

CHARACTERIZATION OF α Ig-LOCALLY CLOSED SETS IN IDEAL TOPOLOGICAL SPACES

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Abstract

We introduce new class of sets namely α IgIc-set, α IgIc*-set, α IgIc**-set and discuss about their properties. The concept of α IgIc-set, α IgIc*-set, α IgIc**-set are used to define α IgIc-continuous functions, α IgIc*-continuous functions and α IgIc**-continuous functions.

Keywords: α Ig-closed sets, α Ig-continuous function, α Ig-irresolute function, α IgIc-set, α IgIc*-set, α IgIc**-set, α IgIc-set, α IgIc*-set, α IgIc**-set.

I. INTRODUCTION

An Ideal I on a topological space (X, τ, I) is defined as a non-empty collection I of subsets of X satisfying the following two conditions (i) if $A \in I$ and $B \subset A$, then $B \in I$ (ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$. We will make use of the basic facts about the local functions without mentioning it explicitly. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(\tau, I)$ called the $*$ -topology, finer than τ is defined by $cl^*(A) = A \cup A^*(\tau, I)$. When there is no chance for confusion, we simply write A^* instead of $A^*(\tau, I)$ and τ^* for $\tau^*(\tau, I)$. For a subset $A \subset X$, $A^*(\tau, I) = \{x \in X / U \cap A \notin I \text{ for each neighborhood } U \text{ of } x\}$. For every ideal topological space (X, τ, I) , there exists a topology generated by $\beta(I, \tau) = \{U - J / U \in \tau \text{ and } J \in I\}$. In general, $\beta(I, \tau)$ is not always a topology [1]. If I is an ideal on X , then it is called ideal space. By an ideal space, we always mean an ideal topological space with no separation properties assumed.

In 1898, locally closed sets are investigate by Ganster and Reilly [4]. In topological spaces locally closed sets were studied more by Bourbaki [2] in 1966, which is the intersection of an open set and a closed set. Kuratowski [6] was introduced the local function in ideal spaces. Vaidyanathaswamy [15] was given much importance to the topic and ideal topological space. Balachandran, Sundaram and Maki [1] introduced and investigated the concept of generalized locally closed sets. Navaneethakrishnan and Sivaraj [14] were introduced the concept of Ig-locally $*$ -closed sets in ideal topological spaces. M. Navaneethakrishnan, P. Periyasamy, S. PiousMissier [13] introduced locally closed sets, normal space and connectedness in ideal topological spaces. Also introduced $\widehat{\delta}$ s-locally closed sets, $\widehat{\delta}$ s-normal space, $\widehat{\delta}$ s-separated sets and $\widehat{\delta}$ s-connectedness in ideal topological spaces. Sakthi alias sathya B and Murugesan S [12] introduced new class of sets namely rpsIlc-set, rpsIlc*-set, rpsIlc**-set. In 1999, Dontchev [3] introduced I locally closed subsets in an ideal topological spaces. In 2000, Han Park [5] introduced sg-locally closed sets and SGLC-continuous functions in topological spaces. In this paper, we

introduce the concept of α Iglc-set, α Iglc*-set, α Iglc**-set and discuss about their properties. The concept of α Iglc-set, α Iglc*-set, α Iglc**-set are used to define α Iglc-continuous functions, α Iglc*-continuous functions and α Iglc**-continuous functions.

II. PRELIMINARIES

Definition 2.1[12]: A subset A of a topological space (X, τ) is called locally closed, if A is the intersection of an open set and a closed set.

Definition 2.2[12]: A function $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is called ILC-continuous, if $f^{-1}(V)$ is I-locally closed in X for each closed set V in Y .

Definition 2.3[13]: A subset A of an ideal space (X, τ, I) is said to be I-locally *-closed if there exist an open set U and a *-closed set F such that $A = U \cap F$.

Definition 2.4[13]: A subset A of an ideal space (X, τ, I) is said to be Ig-locally *-closed if there exist an Ig-open set U and a *-closed set F such that $A = U \cap F$.

Definition 2.5[12]: A subset A of an ideal topological space (X, τ, I) is called

- (i) rpsIlc-set if there exists an rpsI-open set U and a rpsI-closed set V of X such that $A = U \cap V$.
- (ii) rpsIlc*-set if there exists an rpsI-open set U and a closed set F of X such that $A = U \cap F$.
- (iii) rpsIlc**-set if there exists an open set U and a rpsI-closed set F of X such that $A = U \cap F$.

Definition 2.6[12]: A function $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is called

- (i) RPSILC-continuous if $f^{-1}(A)$ is rpsIlc-set in X , for every open subset A of Y .
- (ii) RPSILC*-continuous if $f^{-1}(A)$ is rpsIlc*-set in X , for every open subset A of Y .
- (iii) RPSILC**-continuous if $f^{-1}(A)$ is rpsIlc**-set in X , for every open subset A of Y .

Definition 2.7[12]: A function $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is called

- (i) RPSILC-irresolute if $f^{-1}(A)$ is rpsIlc-set in X , for every rpsIlc-set A of Y .
- (ii) RPSILC*-irresolute if $f^{-1}(A)$ is rpsIlc*-set in X , for every rpsIlc*-set A of Y .
- (iii) RPSILC**-irresolute if $f^{-1}(A)$ is rpsIlc**-set in X , for every rpsIlc*-set A of Y .

Definition 2.8[7]: Let (X, τ) be a topological space and I be an ideal on X . A subset A of X is said to be an α -Ideal generalized closed set (α Ig-closed set) if $A^* \subseteq U$ whenever $A \subseteq U$ and U is α -open.

Definition 2.9[8]: A function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is called α Ig-continuous, if the inverse image of every closed set in Y is α Ig-closed set in X .

Definition 2.10[8]: A function $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is said to be α Ig-irresolute, if $f^{-1}(A)$ is α Ig-closed set in (X, τ, I) , for every α Ig-closed set A in (Y, σ, J) .

Definition 2.11[7]: A Subset A of an ideal space (X, τ, I) is said to be *-closed, if $A^* \subseteq A$.

III. α IgIc-SETS, α IgIc*-SETS AND α IgIc**-SETS

In this section, we introduce α IgIc-set, α IgIc*-set and α IgIc**-set each of which is stronger than α IgIc-set and is weaker than I-locally closed set and study their relations with existing ones.

Definition 3.1: A subset A of an ideal topological space (X, τ, I) is called an

- (i) α IgIc-set if there exists an α Ig-open set U and an α Ig-closed set V of X such that $A = U \cap V$.
- (ii) α IgIc*-set if there exists an α Ig-open set U and a closed set F of X such that $A = U \cap F$.
- (iii) α IgIc**-set if there exists an open set U and an α Ig-closed set F of X such that $A = U \cap F$.

The collection of all α IgIc-sets (resp. α IgIc*-sets, α IgIc**-sets) of (X, τ, I) will be denoted by α IgLC (X, τ) (resp. α IgLC*(X, τ), α IgLC**(X, τ)). The following theorem follows from the definitions.

Theorem 3.2: For an ideal topological space (X, τ, I) the following implications hold:

- (i) $ILC(X, \tau) \subseteq \alpha$ IgLC*(X, τ) \subseteq α IgLC(X, τ).
- (ii) $ILC(X, \tau) \subseteq \alpha$ IgLC**(X, τ) \subseteq α IgLC(X, τ).

The reverse inclusion need not be true as seen from the following example.

Example 3.3: Consider the ideal topological space (X, τ, I) , where $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then $ILC(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$, α IgLC(X, τ) = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{b, c, d\}, X\}$. In this ideal space, the set $\{d\}$, $\{c, d\}$, $\{b, c, d\}$ are α IgIc-set but not Ilc-set. Also, α IgLC*(X, τ) = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{c, d\}, \{b, c, d\}, X\}$. The set $\{d\}$, $\{c, d\}$, $\{b, c, d\}$ are α IgIc*-set but not Ilcset. Here, α IgLC**(X, τ) = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$. In this ideal space, the set $\{d\}$, $\{c, d\}$, $\{b, c, d\}$ are α IgIc-set but not α IgIc**-set.

Remark 3.4: The following example shows that α IgIc-sets, α IgIc*-sets and α IgIc**-sets are independent of each other.

Example 3.5: Consider the ideal topological space (X, τ, I) , where $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a, c\}, \{d\}, \{a, c, d\}, X\}$ and $I = \{\emptyset, \{c\}, \{d\}, \{c, d\}\}$. Then α IgLC(X, τ) = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, X\}$. α IgLC*(X, τ) = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, X\}$. α IgLC**(X, τ) = $\{\emptyset, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, X\}$. In this ideal space, the set $\{c, d\}$ is α IgIc**-set but not α IgIc*-set and the set $\{b\}$ is α IgIc*-set but not α IgIc**-set. Also the set $\{c, d\}$ is α IgIc-set but not α IgIc*-set and the set $\{a, b\}$ is α IgIc-set but not α IgIc**-set

Remark 3.6: The following example shows that the union of two α IgIc*-sets need not be α IgIc* set.

Example 3.7: Consider the ideal topological space (X, τ, I) , where $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a, c\}, \{d\}, \{a, c, d\}, X\}$ and $I = \{\emptyset, \{c\}, \{d\}, \{c, d\}\}$. Then α IgLC*(X, τ) = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, d\}, \{a, b, c\}, X\}$. In this ideal space, $\{a\} \cup \{b, d\} = \{a, b, d\}$ is not an α IgIc*-set.

Remark 3.8: The following example shows that the union of two αIglc^{**} -sets need not be an αIglc^{**} -set.

Example 3.9: Consider the ideal topological space (X, τ, I) , where $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, \{b, d\}, \{a, b, d\}, X\}$ and $I = \{\emptyset, \{a\}\}$. Then $\alpha\text{Iglc}^{**}(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{d\}, \{b, d\}, \{a, d\}, \{a, b, d\}, X\}$. In this ideal space, the sets $\{a\} \cup \{b\} = \{a, b\}$ is not an αIglc^{**} -set.

Theorem 3.10: Let A and B be any two subsets of an ideal topological space (X, τ, I) . If $A \in \alpha\text{Iglc}^*(X, \tau)$ and B is closed then $A \cap B \in \alpha\text{Iglc}^*(X, \tau)$.

Proof: If $A \in \alpha\text{Iglc}^*(X, \tau)$, then there exists an αIgl -open set U and a closed set F in X such that $A = U \cap F$. Now, $A \cap B = (U \cap F) \cap B = U \cap (F \cap B) \in \alpha\text{Iglc}^*(X, \tau)$.

Theorem 3.11: Let A and B be any two subsets of an ideal topological space (X, τ, I) . If $A \in \alpha\text{Iglc}^{**}(X, \tau)$ and B is open then $A \cap B \in \alpha\text{Iglc}^{**}(X, \tau)$.

Proof: If $A \in \alpha\text{Iglc}^{**}(X, \tau)$, then there exists an αIgl -closed set V and a open set G in X such that $A = V \cap G$. Now $A \cap B = (V \cap G) \cap B = V \cap (G \cap B) \in \alpha\text{Iglc}^{**}(X, \tau)$.

Theorem 3.12: Let (X, τ, I) be an ideal topological space and $A \subseteq X$. Then $A \in \alpha\text{Iglc}^{**}(X, \tau)$ if and only if $A = U \cap \alpha\text{Igl-cl}(A)$ for some open set U .

Proof: Let $A \in \alpha\text{Iglc}^{**}(X, \tau)$. Then $A = U \cap F$, where U is open and F is αIgl -closed. Since $A \subseteq F$ implies $\alpha\text{Igl-cl}(A) \subseteq F$. Now $A = A \cap \alpha\text{Igl-cl}(A) = (U \cap F) \cap \alpha\text{Igl-cl}(A) = U \cap \alpha\text{Igl-cl}(A)$.

Converse: Let $A = U \cap \alpha\text{Igl-cl}(A)$, where U is open. Since $\alpha\text{Igl-cl}(A) \subseteq F$, we have $A = U \cap \alpha\text{Igl-cl}(A) \subseteq U \cap F$. This implies that $A \in \alpha\text{Iglc}^{**}(X, \tau)$.

Theorem 3.13: Let (X, τ, I) be an ideal topological space and subsets of X . Then the following hold.

- (i) If $A, B \in \alpha\text{Iglc}(X, \tau)$, then $A \cap B \in \alpha\text{Iglc}(X, \tau)$.
- (ii) If $A, B \in \alpha\text{Iglc}^*(X, \tau)$, then $A \cap B \in \alpha\text{Iglc}^*(X, \tau)$.
- (iii) If $A, B \in \alpha\text{Iglc}^{**}(X, \tau)$, then $A \cap B \in \alpha\text{Iglc}^{**}(X, \tau)$.

Proof: (i) Since $A, B \in \alpha\text{Iglc}(X, \tau)$, there exist αIgl -open sets U, V and αIgl -closed sets F, G such that $A = U \cap F$ and $B = V \cap G$. Now $A \cap B = (U \cap F) \cap (V \cap G) \in \alpha\text{Iglc}(X, \tau)$.

(ii) Since $A, B \in \alpha\text{Iglc}^*(X, \tau)$, there exist αIgl -open sets U, V and closed sets F, G such that $A = U \cap F$ and $B = V \cap G$. Now $A \cap B = (U \cap F) \cap (V \cap G) \in \alpha\text{Iglc}^*(X, \tau)$.

(iii) Since $A, B \in \alpha\text{Iglc}^{**}(X, \tau)$, there exists open set U, V and αIgl -closed sets F, G such that $A = U \cap F$ and $B = V \cap G$. Now $A \cap B = (U \cap F) \cap (V \cap G) \in \alpha\text{Iglc}^{**}(X, \tau)$.

IV. $\alpha\text{Iglc}, \alpha\text{Iglc}^*$ AND αIglc^{**} -CONTINUOUS FUNCTIONS

In this section, we define αIglc -continuous, αIglc^* -continuous and αIglc^{**} -continuous functions in ideal topological spaces and discuss their properties.

Definition 4.1: A function $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ is called

- (i) α IgIc-continuous if $f^{-1}(A)$ is an α IgIc-set in X , for every open subset A of Y .
- (ii) α IgIc*-continuous if $f^{-1}(A)$ is an α IgIc*-set in X , for every open subset A of Y .
- (iii) α IgIc**-continuous if $f^{-1}(A)$ is an α IgIc**-set in X , for every open subset A of Y .

Theorem 4.2: Let $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ be a function.

- (i) If f is ILC-continuous, then f is α IgIc*-continuous.
- (ii) If f is α IgIc*-continuous, then f is α IgIc-continuous.
- (iii) If f is α IgIc**-continuous, then f is α IgIc-continuous.
- (iv) If f is ILC-continuous, then f is α IgIc-continuous.

Proof: The proof follows from Theorem [3.2].

Remark 4.3: The converse of the above theorem need not be true as seen from the following example.

Example 4.4: Consider $X = Y = \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$, $\sigma = \{\Phi, \{a, c\}, \{d\}, \{a, c, d\}, Y\}$, $I = \{\Phi, \{a\}\}$. Define $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$. Then f is α IgIc*-continuous but not ILC-continuous. Because for the closed set $\{a, c, d\}$ in Y , $f^{-1}(\{a, c, d\}) = \{b, c, d\}$ which is not a I-locally closed set.

Example 4.5: Consider $X = Y = \{a, b, c, d\}$, $\tau = \{\Phi, \{a, c\}, \{d\}, \{a, c, d\}, X\}$, $\sigma = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, Y\}$, $I = \{\Phi, \{c\}, \{d\}, \{c, d\}\}$. Define a function $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$, $f(d) = d$. Then f is α IgIc-continuous but not α IgIc*-continuous. Because for the subset $\{b, c\}$ is open in Y , $f^{-1}(\{b, c\}) = \{b, c\}$ which is not an α IgIc*-set.

Example 4.6: Consider $X = Y = \{a, b, c, d\}$, $\tau = \{\Phi, \{a, c\}, \{d\}, \{a, c, d\}, X\}$, $\sigma = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, Y\}$, $I = \{\Phi, \{c\}, \{d\}, \{c, d\}\}$. Define a function $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ by $f(a) = a$, $f(b) = c$, $f(c) = b$, $f(d) = d$. Then f is α IgIc-continuous but not α IgIc**-continuous. Because for the subset $\{b, c\}$ is open in Y , $f^{-1}(\{b, c\}) = \{b, c\}$ which is not an α IgIc**-set.

Example 4.7: Consider $X = Y = \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$, $\sigma = \{\Phi, \{a, c\}, \{d\}, \{a, c, d\}, Y\}$, $I = \{\Phi, \{a\}\}$. Define $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$. Then, f is α IgIc-continuous but not ILC-continuous. Because for the closed set $\{a, c, d\}$ in Y , $f^{-1}(\{a, c, d\}) = \{b, c, d\}$ which is not a I-locally closed set.

Remark 4.8: The following example shows that the concept of α IgIc*-continuous and α IgIc**-continuous are independent of each other.

Example 4.9: Consider $X = Y = \{a, b, c, d\}$, $\tau = \{\Phi, \{a, c\}, \{d\}, \{a, c, d\}, X\}$, $\sigma = \{\Phi, \{a\}, \{b\}, \{a, b\}, Y\}$, $I = \{\Phi, \{c\}, \{d\}, \{c, d\}\}$.

- (i) Define a function $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ by $f(a) = a$, $f(b) = c$, $f(c) = d$, $f(d) = b$. Then, f is α IgIc**-continuous but not α IgIc*-continuous. Because for the subset $\{a, b\}$ is open in Y , $f^{-1}(\{a, b\}) = \{a, d\}$ which is not an α IgIc*-set.

- (ii) Define a function $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ by $f(a) = a, f(b) = b, f(c) = c, f(d) = d$. Then, f is αIglc^* -continuous but not αIglc^{**} -continuous. Because for the subset $\{a, b\}$ is open in Y , $f^{-1}(\{a, b\}) = \{a, b\}$ which is not an αIglc^{**} -set.

Theorem 4.10: Let $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ and $g : (Y, \sigma, J) \rightarrow (Z, \eta, K)$ be two functions. Then the followings are hold

- (i) $g \circ f$ is αIglc -continuous if f is αIglc -continuous and g is continuous.
(ii) $g \circ f$ is αIglc^* -continuous if f is αIglc^* -continuous and g is continuous.
(iii) $g \circ f$ is αIglc^{**} -continuous if f is αIglc^{**} -continuous and g is continuous.

Proof: (i) Let g be a continuous function and V be any closed set in (Z, η, K) . Then, $g^{-1}(V)$ is an open set in (Y, σ, J) . Since f is αIglc -continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is an αIglc -set in (X, τ, I) . Hence $g \circ f$ is αIglc -continuous.

(ii) Let g be a continuous function and V be any closed set in (Z, η, K) . Then, $g^{-1}(V)$ is an open set in (Y, σ, J) . Since f is αIglc^* -continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is an αIglc^* -set in (X, τ, I) . Hence $g \circ f$ is αIglc^* -continuous.

(iii) Let g be a continuous function and V be any closed set in (Z, η, K) . Then, $g^{-1}(V)$ is an open set in (Y, σ, J) . Since f is αIglc^{**} -continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is an αIglc^{**} -set in (X, τ, I) . Hence $g \circ f$ is αIglc^{**} -continuous.

V. αIglc , αIglc^* and αIglc^{**} -IRRESOLUTE FUNCTIONS

In this section, we define αIglc -irresolute function, αIglc^* -irresolute function and αIglc^{**} -irresolute function in ideal topological spaces and discuss about their properties.

Definition 5.1: A function $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is called

- (i) αIglc -irresolute, if $f^{-1}(A)$ is an αIglc -set in X , for every αIglc -set A of Y .
(ii) αIglc^* -irresolute, if $f^{-1}(A)$ is an αIglc^* -set in X , for every αIglc^* -set A of Y .
(iii) αIglc^{**} -irresolute, if $f^{-1}(A)$ is an αIglc^{**} -set in X , for every αIglc^{**} -set A of Y .

The following theorem gives the properties of αIglc^* -irresolute and αIglc^{**} -irresolute.

Theorem 5.2: Let $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ be a function.

- (i) If f is αIglc^* -irresolute, then f is an αIglc -irresolute function.
(ii) If f is αIglc^{**} -irresolute, then f is an αIglc -irresolute function.

Proof: It follows from Theorem [3.2]

Remark 5.3: The converse of the above theorem need not be true as seen from the following example.

Example 5.4: Consider $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a, c\}, \{d\}, \{a, c, d\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{a, b\}, \{b, d\}, \{a, b, d\}, Y\}$, $I = \{\emptyset, \{c\}, \{d\}, \{c, d\}\}$ and $J = \{\emptyset, \{a\}\}$. Define $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ by $f(a) = d$,

$f(b) = b, f(c) = c, f(d) = a$. Then, f is αIglc -irresolute but not αIglc^* -irresolute and αIglc^{**} -irresolute. Because for the subset $\{a, b, d\}$ which is an αIglc^* -set in Y , we have $f^{-1}\{a, b, d\} = \{a, b, d\}$ is not an αIglc^* -set in X and for the subset $\{b\}$ which is an αIglc^{**} -set in Y , we have $f^{-1}\{b\} = \{b\}$ is not an αIglc^{**} -set in X .

Theorem 5.5: Let $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$ and $g: (Y, \sigma, J) \rightarrow (Z, \eta, K)$ be any two functions. Then the following are hold

- (i) $g \circ f$ is αIglc -irresolute if f and g are αIglc -irresolute.
- (ii) $g \circ f$ is αIglc^* -irresolute if f and g are αIglc^* -irresolute.
- (iii) $g \circ f$ is αIglc^{**} -irresolute if f and g are αIglc^{**} -irresolute.

Proof: (i) Let g be αIglc -irresolute and V be any αIglc -set in (Z, η, K) . Then, $g^{-1}(V)$ is an αIglc -set in (Y, σ, J) . Since f is αIglc -irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is αIglc -set in (X, τ, I) . Hence $g \circ f$ is αIglc -irresolute.

(ii) Let g be αIglc^* -irresolute and V be any αIglc^* -set in (Z, η, K) . Then, $g^{-1}(V)$ is an αIglc^* -set in (Y, σ, J) . Since f is αIglc^* -irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is an αIglc^* -set in (X, τ, I) . Hence $g \circ f$ is αIglc^* -irresolute.

(iii) Let g be αIglc^{**} -irresolute and V be any αIglc^{**} -set in (Z, η, K) . Then, $g^{-1}(V)$ is an αIglc^{**} -set in (Y, σ, J) . Since f is αIglc^{**} -irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is an αIglc^{**} -set in (X, τ, I) . Hence $g \circ f$ is αIglc^{**} -irresolute.

Reference:

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