$$

4.2: Bayesian Estimation Of Inverse Ailamujia Distribution Under The Assumption Of Gamma Distribution:

We assume the prior distribution of α to be gamma distribution i.e $g(\alpha) \propto \frac{a^b}{\Gamma(b)} e^{-a\alpha} \, \alpha^{b-1}$

Now under the assumption of gamma prior. The posterior distribution of α can be obtained as

$$\pi(\alpha|y) \propto l(\alpha|y) g(\alpha)$$

$$\Rightarrow \pi(\alpha|y) \propto \left(4^n \prod_{i}^{n} \frac{1}{y_i^3} \right) \alpha^{2n} \, e^{-2\alpha \sum_i^n \frac{1}{y_i}} \, \frac{a^b}{\Gamma(b)} e^{-a\alpha} \, \alpha^{b-1}$$

$$\Rightarrow \pi(\alpha|y) = k \, \alpha^{2n+b-1} e^{-\alpha\left(a+2\sum_i^n \frac{1}{y_i}\right)}$$

Where k is independent of α and

$$k^{-1} = \int_0^\infty \alpha^{2n+b-1} e^{-\alpha\left(a+2\sum_i^n \frac{1}{y_i}\right)} d\alpha$$

$$= \frac{\Gamma(2n+b)}{\left(a+2\sum_i^n \frac{1}{y_i}\right)^{2n+b}}$$

So that

$$k = \frac{\left(a+2\sum_i^n \frac{1}{y_i}\right)^{2n+b}}{\Gamma(2n+b)} = \frac{(a+2S)^{2n+b}}{\Gamma(2n+b)}$$

Where $S = \sum_i^n \frac{1}{y_i}$

Hence the posterior distribution of α is given as

$$\pi(\alpha|y) = \frac{(a+2S)^{2n+b}}{\Gamma(2n+b)} \alpha^{2n+b-1} e^{-\alpha(a+2S)}$$

Where $S = \sum_i^n \frac{1}{y_i}$

4.2.1: Estimation Under Squared Error Loss Function

The squared error loss function is defined as $l(\hat{\alpha}, \alpha) = c_1(\hat{\alpha} - \alpha)^2$ for some constantant c_1 the risk function is given as

$$R(\hat{\alpha}, \alpha) = E[I(\hat{\alpha}, \alpha)]$$

$$= \int_0^\infty c_1(\hat{\alpha} - \alpha)^2 \frac{(a+2S)^{2n+b}}{\Gamma(2n+b)} \alpha^{2n+b-1} e^{-\alpha(a+2S)} d\alpha$$

$$= c_1 \frac{(a+2S)^{2n+b}}{\Gamma(2n+b)} \int_0^\infty (\hat{\alpha} - \alpha)^2 \alpha^{2n+b-1} e^{-\alpha(a+2S)} d\alpha$$

After solving the integral, we obtain

$$R(\hat{\alpha}, \alpha) = c_1 \left[\hat{\alpha}^2 + \frac{(2n+b)(2n+b+1)}{(a+2S)^2} - 2\hat{\alpha}\frac{(2n+b)}{(a+2S)}\right]$$

Now solving $\frac{\partial R(\hat{\alpha}, \alpha)}{\partial \hat{\alpha}} = 0$, we get

$$\hat{\alpha}_s = \frac{2n+b}{a+2S}$$

Where $S = \sum_i^n \frac{1}{y_i}$

4.2.2: Estimation Under Entropy Loss Function

The entropy loss function is defined as $L(\delta) = b[\delta - \log(\delta) - 1]; b > 0\,, \delta = \frac{\hat{\alpha}}{\alpha}$ the risk functions given as

$$R(\hat{\alpha}, \alpha) = \int_0^\infty b[\delta - \log(\delta) - 1] \frac{(a + 2S)^{2n+b}}{\Gamma(2n + b)} \alpha^{2n+b-1} e^{-\alpha(a+2S)}\, d\alpha$$

$$= b \frac{(a + 2S)^{2n+b}}{\Gamma(2n + b)} \int_0^\infty \left[\frac{\hat{\alpha}}{\alpha} - \log \hat{\alpha} + \log \alpha - 1\right] \alpha^{2n+b-1} e^{-\alpha(a+2S)} d\alpha$$

After solving the integral, we obtain

$$R(\hat{\alpha}, \alpha) = b\left[\hat{\alpha} \frac{(a + 2S)}{(2n + b - 1)} - \log \hat{\alpha} + \frac{\Gamma'(2n + b)}{\Gamma(2n + b)} - 1\right]$$

Now solving $\frac{\partial R(\hat{\alpha},\alpha)}{\partial \hat{\alpha}} = 0$, we get

$$\hat{\alpha}_e = \frac{2n + b - 1}{a + 2S}$$

Where $S = \sum_i^n \frac{1}{y_i}$

4.2.3: Estimation Under Precautionary Loss Function

The precautionary loss function is defined as$l(\hat{\alpha}, \alpha) = \frac{(\hat{\alpha}-\alpha)^2}{\hat{\alpha}}$, the risk function is given as

$$R(\hat{\alpha}, \alpha) = \frac{(a + 2S)^{2n+b}}{\Gamma(2n + b)} \int_0^\infty \frac{(\hat{\alpha} - \alpha)^2}{\hat{\alpha}} \alpha^{2n+b-1} e^{-\alpha(a+2S)} d\alpha$$

After solving the integral, we get

$$= \left[\hat{\alpha} + \frac{(2n + b)(2n + b - 1)}{\hat{\alpha}(a + 2S)^2} - 2\frac{(2n + b)}{(a + 2S)}\right]$$

Now solving $\frac{\partial R(\hat{\alpha},\alpha)}{\partial \hat{\alpha}} = 0$, we get

$$\hat{\alpha}_p = \frac{[(2n + b)(2n + b - 1)]^{\frac{1}{2}}}{(a + 2S)}$$

Where $S = \sum_i^n \frac{1}{y_i}$

4.2.4: Estimation Under Linex Loss Function

The linex loss function is defined as $L(\hat{\alpha}, \alpha) = exp\{b_1(\hat{\alpha} - \alpha)\} - b_1(\hat{\alpha} - \alpha) - 1$, the risk function is given as

$$R(\hat{\alpha},\alpha)=\frac{(a+2S)^{2n+b}}{\Gamma(2n+b)}\int_0^\infty\{e^{(b_1(\hat{\alpha}-\alpha))}-b_1(\hat{\alpha}-\alpha)-1\}\alpha^{2n+b-1}\,e^{-\alpha(a+2S)}d\alpha$$

$$=\frac{(a+2S)^{2n+b}}{\Gamma(2n+b)}\Bigg[e^{b_1\hat{\alpha}}\int_0^\infty\alpha^{2n+b-1}\,e^{-\alpha(a+b_1+2S)}d\alpha - b_1\hat{\alpha}\int_0^\infty\alpha^{2n+b-1}\,e^{-\alpha(a+2S)}d\alpha + b_1\int_0^\infty\alpha^{2n+b}\,e^{-\alpha(a+2S)}d\alpha-\int_0^\infty\alpha^{2n+b-1}\,e^{-\alpha(a+2S)}d\alpha\Bigg]$$

After solving the integrals, we obtain

$$R(\hat{\alpha},\alpha)=\frac{(a+2S)^{2n+b}}{\Gamma(2n+b)}\Bigg[e^{b_1\hat{\alpha}}\frac{\Gamma(2n+b)}{(a+b_1+2S)^{2n+b}}-b_1\hat{\alpha}\frac{\Gamma(2n+b)}{(a+2S)^{2n+b}} + b_1\frac{\Gamma(2n+b+1)}{(a+2S)^{2n+b+1}}-\frac{\Gamma(2n+b)}{(a+2S)^{2n+b}}\Bigg]$$

$$=\left[e^{b_1\hat{\alpha}}\left(\frac{a+2S}{a+b_1+2S}\right)^{2n+b}-b_1\hat{\alpha}+b_1\frac{(2n+b)}{(a+2S)}-1\right]$$

Now solving $\frac{\partial R(\hat{\alpha},\alpha)}{\partial\hat{\alpha}}=0$, we get

$$\hat{\alpha}_l=\frac{1}{b_1}\log\left(\frac{a+b_1+2S}{a+2S}\right)^{2n+b}$$

Where $S=\sum_i^n\frac{1}{y_i}$

5: Application

In this section we provide an application through which the performance of the estimators and posterior risk of different loss function has been obtained. The data set are follows:

Data set 1: The data set represents the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal (1960). The data are follows

0.1, 0.33, 0.44, 0.56, 0.59, 0.59, 0.72, 0.74, 0.92, 0.93, 0.96, 1, 1, 1.02, 1.05, 1.07, 1.07, 1.08, 1.08, 1.08, 1.09, 1.12, 1.13, 1.15, 1.16, 1.2, 1.21, 1.22, 1.22, 1.24, 1.3, 1.34, 1.36, 1.39, 1.44, 1.46, 1.53, 1.59, 1.6, 1.63, 1.68, 1.71, 1.72, 1.76, 1.83, 1.95, 1.96, 1.97, 2.02, 2.13, 2.15, 2.16, 2.22, 2.3, 2.31, 2.4, 2.45, 2.51, 2.53, 2.54, 2.78, 2.93, 3.27, 3.42, 3.47, 3.61, 4.02, 4.32, 4.58, 5.55, 2.54, 0.77.

By using different loss functions the Bayes estimates and posterior risks of the posterior distribution through both prior's are as follows where posterior risk are in parenthesis.

Table 5.1
Bayes Estimation and Posterior Risks Using Jeffery's Prior

α	C	$\hat{\alpha}$	$\hat{\alpha}_S$	$\hat{\alpha}_E$	$\hat{\alpha}_P$	$\hat{\alpha}_L$	
						$b_1 = 0.01$	$b_1 = 0.05$
1.0	0.5	0.5583	0.5583 (1.260)	1.109 (4.862)	2.241 (17.97)	1.116 (0.0111)	1.116 (0.0558)
	1.0	0.5583	0.5545 (1.247)	1.101 (4.862)	2.225 (17.85)	1.108 (0.0110)	1.108 (0.0554)
	1.5	0.5583	0.5506 (1.234)	1.093 (4.862)	2.210 (17.73)	1.1012 (0.0110)	1.1010 (0.0550)
2.0	0.5	0.5583	0.5583 (1.260)	1.1090 (4.862)	2.2413 (17.97)	1.1167 (0.0111)	1.1165 (0.0558)
	1.0	0.5583	0.5545 (1.247)	1.1012 (4.862)	2.2258 (17.85)	1.1089 (0.0110)	1.1088 (0.0554)
	1.5	0.5583	0.5506 (1.234)	1.093 (4.862)	2.2102 (17.73)	1.1012 (0.0110)	1.1010 (0.0550)
3.0	0.5	0.5583	0.5583 (1.260)	1.1090 (4.862)	2.2413 (17.97)	1.1167 (0.0111)	1.1165 (0.0558)
	1.0	0.5583	0.5545 (1.247)	1.1012 (4.862)	2.2258 (17.85)	1.1089 (0.0110)	1.1088 (0.0554)
	1.5	0.5583	0.5506 (1.234)	1.0935 (4.862)	2.2102 (17.73)	1.1012 (0.0110)	1.1010 (05506)

$\hat{\alpha}$ = MLE, $\hat{\alpha}_S$ = Estimation under SELF, $\hat{\alpha}_E$ = Estimation under Entropy,

$\hat{\alpha}_P$ = Estimation under Precautionary, $\hat{\alpha}_L$ = Estimation under LINEX

Table 5.2
Bayes Estimation and Posterior Risks Using Gamma Prior

α	a	b	$\hat{\alpha}$	$\hat{\alpha}_S$	$\hat{\alpha}_E$	$\hat{\alpha}_P$	$\hat{\alpha}_L$	
							$b_1 = 0.01$	$b_1 = 0.05$
1.0	0.5	0.5	0.5583	1.1163 (0.0086)	1.1240 (4.8667)	1.1124 (1.1085)	1.1162 (0.0111)	1.1161 (0.0558)
	0.5	1.0	0.5583	1.1201 (0.0086)	1.1279 (4.8666)	1.1163 (1.1124)	1.1201 (0.0112)	1.1199 (0.0560)
	1.0	0.5	0.5583	1.1120 (0.0085)	1.119 (4.8705)	1.1081 (1.1043)	1.1119 (0.0111)	1.1118 (0.0556)
2.0	0.5	0.5	0.5583	1.1163 (0.0086)	1.1240 (4.8667)	1.1124 (1.1085)	1.1162 (0.0111)	1.1161 (0.0558)
	0.5	1.0	0.5583	1.1201 (0.0086)	1.1279 (4.8666)	1.1163 (1.1124)	1.1201 (0.0112)	1.1199 (0.0560)
	1.0	0.5	0.5583	1.1120 (0.0085)	1.1197 (4.8705)	1.1081 (1.1043)	1.1119 (0.0111)	1.1118 (0.0556)
3.0	0.5	0.5	0.5583	1.1163 (0.0086)	1.1240 (4.8667)	1.1124 (1.1085)	1.1162 (0.0111)	1.1161 (0.0558)
	0.5	1.0	0.5583	1.1201 (0.0086)	1.1279 (4.8666)	1.1163 (1.1124)	1.1201 (0.0112)	1.1199 (0.05600)
	1.0	0.5	0.5583	1.1120 (0.0085)	1.1197 (4.8705)	1.1081 (1.1043)	1.1119 (0.0111)	1.1118 (0.0556)

Among other loss functions, it is evident from Table 5.1 and Table 5.2. That the Linex loss function shows smaller Bayes posterior risk under the both assumptions (extended Jeffery's prior and gamma prior). According to decision rule of less Bayes posterior risk, we accomplish that Linex loss function is more useful than others.

Conclusion:

In this paper, we have initially obtained the Bayes posterior distribution and estimation of parameter of the inverse Ailamujia distribution under both informative and non-informative priors. We have discussed different loss functions among them Linex loss function provide less Bayes posterior risk. Eventually through an application, the performance of the estimators has been achieved.

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