Numerical parametric study on lateral torsional buckling of T-shaped beams containing openings

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Abstract - T-shaped beams are highly susceptible to failure in lateral-torsional buckling when subjected to bending moments. However, there is a lack of previous research on the lateral-torsional buckling of T-shaped beams, particularly those containing stem openings. This research aims to numerically study T-shaped beams' behavior containing stem openings and subjected to mid-span concentrated load and uniformly distributed load. It also focuses on the impact of beam geometric parameters and openings on the moment gradient factor of simply supported beams. To that aim, a 3D finite element model for a 324 T-beam was established using a finite element program. Both the beam geometric imperfection and material nonlinearity are considered in this study. The numerical modeling technique and parameters were verified using theoretical, numerical, and experimental work from the literature. A comprehensive parametric study was then performed to investigate various parameters on the studied beams' moment gradient factor. It was found that the studied parameters have a considerable impact on the behavior of the T-shaped beams. Also, the openings adversely affect the moment gradient factor used in LTB calculation. Moreover, the moment gradient factor for beams of short span and small stem thickness must be carefully estimated when using gradient factors recommended by design codes.

Keywords – Finite element, Web openings, Lateral torsional buckling, T-shaped beams, moment gradient factor

I. INTRODUCTION

T-shaped beams are typically isolated from I-section and can be used for all load applications similar to I-section. However, T-section is more susceptible to lateral-torsional buckling (LTB), which occurs when the compression flange of the beam deflects vertically and then deflects laterally and rotates simultaneously. For decades, several types of research studying the LTB of beams have been carried out on double and single symmetric I-shaped beams [1-4]; however, there is limited research on LTB T-shaped beams, particularly those containing openings. Closed-form solutions for LTB critical moment could be attained for I-section beams under a constant moment. This results in the implementation of a moment gradient factor to account for other load scenarios.

There are limited researches on the evaluation of the moment gradient factor of T-shaped beams. It is worth mentioning that the only researches in this regard were carried out by [5,6]. In [5] simple beam subjected to moment gradient at its ends was investigated using an energy approach. While [6] studied the adequacy of the moment gradient factor of T-shaped beams in the elastic and inelastic stage. They also consider geometric imperfection and residual stress in analysis and compared their results with those used different design codes. They concluded that including residual stress does not significantly affect the LTB moment. Accordingly, this section presents a review of relevant researches conducted on steel beams containing web openings.

Different approaches were used in the evaluation of LTB of steel beams. The approaches include theoretical, numerical, and experimental work. [7] presented an exact formula for calculating the buckling load of thin-walled beams by involving coupling between LTB and the beams' failure modes like distortional and local buckling. Complex phenomena, such as the load height effect arising from the interaction between the relative rotation of the wall and the cross-section rotation, are captured accurately. [8] presented an extensive analytical study of the lateral-torsional buckling of simply supported beams with openings subject to moment and uniformly distributed load. They derived an analytical expression for the LTB moment using the principle of total potential energy. A three-dimensional model was used to verify the analytical model. The conclusion was that the critical moment should be calculated on the average torsional constant of the full and reduced sections instead of simply taking the average of
the critical moments or loads calculated from the full and reduced section properties. The strength of steel beams with web openings was studied by [9-12], where they suggested some formulas to design such beams.

An experimental test was used to evaluate the LTB of steel beams with web openings [13-17]. [13] conducted eight experiments to study the behavior of LTB capacity of welded beams. All beams were tested under mid-span concentrated load. They established a parametric study to investigate the effect of cross-section geometric dimensions and beam span on the LTB of the beam. A comprehensive parametric study was carried out using a validated numerical model. Then, they compared the results of the analysis with the LTB curve of the Chinese design code. It was concluded that the code equation is not conservative in the LTB evaluation of HSS beams and presented an alternative equation. [14] conducted experimental tests on seven cellular beams to investigate the different failure modes of beams subjected to two-point loads. Most of the tests were conducted on beams without web stiffeners to capture the web-post buckling mode. Several lateral supports were provided to the test beams at short intervals to avoid LTB effects. It was concluded from tests that web bearing stiffeners enhance beams' load-carrying capacity with web openings. [18] carried out an experimental study on eight cellular specimens with different geometries, spans, and layouts of circular openings. They investigated the ultimate load and deflection behavior of the beams. Test specimens were configured to avoid the possibility of a web-post buckling mechanism. The beams were tested based on two loading conditions; three-point and four-point loading. They reported that the Vierendeel mechanism failed seven beams while only one specimen exhibited web-post buckling failure.

Extensive numerical studies were carried out to assess the LTB behavior steel beam with web openings [19-23]. [20] investigated the LTB buckling of steel beams with web openings considering the distribution of residual stresses after the manufacturing. They analyzed simple beams subjected to mid-span concentrated load and uniformly distributed load using a finite element model. They concluded that the new proposal's prescription calculation is reliable, accurate, and conservative for LTB resistance in inelastic and elastic behavior, taking into account the imperfection factor that meets the residual stresses' magnitude after the cellular steel production beam. [24] employed the finite element method to study the behavior of steel beams with circular web openings. They established a three-dimensional model that considers possible failure modes of the beam and conducted an extensive parametric study to evaluate various geometrical parameters' effect on the beam's stability. Moment gradient factor that corresponds to all possible mode failure were reported. [21] studied the LTB resistance of steel beams with web openings considering load and resistance factor design. They conducted geometric and material nonlinearity in their analysis. They concluded that design strength is ineffective under inelastic buckling, where the interaction of failure modes can occur. [25] investigated the influence of cellular web holes on steel beams' elastic lateral stability numerically. A comprehensive parametric analysis revealed that the beam geometry and slenderness significantly influence the moment gradient factor. The study also presented that as the beam slenderness decreases, web distortion increases, leading to lateral distortional buckling mode associated with a lower moment gradient factor than those recommended by design codes for solid beams. [26] performed nonlinear numerical modeling to explore the failure loads and load-deflection relations of normal and high strength cellular steel beams. A parametric study was carried out on 120 cellular beams to identify the effects of variation in beam geometry, span, and material on the LTB of the beams. Midspan concentrated load was used throughout the study. It was concluded that the combined effect of web distortion and web-post buckling leads to a considerable reduction in the beams' load-carrying capacity. He also presented that high strength steel results in a considerable increase in the load-carrying capacity of cellular steel beams with low slenderness. [19] numerically investigated the LTB of steel beams with web openings. They conducted a parametric study to evaluate the variation of moment gradient factor with different span lengths. They suggested a reduction factor to the gradient factor used by the Indian code. [27] investigated the elastic LTB of simply supported I-shaped steel beams under mid-span concentrated load and moment gradient finite element analysis. Several unbraced member lengths and end moment values were considered to compare and evaluate the numerical model in terms of elastic critical moment and end moment ratios. Analysis results showed that LTB is an essential stability problem for I-shaped members that are under flexure. They concluded that those beams are reflected with adequate safety in the design codes and standards considering finite element analysis outcomes. [28] reported a comprehensive numerical study on steel beams with web openings of different shapes and sizes. The beams are examined in detail. It was found that the failure mode is similar in all beams. Also, steel beams with large web openings of different shapes behave similarly under a wide range of applied moments. Based on the numerical results, they proposed an empirical design approach for the beams with web openings. It was also shown that the length of the tee-sections above and below the web opening length is the most important parameter in evaluating perforated beams.
The literature reviewed above reveals a lack of experimental test results and finite element analysis on T-shaped beams, particularly T-shaped beams with stem openings. This paper's objective is:
- Determining the moment gradient factor of the beam with openings through the investigation of LTB.
- Investigate the effect of opening size, flange thickness, and web thickness on the inelastic LTB of the T-shaped beams.
- Comparing the moment gradient factor results from the current research with the design codes' values.

In this paper, a brief background on the T-shaped beams' elastic critical moment is presented first. A detailed 3D finite element model for T-shaped beams, including stem openings with different geometric dimensions, was then established using finite element ABAQUS software [29]. The model parameters were validated with theoretical, numerical, and experimental test results from the literature. The validated model was used to conduct a comprehensive parametric study where 324 models were established to achieve this goal. It was found that the moment gradient factor for beams of short span and low stem thickness has to be carefully estimated when using gradient factors recommended by design codes.

![Figure 1. Simply supported T-beam under a uniform moment](image)

1.1 Theoretical overview

Since the warping constant of the T-shape is zero and the values $\theta(x)$ and $\frac{dy}{dx}$ are small, the governing differential equations of LTB of T-beam can be written as follows:

$$y''(x) = \frac{M}{EI_y} \theta(x) \quad (1)$$

$$\left(GI_y + M\beta_y\right)\theta''(x) + My''(x) = 0 \quad (2)$$

Substitute (1) into (2) yields

$$\theta''(x) + \frac{M^2}{(GI_y + M\beta_y)EI_y} \theta(x) = 0 \quad (3)$$

Equation (3) is a linear second-order differential equation, and its general solution is

$$\theta(x) = C_1 \cos \left(\frac{Mx}{\sqrt{(GI_y + M\beta_y)EI_y}}\right) + C_2 \sin \left(\frac{Mx}{\sqrt{(GI_y + M\beta_y)EI_y}}\right) \quad (4)$$

Boundary conditions

$$\theta(0) = 0 \quad (5)$$

$$\theta(L) = 0 \quad (6)$$

Substituting the first boundary condition (5) into (4) gives $C_1 = 0$

Now substitute the second boundary condition (6) into (4) gives
\[ \theta(L) = C_2 \sin \left( \frac{ML}{(GJ + M\beta_y)EI_y} \right) = 0 \]  
(7)

Which requires that

\[ \sin \left( \frac{ML}{(GJ + M\beta_y)EI_y} \right) = 0 \]  
(8)

hence,

\[ \frac{ML}{(GJ + M\beta_y)EI_y} = n\pi \]  
(9)

n=1 yields the lowest critical value for M

\[ M^2L^2 = \pi^2(GJ + M\beta_y)EI_y \]  
(10)

Rearrange equation (10) gives

\[ L^2M^2 - \pi^2\beta_yEI_yM - \pi^2GJEI_y = 0 \]  
(11)

Solve the second-degree equation yields a simply supported T-shape beam's critical buckling moment under a uniform moment.

\[ M_{cr} = \frac{\pi}{L} (EI_yGJ)^{1/2} \times \left[ (1 + \frac{\pi^2\beta_yEI_y}{4GJL^2})^{1/2} + \frac{\pi\beta_y}{2L} \left( \frac{EI_y}{GJ} \right)^{1/2} \right] \]  
(12)

\[ \beta_y = 0.9d' \]  
(13)

\[ d' = d - \frac{t_f}{2} \]  
(14)

\( E\ell \) is the minor axis flexural rigidity, \( GJ \) is the torsional rigidity, \( L \) is the length of the beam, \( d \) is the section's depth, and \( t_f \) is the flange's thickness. When the moment is not uniform and the beam containing openings, numerical methods are used to predict the critical buckling moment. Among these methods, the method of finite element, finite integrals, and finite differences. In this study, the finite element method is used to investigate the LTB of T-shaped beams containing openings. In practice, the beam may be subjected to a non-uniform moment with a greater critical moment than equation (12). This is reflected in the design codes using the moment gradient factor \( (C_b) \), which relates the critical buckling moment of a non-uniform moment to the uniform moment's critical buckling moment.

<table>
<thead>
<tr>
<th>Design Code</th>
<th>Moment gradient factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Egyptian code of practice ASD [30]</td>
<td>For cases of unequal end moments without transversal loads: ( C_b = 1.75 + 1.05\beta + 0.3\beta^2 ) ( (16) ) Where ( \beta ) is the smaller to the bigger end moment ratio.</td>
</tr>
<tr>
<td>Egyptian code of practice LRFD [31]</td>
<td>( L'_{b} = \frac{M_2}{2.25M_2^2 + 2M_2M_0 + M_0^2} ) ( (17) ) Where ( M_2 ) absolute value of the maximum moment and ( M_0, M_0, ) and ( M_c ) are the absolute value of the moment at the quarter, centerline, and three-quarter points of the beam.</td>
</tr>
<tr>
<td>AISC 360-16 [32]</td>
<td>The ( C_b ) factor is not included in the code equation to calculate T-shaped beams' critical moment where its value is implicitly considered as 1.0.</td>
</tr>
<tr>
<td>CSA S16-14 [33]</td>
<td>( C_b = \frac{4M_{max}}{\sqrt{4M_{max}^2 + 4M_0^2}} \leq 2.5 ) ( (18) ) The moments' definition is as per in ECP-LRFD equation.</td>
</tr>
</tbody>
</table>
1.2 Code provisions

Design codes relate the buckling moment capacity $M_{LTB}$ of steel beams with different loads and support conditions under moment gradient to its elastic critical buckling capacity (without the effect of imperfections or residual stresses) $M_{cr}$ using moment gradient factor $C_b$ such that:

$$M_{LTB} = C_b \times M_{cr}$$

The moment gradient factor $C_b$ is calculated in different design specifications, as listed in Table 1. It can be seen that moment gradient factor is not explicitly

II. PROBLEM DESCRIPTION

The analysis considers a T-shaped beam’s geometry distinguished by its span $L$, flange width $b_f$, flange thickness $t_f$, stem height $h_s$, and stem thickness $t_s$. Circular openings with a diameter $d_o$ equally spaced at distance $S$ along the span. The beam has an integer-valued length-to-opening spacing ratio $L/S$, which corresponds to having several standard panels of width $S$, each with a single opening. Figure 2 shows the geometry of a typical T-shaped beam containing openings in the stem. A simply supported beam is utilized with a hinge at its left end and a roller at its right end. The supports are located at the center of gravity of the T-shaped section. A cartesian coordinate system is also used with Y-axis down with the beam depth and Z-axis along the beam length. Localized web yielding due to concentrated reaction is avoided by using stiffener plates at beam ends. A comprehensive survey has been undertaken to categorize the widely used practical dimensions of T-shaped beams with openings. A total of 108 different geometrical configurations is considered in the current study; however, 324 finite element analysis is reported since each configuration is studied under three different load cases. The Tee section's dimension is chosen to cover the Egyptian code's three-section category, namely compact, non-compact, and slender. The range of dimensions adopted is summarized in Table 2 using dimensionless parameters that define the T-shaped beam geometry. All the analyzed beams are assigned a single value for flange width $b_f$ and stem depth $d_o$.

![Figure 2. The geometry of the studied problem](image-url)
Table 2 Numerical Parameters of T-shaped Beams Containing Openings

<table>
<thead>
<tr>
<th>L (mm)</th>
<th>5400</th>
<th>6300</th>
<th>7200</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_s ) (mm)</td>
<td>10</td>
<td>8</td>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>( d_o/d_s )</td>
<td>0.5</td>
<td>0.6</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>( t_f ) (mm)</td>
<td>12</td>
<td>10</td>
<td>8</td>
<td>6</td>
</tr>
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</table>

III. NUMERICAL MODELS

Previous studies have proved that the finite element method has been successfully used to evaluate the elastic and inelastic buckling of structural elements. In this study, the numerical finite element model has been established using general-purpose finite element software ABAQUS [29]. The Riks method given by ABAQUS is an important technique that is commonly used to predict buckling loads. The method comprises nonlinear materials and boundary conditions. Evaluate the critical moment of T-shaped beams containing openings through numerical analysis using general purpose. Figure 3 shows the analysis procedure starts with performing linear buckling analysis where buckling load and buckling shapes are the stage's outcomes. Inelastic buckling analysis is then performed using the arc-length method or Riks analysis, considering the first stage results. The Tee shaped beams are modeled using the general-purpose S4 shell element (with full integration). This element has four corner nodes with 6 degrees of freedom and is ideal for a study involving finite membrane strains and large rotations. The flange and the Tee beam's stem have meshed such that six elements per flange width, 16 elements per web height, have meshed to obtain accurate results.

![Figure 3. Analysis procedure of buckling analysis using ABAQUS software](image)

![Figure 4. Linear and bilinear material model for steel ST52 (a. Elastic material and b. Elasto-plastic material)](image)
3.1 Materials

The two material models used in the finite element models to simulate the steel material's elastic and elastoplastic behavior are shown in Fig. 4, in the elastic and nonlinear elastoplastic analyses. For elastic analysis, the T-shaped beam is assumed to have linear elastic material with Young's modulus $E = 210$ GPa and Poisson's ratio $\nu = 0.3$, as shown in Figur 4(a). For nonlinear analysis, the beam material is assumed to be elastoplastic with multi-linear isotropic, kinematic hardening, as shown in Fig. 3(b). This multi-linear material model comprises initial elastic Young's modulus $E$, strain hardening modulus $E_t$, Poisson's ratio $\nu$, yield stress $F_y$, and ultimate stress $F_u$. The values of $E = 210$ GPa, $E_t = 6.3$ GPa, $\nu = 0.3$, $y=250$ MPa, $F_u=400$ MPa, and $\varepsilon = 0.014$ represent the typical characteristics of Carbon steel used in the study unless otherwise noted, and as provided by Salmon et al. [18]

3.2 Boundary condition and constraints

Boundary conditions are shown in Figur 2. Vertical displacement $U_y$, lateral displacement $U_x$, and longitudinal displacement $U_z$ are restrained at the left support, while vertical and lateral displacement is restrained at the right support. Also, torsional rotation is not allowed along the beam length. Kinematic coupling constraints limit the group of nodes' motion in the flange and stem of the section. The finite element factors applied in this model (element types, material models, and mesh size) have been verified against theoretical, numerical, and experimental results from the literature.

![Types of applied load](image)

**Figure 5.** Types of applied load a. mid-span concentrated load b. uniform distributed load c. end moments

<table>
<thead>
<tr>
<th>Beam length (mm)</th>
<th>Critical buckling moment (kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kitiporanchi and Wang [5]</td>
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<tr>
<td></td>
<td>LTBeam program results</td>
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<tr>
<td></td>
<td>Numerical model results</td>
</tr>
<tr>
<td>3000</td>
<td>170.5</td>
</tr>
<tr>
<td>6000</td>
<td>54.1</td>
</tr>
<tr>
<td>9000</td>
<td>31.9</td>
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<tr>
<td>12000</td>
<td>21.81</td>
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<tr>
<td>12000</td>
<td>168.81</td>
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<td>52.33</td>
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<td>29.46</td>
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<td>20.03</td>
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<td>166.95</td>
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<tr>
<td>52.9</td>
<td></td>
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<tr>
<td>29.56</td>
<td></td>
</tr>
<tr>
<td>22.91</td>
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</table>

3.3 Load applications

Figure 5 depicts the three different load types which have been considered in this study. They comprise concentrated mid-span load, uniform distrusted load, and equal and opposite end moments. Concentrated load and uniform load are applied at the T-shaped beam's shear center to avoid stabilizing and destabilizing effects. The concentrated load is applied to a square area to avoid web crippling.
IV. NUMERICAL MODEL VERIFICATION

Many researchers studied I-shaped beams with or without web openings; however, few studies were found for T-shaped beams without openings and none for T-shaped beams with openings. Accordingly, this research's numerical model parameters (material models, element types, mesh size, and analysis technique) were validated against theoretical, numerical, and experimental model of T-shaped without openings and an I-shaped section with and without web openings from literature. The numerical model was constructed using the same geometry and material characteristics reported in the selected literature used in the validation.

4.1 Comparison with Kitipornchai and Wang [5]

The theoretical work performed in 1985 by Kitipornchai and Wang involved investigating the elastic lateral buckling of simply supported T-beams under moment gradient. T-beams with different lengths, namely 3m, 6m, 9m, and 12m, were studied to evaluate these beams' critical buckling moment under a uniform moment. Therefore, these beams' results are used to validate the accuracy of the finite element model adopted in this paper to capture the elastic buckling capacity.
freely available software, LTBeam, is also used to analyze the same beams. The program calculates Elastic critical moment using an iteration process. Table 4 summarizes the comparison between the numerical model results with theoretical and LTBeam findings. It could be noticed that the estimated critical moment using the numerical model is in close agreement with their counterpart values.

4.2 Comparison with Fukumoto et al. [33]

In 1980 Fukumoto et al. carried out a series of tests on simply supported beams under vertical load applied at the compression flange's mid-span. Twenty-five beams were tested for each length of 2.66m, 2.0m, and 1.5m. For each length, there was one load-deflection curve reported. A lateral imperfection of L/1500 is imposed on the initial geometry to trigger the studied beams’ lateral buckling mode. The tested beams were an I-section with 200mm depth, 100mm flange width, 5.5mm web thickness, and 8mm flange thickness. The finite element model is established for 2.66m beam length, and the numerical results from the model were compared to the results of the tested beam. The numerical model was verified with the reported load-deflection curves of the tested beam. Figure 6 shows the comparison of load-deflection curves of the experimental test and the numerical model. Both horizontal and vertical deflection of the tested beam was used in the validation. Also, the reported buckling load from the test was 68.5, while that from the numerical model was 67.07 kN, as shown in Figur 7, which is a 2.1% underestimation compared to the test value.

![Tested beam dimensions](9)

4.3 Comparison with Fukumoto et al. [33]

In 2012 Nsier et al. conducted 3 full-scale tests on simply supported cellular beams of different profiles under two vertical loads, as shown in Figur 8. The beam with section profile IPE 330 was selected to calibrate the current finite element model. The beam had 17 cells with a diameter of 345mm, each having spacing s=395mm as shown in Fig. 8. The beam was exposed to a two-point loading scheme. Four stiffeners with a thickness of 20mm are used at the load application and supports. Also, the top and bottom flange are laterally braced at the point of load application. According to coupon tests, the tested steel beam's yield stress and young's modulus is 373 MPa and 173.4 GPa, respectively. The estimated numerical maximum load is 168.8 kN, which is in close agreement with the reported experimental load of 176.9 kN. The estimated mid-span deflection at failure is 69.8 mm, in good agreement with its experimental counterpart, 62.3 mm.

V. NUMERICAL RESULTS AND DISCUSSIONS

A comprehensive numerical parametric study is conducted using the verified finite element model to evaluate the buckling characteristics of a wide range of T-shaped steel beam dimensions and openings configurations, as shown in Table 2. The maximum moment at buckling MF−P and MF−W resulted from the mid-span concentrated load and uniform load, respectively, is calculated for each case. These values are then utilized to obtain the moment gradient factor, $C_b$, through normalization for the corresponding critical moment that results from a uniform moment case of loading MF−M. Therefore, the moment gradient factor is defined by Eqs. (16) and (17) for the concentrated load case and the uniformly distributed load case, respectively:

$$C_b = \frac{M_{F-P}}{M_{MF-M}}$$  \hspace{1cm} (16)

$$C_b = \frac{M_{F-W}}{M_{MF-M}}$$  \hspace{1cm} (17)
Figur 9 Moment gradient factor for beams with \( t_s = 12\, \text{mm} \) under concentrated mid-span load. a) Opening size \( d_o/h_s = 0.5 \). b) Opening size \( d_o/h_s = 0.6 \). c) Opening size \( d_o/h_s = 0.8 \).
Figur 10 Moment gradient factor for beams with \((t_s=12\text{mm})\) under uniform distributed load. a) Opening size \((d/o=0.5)\). b) Opening size \((d/o=0.6)\). c) Opening size \((d/o=0.8)\).
Fig 11 Moment gradient factor for beams with \((t=6\text{mm})\) under concentrated mid-span load. a) Opening size \(d_0/h=0.5\). b) Opening size \(d_0/h=0.6\). c) Opening size \(d_0/h=0.8\).
Figur 12 Moment gradient factor for beams with (t=6mm) under uniform distributed load. a) Opening size (d_o/h_s=0.5). b) Opening size (d_o/h_s=0.6). c) Opening size (d_o/h_s=0.8)
5.1 Effect of the stem thickness

Figures 9 and 10 demonstrate the variation of moment gradient factor $C_b$ for a set of T-shaped beams of different spans loaded with mid-span concentrated load and beams loaded with uniformly distributed load, respectively. The beams were analyzed considering different stem thicknesses of 10mm, 8mm, and 6mm, while the flange's thickness remained constant at 12mm. The following can be revealed from Figures 9 and 10:
- The $C_b$ values for beams with uniform distributed loads are typically lower than those subjected to mid-span concentrated load.
- In general, $C_b$'s values increase with the increase in the beams' span, where LTB failure mode control.
- On the contrary, for the beams with small lengths, $C_b$'s values are less than 1.0, where LTB unlikely to occur and other mode failure control.
- Similar trends can be observed for T-beams subjected to uniformly distributed load, as shown in Fig. 10 for the studied opening sizes. However, $C_b$'s values are considerably lower than their counterparts under the mid-span concentrated load. This difference in $C_b$ values is attributed to the widespread compression over a longer portion of the span, causing a lower value for the $C_b$.
- $C_b$ value of 1.0, considered by different codes for T-shaped beams, may not be considered an upper bound estimate, particularly for beams with small stem thickness, since $C_b$'s values are less than 1.0.

5.2 Effect of the flange size

Figures 9 and 11 present the variation of moment gradient factor $C_b$ for a set of T-shaped beams of different spans loaded with mid-span concentrated load and flange thickness of 12mm and 6mm, respectively. The beams were studied considering different stem thicknesses of 10mm, 8mm, and 6mm. Also, the size to stem ratio of 0.5, 0.6, and 0.8 taken into consideration. The following can be revealed from Figures 9 and 10:
- There are no significant $C_b$ value changes than the counterparts' values of the thicker flange for beams of the long span since longer beams experience LTB mode of failure.
- However, a reduction is observed in $C_b$ values for shorter span beams with flange thickness 6mm relative to those beams with 12mm flange thickness. This reduction can be attributed to web distortion of the shorter beams.
- On the contrary, for the beams with small lengths, $C_b$'s values are less than 1.0, where LTB unlikely to occur and other mode failure control.
- Similar trends can be observed for T-beams subjected to uniformly distributed load, as shown in Figure 10 and 12 for beams with a flange thickness of 12mm and 6mm, respectively.

5.3 Effect of the opening size

The effect of stem opening size on the moment gradient factor of beams is analyzed for a set of beams with different stem thicknesses; 10mm, 10mm, and 6mm and with stem opening size do/hs =0.5, 0.6, and 0.8. Two flange thicknesses are utilized; 12mm (Figures 9 and 10) and 6mm (Figures 11 and 12). As depicted from Figures 9-12, for both beams subjected to mid-span, concentrated load, and uniformly distributed load, the increase of opening size leads to a slight decrease in $C_b$ values, particularly for web thickness of 10mm and 8mm. However, a considerable reduction is observed for beams with 6mm stem thickness. This behavior is due to a decrease in the web's flexural stiffness compared to the torsional stiffness of the beam

VI.CONCLUSION

The study of lateral-torsional buckling of T-shaped beams containing openings is carried out using a three-dimensional finite element model. Liner and nonlinear material models are considered. The modeling parameters have been validated against theoretical, numerical, and experimental test results from the literature. Various stem thicknesses are utilized; 10mm, 8mm, and 6mm, while two flange thickness values of 12mm and 6mm are considered. Stem opening configuration (d/ds) is taken as 0.5, 0.6, and 0.8. A comprehensive parametric numerical analysis is performed to assess the effect of stem thickness, flange thickness dimensions, and stem opening sizes on lateral-torsional buckling and the associated moment gradient factor on T-shaped beams containing openings. Three different load cases are considered; end moment load, mid-span concentrated load, uniformly distributed load. Those
loads configuration leads to 324 case studies. The main conclusions within the range of the studied models and parameters can be summarized as follows:

1. Long spans beams exhibit moment gradient factor $C_b$ larger than those of short span where LTB controlled the beam's failure mode.
2. Short span beams are shown to experience web distortion, causing a reduction in those beams' $C_b$ values.
3. $C_b$ value of 1.0 for T-shaped beams, considered by different codes, should be carefully estimated, particularly for beams with small stem thickness having openings in the stem.
4. T-shaped beams with big flange thickness resulted in bigger $C_b$ values, where the increase of the flange thickness encounter the effect of stem openings.
5. T-shaped beams subjected to uniformly distributed load are in general experienced lower $C_b$ values than their counterparts' beams subjected to mid-span concentrated load.
6. It is recommended to carry out experimental tests for T-shaped beams with openings to validate the numerical model results. Also, different loadings configurations need to be considered to ensure the $C_b$ values used in design codes.

REFERENCES

[29] ABAQUS 6.12 [Computer software]. Dassault Systèmes, Waltham, MA.
[31] ECP-205 (LRFD), Egyptian Code of Practice for Steel Construction and Bridges. Housing and Building Research Center, 2011