# Extended Roman Domination of Product Related Graphs and Star Related Graphs.

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**Abstract: "An Extended Roman Domination function on a graph G=(V,E) is a function $f : V \rightarrow \{0, 1, 2, 3\}$ satisfying the conditions that (i) every vertex u for which $f(u)$ is either 0 or 1 is adjacent to at least one vertex v for which $f(v) = 3$, (ii) if u and v are two adjacent vertices and if $f(u) = 0$ then $f(v) \neq 0$. The weight of an Extended Roman Domination function is the sum of values assigned to all vertices". [3], [4], [9] "The minimum weight of an Extended Roman Domination function on a graph G is called the Extended Roman Domination number of G, denoted by $\gamma_{erd}$" [3], [4], [9]. In this paper we study the Extended Roman Domination of Prism, M¨obius ladder, Mongolian Tent, Ladder, Star, Firecracker Graph, Book graph and Bistar graph.**

**AMS Subject Classification: 05C69, 05C78**

**Keywords: Roman Domination function, Roman Domination number, Extended Roman Domination function, Extended Roman Domination number.**

## I. INTRODUCTION

Cockayne et al. [5] (2004) defined "a Roman dominating function (RDF) on a graph G(V, E) to be a function $f: V \rightarrow \{0,1,2\}$ satisfying the condition that every vertex u for which f(u)=0 is adjacent to at least one vertex v for which f(v) =2. The weight of a Roman Dominating function is the value f(V)=$\sum_{u \in V} f(u)$. The minimum weight of a Roman Dominating function on a graph G is called the Roman Domination number of G, denoted by $\gamma_{rd}$". [1], [4],[9],[11] The "definition of a Roman dominating function was motivated by an article in Scientific American by Ian Stewart entitled "Defend the Roman Empire" [13] and suggested even earlier by ReVelle (1997). Each vertex in our graph represents a location in the Roman Empire. A location (vertex v) is considered unsecured if no legions are stationed there (i.e., $f(v) = 0$) and secured otherwise (i.e., if f(v) ∈ {1, 2}). An unsecured location (vertex $v$) can be secured by sending a legion to $v$ from an adjacent location (an adjacent vertex $u$). But Constantine the Great (Emperor of Rome) issued a decree in the 4th century A.D. for the defense of his cities. He decreed that a legion cannot be sent from a secured location to an unsecured location if doing so leaves that location unsecured. Thus, two legions must be stationed at a location

( $ie\ f(v) = 2$) before one of the legions can be sent to an adjacent location. In this way, Emperor Constantine the Great can defend the Roman Empire". [1], [4],[9]

## II. DEFINITIONS

"An Extended Roman Domination function (ERD) on a graph G=(V,E) is a function $f: V \to \{0,1,2,3\}$ satisfying the conditions that (i) every vertex $u$ for which $f(u)$ is either 0 or 1 is adjacent to at least one vertex $v$ for which $f(v) = 3$, (ii) if $u$ and $v$ are two adjacent vertices and if $f(u) = 0$ then $f(v) \neq 0$. The weight of an Extended Roman Domination function is the value $f(v) = \sum_{u \in v} f(u)$. The minimum weight of an Extended Roman Domination function on a graph G is called the Extended Roman Domination number of G, denoted by $\gamma_{erd}$". [3],[4],[9]

"For a graph G=(V,E), let $f: V \to \{0,1,2,3\}$, and let $\{V_{0,}V_{1,}V_2, V_3\}$ be the ordered partition of $V$ induced by f, where $V_i$={$v \in V\ | f(v) = i$} and $|V_i| = n_i$, for i= 0,1,2,3. Note that there exists a 1-1 correspondence between the functions $f: V \to \{0,1,2,3\}$ and the ordered partitions $\{V_{0,}V_{1,}V_2, V_3\}$ of V. thus we will write $f = \{V_{0,}V_{1,}V_2, V_3\}$ ". [3],[4],[5], [6],[9]

"A function $f = \{V_{0,}V_{1,}V_2, V_3\}$ is an Extended Roman Domination function if, (i) $V_3 \succ V_0 \cup V_1$, where $\succ$ means that the set $V_3$ dominates the set $V_0 \cup V_1$, i.e., $V_0 \cup V_1 \subseteq$ N $[V_3]$ .The weight of $f$ is $f(V) = \sum_{u \in v} f(u) = 3n_3 + 2n_2 + n_1$ . We say a function $f = \{V_{0,}V_{1,}V_2, V_3\}$ is a $\gamma_{erd}$ function if it is an extended Roman Domination function and $f(V) = \gamma_{erd}$(G)". [3],[4],[5], [6],[9]

## III. RESULTS ON EXTENDED ROMAN DOMINATION

### 3.1 Prism Graph

"A generalized prism graph $Y_{m,n}$ is the graph Cartesian product $Y_{m,n}= C_n \times P_m$, has mn vertices and m(2n-1)". [10]

**Theorem 1:**

For the Prism graph $Y_{m,n}$ Extended Roman Domination number

$$\gamma_{erd} = \begin{cases} \frac{9n+5}{4}, \ for\ n = 2\mathrm{k}+1, \text{where k} = 1,3,5,7, \ldots \ldots. \\ \frac{9n+3}{4}, \ \ for\ \ \mathrm{n} = 2\mathrm{k}+3, \text{where k} = 1,3,5,7, \ldots \\ \frac{9n}{4}, \quad for\ n = 2k+2, where\ k = 1,3,5,7, \ldots \ldots \\ \frac{9n+2}{4}, \ for\ n = 2k+4, where\ k = 1,3,5,7, \ldots \ldots \ldots. \end{cases}$$

**Proof:** Case (i): $for\ n = 2\mathrm{k}+1$ , where $\mathrm{k} = 1,3,5,7, \ldots \ldots$

Here vertices of the Prism graph $Y_{m,n}$ are labeled as 3, 1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n+1}{2}\right)$ i.e., $|V_3| = \left|\left(\frac{n+1}{2}\right)\right|$, 1 are $\left(\frac{3n-1}{4}\right)$ $i, e., |V_1| = \left|\left(\frac{3n-1}{4}\right)\right|$ and rest of the vertices are zero. Therefore the weight of the ERD of Prism graph $Y_{m,n}$ is, $|V_1| + 3|V_3| = \left(\frac{3n-1}{4}\right) + \left(\frac{3(n+1)}{2}\right) = \left(\frac{9n+5}{4}\right)$, this being the minimum weight. Hence $\gamma_{erd}$ ($Y_{m,n}$) = $\left(\frac{9n+5}{4}\right)$.

Case (ii): $for\ n = 2\mathrm{k}+3$ where $\mathrm{k} = 1,3,5,7, \ldots \ldots$

Here vertices of the Prism graph ($Y_{m,n}$) are labeled as 3, 2,1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{(n-1)}{2}\right)$ i.e., $|V_3| = \left|\frac{(n-1)}{2}\right|$, 2 are 2 $i,e., |V_2| = |2|$, 1 are $\left(\frac{3n-7}{4}\right)$ $i,e., |V_1| = |\left(\frac{3n-7}{4}\right)|$ and rest of the vertices are zero. Therefore the weight of the ERD of Prism graph $Y_{m,n}$ is, $|V_1| + 2|V_2| + 3|V_3| = \left(\frac{3n-7}{4}\right) + 4 + \left(\frac{3(n-1)}{2}\right) = \left(\frac{9n+3}{4}\right)$, this being the minimum weight. Hence $\gamma_{erd}$ ($Y_{m,n}$) = $\left(\frac{9n+3}{4}\right)$.

Case (iii): $for\ n = 2k + 2$ where $k = 1,3,5,7, \dots ....$

Here vertices of the Prism graph $Y_{m,n}$ are labeled as 3, 1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n}{2}\right)$ i.e.,$|V_3| = \left|\left(\frac{n}{2}\right)\right|$,1 are $\left(\frac{3n}{4}\right)$ $i,e., |V_1| = \left|\left(\frac{3n}{4}\right)\right|$ and rest of the vertices are zero. Therefore the weight of the ERD of Prism graph $Y_{m,n}$ is, $|V_1| + 3|V_3| = \left(\frac{3n}{4}\right) + \left(\frac{3(n)}{2}\right) = \left(\frac{9n}{4}\right)$, this being the minimum weight. Hence $\gamma_{erd}$ ($Y_{m,n}$) = $\left(\frac{9n}{4}\right)$.

Case (iv): $for\ n = 2k + 4$ where $k = 1,3,5,7, \dots ....$

Here vertices of the Prism graph $Y_{m,n}$ are labeled as 3, 2,1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n}{2}\right)$ i.e., $|V_3| = \left|\left(\frac{n}{2}\right)\right|$, 2 are 1 $i,e., |V_2| = |1|$, 1 are $\left(\frac{3(n-2)}{4}\right)$ $i,e., |V_1| = \left|\left(\frac{3(n-2)}{4}\right)\right|$, and rest of the vertices are zero. Therefore the weight of the ERD of Prism graph $Y_{m,n}$ is, $|V_1| + 2|V_2| + 3|V_3| = \left(\frac{3(n-2)}{4}\right) + 2 + \left(\frac{3(n)}{2}\right) = \left(\frac{9n+2}{4}\right)$, this being the minimum weight. Hence $\gamma_{erd}$ ($Y_{m,n}$ )= $\left(\frac{9n+2}{4}\right)$.

### 3.2 M¨obius ladder

"The M¨obius ladder $M_n$ is the graph obtained from the ladder $P_n \times P_2$ by joining the opposite end points of the two copies of $P_n$". [8]

**Theorem 2:**

For the M¨obius ladder graph $M_n$ Extended Roman Domination number

$$\gamma_{erd} = \begin{cases} \frac{9n+1}{4}, & for\ n = 2k+1 \text{ where } k = 1,3,5,7, \dots .... \\ \frac{9n+3}{4}, & for\ n = 2k+3, \text{ where } k = 1,3,5,7, \dots \\ \frac{9n+4}{4}, & for\ n = 2k+2, where\ k = 1,3,5,7, \dots \dots \\ \frac{9n+6}{4}, & for\ n = 2k+4, where\ k = 1,3,5,7, \dots \dots ... \end{cases}$$

**Proof:** Case (i): $for\ n = 2k + 1$ where $k = 1,3,5,7, \dots ....$

Here vertices of the M¨obius ladder graph $M_n$ are labeled as 3,2, 1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n-1}{2}\right)$ i.e., $|V_3| = \left|\left(\frac{n-1}{2}\right)\right|$, 2 are 2 i.e., $|V_2| = |2|$ ,1 are $\left(\frac{3n-9}{4}\right)$ $i,e., |V_1| = \left|\left(\frac{3n-9}{4}\right)\right|$ and rest of the vertices are zero. Therefore the weight of the ERD of M¨obius ladder graph $M_n$ is , $|V_1| + 2|V_2| + 3|V_3| = \left(\frac{3n-9}{4}\right) + 4 + \left(\frac{3(n-1)}{2}\right) = \left(\frac{9n+1}{4}\right)$, this being the minimum weight.

Hence $\gamma_{erd}$ ($M_n$) = $\left(\frac{9n+1}{4}\right)$.

Case (ii): $for\ n = 2k + 3$ where $k = 1,3,5,7, \dots ....$

Here vertices of the M¨obius ladder ($M_n$) are labeled as 3, 2,1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{(n-1)}{2}\right)$ i.e., $|V_3| = \left|\frac{(n-1)}{2}\right|$, 2 are 3 $i,e., |V_2| = |3|$, 1 are $\left(\frac{3n-15}{4}\right)$ $i,e., |V_1| = \left|\left(\frac{3n-15}{4}\right)\right|$

and rest of the vertices are zero. Therefore the weight of the ERD of M¨obius ladder graph $M_n$ is, $|V_1| + 2|V_2| + 3|V_3| = \left(\frac{3n-15}{4}\right) + 6 + \left(\frac{3(n-1)}{2}\right) = \left(\frac{9n+3}{4}\right)$, this being the minimum weight.

Hence $\gamma_{erd}$ ($M_n$) = $\left(\frac{9n+3}{4}\right)$.

Case (iii): $for\ n = 2k + 2$ where $k = 1,3,5,7, \ldots ....$

Here vertices of the M¨obius ladder graph $M_n$ are labeled as 3,2, 1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n}{2}\right)$ i.e.,$|V_3| = \left|\left(\frac{n}{2}\right)\right|$, 2 are 1 $i, e., |V_2| = |1|$,1 are $\left(\frac{3n-4}{4}\right)$ $i, e., |V_1| = \left|\left(\frac{3n-4}{4}\right)\right|$ and rest of the vertices are zero. Therefore the weight of the ERD of M¨obius ladder graph is, $|V_1| + 3|V_3| = \left(\frac{3n-4}{4}\right) + 2 + \left(\frac{3(n)}{2}\right) = \left(\frac{9n+4}{4}\right)$, this being the minimum weight.

Hence $\gamma_{erd}$ ($M_n$) = $\left(\frac{9n+4}{4}\right)$.

Case (iv): $for\ n = 2k + 4$ where $k = 1,3,5,7, \ldots ....$

Here vertices of the M¨obius ladder graph $M_n$ are labeled as 3, 2,1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n}{2}\right)$ i.e., $|V_3| = \left|\left(\frac{n}{2}\right)\right|$, 2 are 2 $i, e., |V_2| = |2|$, 1 are $\left(\frac{3n-10)}{4}\right)$ $i, e., |V_1| = \left|\left(\frac{3n-10)}{4}\right)\right|$, and rest of the vertices are zero. Therefore the weight of the ERD of M¨obius ladder graph $M_n$ is, $|V_1| + 2|V_2| + 3|V_3| = \left(\frac{3n-10)}{4}\right) + 4 + \left(\frac{3(n)}{2}\right) = \left(\frac{9n+6}{4}\right)$, this being the minimum weight.

Hence $\gamma_{erd}$ ($M_n$ )= $\left(\frac{9n+6}{4}\right)$.

### 3.3 Ladder Graph

"The n-Ladder Graph can be defined as $L_n = P_2 \times P_n$ where $P_n$ is a path graph". [16]

**Theorem 3:**

For the Ladder graph ($L_n$) Extended Roman Domination number $\gamma_{erd} = \begin{cases} \frac{5n-1}{2}, & if\ (n \equiv 1\ mod\ 2) \\ \frac{5n}{2}, & (\ if\ n \equiv 0\ mod\ 2) \end{cases}$

**Proof:** Case (i): $if\ (n \equiv 1\ mod\ 2)$

Here vertices of the Ladder graph $L_n$ are labeled as 3, 2, and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n-1}{2}\right)$ i.e., $|V_3| = \left(\frac{n-1}{2}\right)$, 2 are $\left(\frac{n+1}{2}\right)$ $i, e., |V_2| = \left(\frac{n+1}{2}\right)$ and rest of the vertices are zero. Therefore the weight of the ERD of $L_n$ is, $2|V_2| + 3|V_3| = (n + 1) + \left(\frac{3(n-1)}{2}\right) = \left(\frac{5n-1}{2}\right)$, this being the minimum weight. Hence $\gamma_{erd}$ ($L_n$) = $\left(\frac{5n-1}{2}\right)$.

Case (ii): $if\ (n\ \equiv 0\ mod\ 2)$

Here vertices of the Ladder graph $L_n$ are labeled as 3, 2, and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n}{2}\right)$ i.e., $|V_3| = \left(\frac{n}{2}\right)$, 2 are $\left(\frac{n}{2}\right)$ $i,e.,|V_2| = \left(\frac{n}{2}\right)$ and rest of the vertices are zero. Therefore the weight of the ERD of $L_n$ is, $2|V_2| + 3|V_3| = n + \left(\frac{3n}{2}\right) = \left(\frac{5n}{2}\right)$, this being the minimum weight.

Hence $\gamma_{erd}$ ($L_n$) = $\left(\frac{5n}{2}\right)$.

**3.4 Mongolian Tent Graph**

"A Mongolian tent is defined as a graph obtained from $P_m \times P_n$, n odd, by adding one extra vertex above the grid and joining every other vertex of the top row of $P_m \times P_n$ to the new vertex".[8]

**Theorem 4:**

For the $M_{m,n}$ Mongolian tent graph $\gamma_{erd}(M_{m,n})$ = (m n + m), m ≥ 2, n is odd.

Case (i): When m=2 and n=3,5,7…. Then $\gamma_{erd}(M_{2,n})$ = (2n+2).

**Proof:** Here vertices of the Mongolian tent graph $M_{2,n}$ are labeled as 3,1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n+1}{2}\right)$ i.e., $|V_3| = \left|\left(\frac{n+1}{2}\right)\right|$, 1 are $\left(\frac{n+1)}{2}\right)$ $i,e.,|V_1| = \left|\left(\frac{n+1)}{2}\right)\right|$, and rest of the vertices are zero. Therefore the weight of the ERD of Mongolian tent graph $M_{2,n}$ is, $|V_1| + 3|V_3| = \left(\frac{n+1}{2}\right) + \left(\frac{3(n+1)}{2}\right) = (2n+2)$, this being the minimum weight. Hence $\gamma_{erd}$ ($M_{2,n}$)= $(2n+2)$ .

Case (ii): When m=3 and n=3, 5,7…. Then $\gamma_{erd}(M_{3,n})$= $(3n+3)$.

**Proof:** Here vertices of the Mongolian tent graph $M_{3,n}$ are labeled as 3, 2,1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n+1}{2}\right)$ i.e., $|V_3| = \left|\left(\frac{n+1}{2}\right)\right|$, 2 are $\left(\frac{n+1}{2}\right)$ $i,e.,|V_2| = \left|\left(\frac{n+1}{2}\right)\right|$, 1 are $\left(\frac{n+1}{2}\right)$ $i,e.,|V_1| = \left|\left(\frac{n+1}{2}\right)\right|$, and rest of the vertices are zero. Therefore the weight of the ERD of Mongolian tent graph $M_{3,n}$ is, $|V_1| + 2|V_2| + 3|V_3| = \left(\frac{n+1}{2}\right) + (n+1) + \left(\frac{3(n+1)}{2}\right) = \left(\frac{9n+6}{4}\right)$, this being the minimum weight. Hence $\gamma_{erd}$ ($M_{3,n}$)= $(3n+3)$ .

Case (iii): When m=4 and n=3, 5,7…. Then $\gamma_{erd}(M_{4,n})$ = (4n+4),

**Proof:** Here vertices of the Mongolian tent graph $M_{4,n}$ are labeled as 3, 2,1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $(n-1)$ i.e., $|V_3| = |(n-1)|$, 2 are 4 $i,e.,|V_2| = |4|$, 1 are $(n-1)$ $i,e.,|V_1| = |(n-1)|$, and rest of the vertices are zero. Therefore the weight of the ERD of Mongolian tent graph $M_{4,n}$ is, $|V_1| + 2|V_2| + 3|V_3| = (n-1) + 8 + (3(n-1) = (4n+4)$, this being the minimum weight. Hence $\gamma_{erd}$ ($M_{4,n}$)= $(4n+4)$ .

Thus, we have $\gamma_{erd}(M_{m,n})$ = (mn+m).

### 3.5 Star graph

"Star graph ($S_n$) is a special type of graph in which (n-1) vertices have degree 1 and a single vertex have degree (n – 1)".[17]

**Theorem 5:**

For the Star graph ($S_n$) Extended Roman Domination number for n ≥ 2 , $\gamma_{erd} = 3$

**Proof :** Here vertices of the Star graph ($S_n$) are labeled as 3and 0 to obtain the minimum weight. The vertex which is labeled 3 is 1 i.e., $|V_3| = |1|$ and rest of the vertices are zero. Therefore the weight of the ERD of ($S_n$) is, $3|V_3| = 3$ , this being the minimum weight.

Hence $\gamma_{erd}$ ($S_n$) = 3.

### 3.6 Bistar graph

"Bistar $B_{n,n}$ is the graph obtained by joining the apex vertices of two copies of stars $K_{1,n}$ by an edge. It has 2n+2 vertices and 2n+1 edges". [2]

**Theorem 6:**

For the Bistar graph , $B_{n,n}$ , n ≥ 2 Extended Roman Domination number is $\gamma_{erd}(B_{n,n}) = (2n + 3)$ .

**Proof:** Here vertices of the Bistar graph $B_{n,n}$ are labeled as 3, 2 and 0 to obtain the minimum weight. The vertices which are labeled 3 is only one i.e., $|V_3| = |1|$ , 2 are $(n)$ $i, e., |V_2| = |n|$ and rest of the vertices are zero. Therefore the weight of the ERD of $B_{n,n}$ is, $|V_2| + 3|V_3| = 2n + 3 = (2n + 3)$, this being the minimum weight.

Hence $\gamma_{erd}(B_{n,n}) = (2n + 3)$.

### 3.7 Book Graph

"The Book graph $B_m$ is the graph $S_m$ x $P_2$ where $S_m$ is the star with m+1 vertices".[8]

**Theorem 7:**

For the Book graph $B_m$ Extended Roman Domination number is $\gamma_{erd}(B_m) = (2m + 3)$ .

**Proof:** Here vertices of the Book graph $B_m$ are labeled as 3, 2 and 0 to obtain the minimum weight. The vertices which are labeled 3 is only one i.e., $|V_3| = |1|$ , 2 are $(m)$ $i, e., |V_2| = |m|$ and rest of the vertices are zero. Therefore the weight of the ERD of $B_m$ is, $|V_2| + 3|V_3| = 2m + 3 = (2m + 3)$, this being the minimum weight. Hence $\gamma_{erd}(B_m) = (2m + 3)$.

### 3.8 Firecracker Graph

"An (n,k)-Firecracker is a graph obtained by the concatenation of nk stars by linking one leaf from each".[15]

**Theorem 8:**

For the Firecracker F(n,k) , Extended Roman Domination number is

$$\gamma_{erd} = \begin{cases} \frac{5n-1}{2}, & for\ k = 2 \text{ where n} = 3,5,7, \dots\dots. \\ \frac{7n}{2}, & for\ \text{k} \geq 3, \text{where n} = 3,5,7, \dots \\ \frac{5n}{2}, & for\ k = 2\ , where\ \text{n} = 2,4,6,8, \dots\dots \\ \frac{7n-1}{2}, & for, \text{k} \geq 3, where\ \text{n} = 2,4,6,8, \dots\dots\dots. \end{cases}$$

Case (i): if n is odd and for k = 2

**Proof:** Here vertices of the Firecracker graph F(n,k) are labeled as 3, 2 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n-1}{2}\right)$ i.e., $|V_3| = \left|\left(\frac{n-1}{2}\right)\right|$, 2 are $\left(\frac{n+1}{2}\right)$ $i, e., |V_2| = \left|\left(\frac{n+1}{2}\right)\right|$ and rest of the vertices are zero. Therefore the weight of the ERD of F(n,k) is, $|V_2| + 3|V_3| = (n+1) + \left(\frac{3(n-1)}{2}\right) = \left(\frac{5n-1}{2}\right)$, this being the minimum weight. Hence $\gamma_{erd}$ (F(n,k)) = $\left(\frac{5n-1}{2}\right)$.

Case (ii): if n is odd and for k ≥ 3

**Proof:** Here vertices of the Firecracker graph F(n,k) are labeled as 3, 1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $n$ i.e., $|V_3| = |n|$, 1 are $\left(\frac{n}{2}\right)$ $i, e., |V_1| = \left|\left(\frac{n}{2}\right)\right|$ and rest of the vertices are zero. Therefore the weight of the ERD of F(n,k) is, $|V_1| + 3|V_3| = \left(\frac{n}{2}\right) + 3n = \left(\frac{7n}{2}\right)$, this being the minimum weight. Hence $\gamma_{erd}$ (F(n,k)) = $\left(\frac{7n}{2}\right)$.

Case (iii): if n is even and for k = 2

**Proof:** Here vertices of the Firecracker graph F(n,k) are labeled as 3, 2 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $\left(\frac{n}{2}\right)$ i.e., $|V_3| = \left|\left(\frac{n}{2}\right)\right|$, 2 are $\left(\frac{n}{2}\right)$ $i, e., |V_2| = \left|\left(\frac{n}{2}\right)\right|$ and rest of the vertices are zero. Therefore the weight of the ERD of F(n,k) is, $|V_2| + 3|V_3| = n + \left(\frac{3n}{2}\right) = \left(\frac{5n}{2}\right)$, this being the minimum weight. Hence $\gamma_{erd}$ (F(n,k)) = $\left(\frac{5n}{2}\right)$.

Case (iv): if n is even and for k ≥ 3

**Proof:** Here vertices of the Firecracker graph F(n,k) are labeled as 3, 1 and 0 to obtain the minimum weight. The vertices which are labeled 3 are $n$ i.e., $|V_3| = |n|$, 1 are $\left(\frac{n-1}{2}\right)$ $i, e., |V_1| = \left|\left(\frac{n-1}{2}\right)\right|$ and rest of the vertices are zero. Therefore the weight of the ERD of F(n,k) is, $|V_1| + 3|V_3| = \left(\frac{n-1}{2}\right) + 3n = \left(\frac{7n-1}{2}\right)$, this being the minimum weight. Hence $\gamma_{erd}$ (F(n,k)) = $\left(\frac{7n-1}{2}\right)$.

## IV. CONCLUSION

We have computed Extended Roman domination number of some Product related and Star related graphs. We can also study this for other class of graphs.

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