Somewhat Continuous Functions in Ideal Topological Spaces: generalized concepts

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Abstract- Al-Omeri and Noiri [2] have introduced the concept of somewhat e-I-continuous function in an ideal topological space and studied certain properties of these functions. Several generalizations of somewhat continuous functions have been introduced in the present paper viz. somewhat-r-continuous functions, somewhat-β-continuous functions and Almost-somewhat continuous functions and their further generalizations with reference to regular open sets, β-open sets and $\delta\beta_I$-open sets. The concept of somewhat e-I-continuous functions have been also generalized as somewhat-$e_I r$-continuous functions. The relationship amongst these somewhat continuous functions have been established and justified by giving relevant illustrative examples. Certain characterizations of these functions have been also explored. Finally, significant property of β-open sets has been investigated in this work.

Keywords- regular open set; β-open set; e_I-open set; $\delta\beta_I$-open sets; Almost somewhat continuous function; somewhat-$\delta\beta_I$-continuous function; somewhat-$e_I r$-continuous function.

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I INTRODUCTION

In 1933, Kuratowski [11] has originated the concept of an ideal in topological spaces. In 1992, Jankovik and Hamlett [8] introduced and studied the notion of I-open sets. Further, Abd El-Monsef et al. [1] have studied certain properties of I-open sets and by using the concept of I-open sets they have defined I-continuous functions. Gentry and Hoyle [7] initiated the concept of somewhat continuous functions and explored the characterizations of these functions. In 1979, Piotrowski [17] has applied the idea of somewhat continuous funcion on linear topology and investigated certain important results.

In 2011, Noiri and Rajesh[14] have introduced the concept of somewhat-b-continuous functions and studied extensively certain basic results. Balasubramanian and Sandhya [4] have given the idea of somewhat almost sg-continuous functions and obtained some fundamental results. The concept of somewhat λ-continuous functions have been defined and investigated by Duraismy and Vennila [6](see also [5],[9],[10],[12],[13],[15],[16],[19]). Recently, Al-Omeri and Noiri [2] have introduced the idea of somewhat-e-I-continuous functions using e_I-open sets and they have defined several other continuous function and studied their relationships pairwise.

In the present paper, the authors have generalized the concept of somewhat continuous function viz. somewhat-r-continuous function, somewhat-β-continuous functions and Almost-somewhat continuous function, These functions have been further generalized in the context of regular open sets, β-open sets and $\delta\beta_I$-open sets. The concept of somewhat e-I-continuous functions has been also generalized to somewhat-$e_I r$-continuous functions. The relationship amongst these generalized somewhat continuous functions have been established and justified by discussing some illustrative examples. Moreover, various composition of somewhat continuous functions have been discussed and finally a remarkable property of β-open sets has been also explored.

II PRE-REQUISITES

Let (X, Ω, I) be an ideal topological space. The collection of closed sets of X is denoted by F. The closure and interior of a subset A of X are denoted by $cl(A)$ and $Int(A)$ respectively.

Definition 2.1 [2] An ideal I on topological space (X, Ω) is a nonempty collection of subsets of X, which satisfies the following conditions:

• $A \in I$ and $B \subset A$ implies $B \in I$,
• $A \in I$ and $B \in I$ implies $A \cup B \in I$.

Then the triplet (X, Ω, I) is called an **ideal topological space.**

Definition 2.2 [2] (see also [1],[3]) Let (X, Ω, I) be an ideal topological space. For a set $A \subset X$,

$$A^*(I, \Omega) = \{x \in X | U \cap A \notin I \text{ for every } U \in \Omega(x)\}$$

where $\Omega(x) = \{U \in \Omega | x \in U\}$, is called the **local function** of A with respect to Ω and I. $A^*(I, \Omega)$ is simply denoted by A^*. The cl^* operator is defined as $cl^*(A) = A \cup A^*$ and is known as the Kuratowski closure operator. X^* is often a proper subset of X.

Definition 2.3 [2] A subset A of an ideal topological space (X, Ω, I) is said to be **R-I-open**(resp. R-I-closed) if $A = Int(cl^*(A))$(resp. $A = cl^*(Int(A))$). The **δ-I-interior** of A is the union of all R-I-open sets of X contained in A and is denoted by $\delta Int_I(A)$.

Definition 2.4 [2] Let (X, Ω, I) be an ideal topological space. Then a point $x \in X$ is called a **δ-I-cluster point** of A if $Int(cl^*(U)) \cap A \neq \phi$ for each open set U containing x. The family of all δ-I-cluster points of A is called the **δ-I-closure** of A and is denoted by $\delta cl_I(A)$.

Definition 2.5 A subset A of an ideal topological space (X, Ω, I) is said to be:

1. regular open [18] if $A = Int(cl(A))$,
2. β-open [18] if $A \subset cl(Int(cl(A)))$,
3. e_I-open [2] $A \subset cl(\delta Int_I(A)) \cup Int(\delta cl_I(A))$,
4. $\delta\beta_I$-open [3] if $A \subset cl(Int(\delta cl_I(A)))$.

The family of regular open sets, β-open sets, e_I-open sets and $\delta\beta_I$-open sets are denoted by $RO(X)$, $\beta O(X)$, $e_I O(X)$ and $\delta\beta_I O(X)$, respectively.

Definition 2.6 [2] A function $f: (X_1, \Omega_1) \to (X_2, \Omega_2)$ is said to be **somewhat continuous** if for each $V \in \Omega_2$ and $f^{-1}(V) \neq \phi$ there exist $U \in \Omega_1$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$.

Definition 2.7 [2] A function $f: (X_1, \Omega_1, I_1) \to (X_2, \Omega_2)$ is said to be **somewhat e-I-continuous** if for every $V \in \Omega_2$ and $f^{-1}(V) \neq \phi$ there exists $U \in e_I O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$.

III GENERALIZED SOMEWHAT CONTINUOUS FUNCTIONS

In this section, we have introduced the generalized versions of somewhat continuous function with illustrative examples. The inter relations of these functions have been also established and shown in figure 1.

• **Somewhat-r-continuous functions**

Definition 3.1 A function $f: (X_1, \Omega_1) \to (X_2, \Omega_2)$ is said to be **somewhat-r-continuous** if for each $V \in RO(X_2)$ and $f^{-1}(V) \neq \phi$, $\exists$ $U \in \Omega_1$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$.

Remark 3.1 By referring Definition 2.6 and Definition 3.1, it is clear that every somewhat continuous function is somewhat-r-continuous but the converse may not hold necessarily. For the converse part following example may be referred.

Example 3.1 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_1, a_2\}\}$. Consider $X_2 = \{b_1, b_2, b_3, b_4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{b_1\}, \{b_2\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}\}$. The collection of closed sets is $F_2 = \{\phi, X_2, \{b_2, b_3, b_4\}, \{b_1, b_3, b_4\}, \{b_3, b_4\}, \{b_1, b_4\}, \{b_4\}\}$. Now, applying Definition

2.5, we compute, $RO(X_2) = \{\phi, X_2, \{b_1\}, \{b_2, b_3\}\}$. The function $f: (X_1, \Omega_1) \rightarrow (X_2, \Omega_2)$ is defined by $f(a_1) = b_1, f(a_2) = b_3, f(a_3) = b_2, f(a_4) = b_4$. First, we show that the function f is somewhat-r-continuous. Let $V = \{b_1\} \in RO(X_2)$ and $f^{-1}(\{b_1\}) = \{a_1\} \neq \phi$ and there exists $U = \{a_1\} \in \Omega_1$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, for the other regular open sets of X_2, there exists $U \in \Omega_1$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the function f is somewhat-r-continuous.** Next, we show that the function f is not somewhat continuous function. For $V = \{b_2\} \in \Omega_2$, $f^{-1}(V) = \{a_3\}$ but there **does not exist** any $U \in \Omega_1$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. **Therefore, the function f is not somewhat continuous.**

Remark 3.2 Referring Definition 3.1 and Definition 2.7, it has been noted that somewhat-r-continuous functions and somewhat e-I-continuous functions are independent of one another. The examples below supports this statement.

Example 3.2 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_1, a_2\}\}$. Let $I_1 = \{\phi, \{a_1\}\}$ and referring Definition 2.5, we obtain, $e_IO(X_1) = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_1, a_4\}, \{a_2, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}, \{a_4, a_1, a_2\}\}$. Consider $X_2 = \{b_1, b_2, b_3, b_4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{b_1\}, \{b_2\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}\}$. The collection of closed sets is $F_2 = \{\phi, X_2, \{b_2, b_3, b_4\}, \{b_1, b_3, b_4\}, \{b_3, b_4\}, \{b_1, b_4\}, \{b_4\}\}$. Now, again applying Definition 2.5, we compute, $RO(X_2) = \{\phi, X_2, \{b_1\}, \{b_2, b_3\}\}$. The function $f: (X_1, \Omega_1) \rightarrow (X_2, \Omega_2)$ is defined by $f(a_1) = b_1, f(a_2) = b_3, f(a_3) = b_2, f(a_4) = b_4$. Now, we show that the function f is somewhat-r-continuous. Let $V = \{b_2, b_3\} \in RO(X_2)$ and $f^{-1}(\{b_2, b_3\}) = \{a_3, a_2\} \neq \phi$ and there exists $U = \{a_2\} \in \Omega_1$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, it has been verified for the other regular open sets V of X_2, there exists $U(\neq \phi) \in \Omega_1$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the function f is somewhat-r-continuous.** But for $V = \{b_2\} \in \Omega_2$, $f^{-1}(\{b_2\}) = \{a_3\}$, there **does not exist** any $U \in e_IO(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. **Therefore, the function f is not somewhat e-I-continuous.**

Example 3.3 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_2\}, \{a_4\}, \{a_2, a_4\}, \{a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_4, a_1, a_2\}\}$. The collection of closed sets is $F_1 = \{\phi, X_1, \{a_1, a_3, a_4\}, \{a_1, a_2, a_3\}, \{a_1, a_3\}, \{a_1, a_4\}, \{a_1\}, \{a_3\}\}$. Considering $I_1 = \{\phi, \{a_1\}\}$ and referring Definition 2.5, we obtain, $e_IO(X_1) = \{\phi, X_1, \{a_2\}, \{a_3\}, \{a_4\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_2, a_4\}, \{a_1, a_4\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_1, a_3, a_4\}, \{a_4, a_1, a_2\}\}$. Let $X_2 = \{b_1, b_2, b_3, b_4\}$ and topology $\Omega_2 = \{\phi, X_1, \{b_2\}, \{b_4\}, \{b_2, b_4\}, \{b_2, b_3\}, \{b_2, b_3, b_4\}, \{b_4, b_1, b_2\}\}$. Applying Definition 2.5, we compute, $RO(X_2) = \{\phi, X_2, \{b_4\}, \{b_2, b_3\}\}$. The function $f: (X_1, \Omega_1) \rightarrow (X_2, \Omega_2)$ is defined by $f(a_3) = b_4, f(a_1) = b_3, f(a_2) = b_2, f(a_4) = b_1$. First, we show the function f is somewhat e-I-continuous function. For $V = \{b_4\} \in \Omega_2$, $f^{-1}(\{b_4\}) = \{a_3\} \neq \phi$ and there exists $U = \{a_1\} \in e_IO(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, for the other regular open sets of X_2, $\exists\, U(\neq \phi) \in e_IO(X_1)$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the function f is somewhat e-I-continuous function.** Next, we show that the function f is not somewhat-r-continuous. For $V = \{b_4\} \in RO(X_2)$, $f^{-1}(\{b_4\}) = \{a_3\}$ but there **exists no $U(\neq \phi) \in \Omega_1$** satisfying the condition $U \subset f^{-1}(V)$. **Therefore, the function f is not somewhat-r-continuous.**

We now define somewhat-β-continuous functions, which is the generalization of somewhat continuous functions (cf. Definition 2.6).

• **Somewhat-β-continuous functions**

Definition 3.2 A function $f: (X_1, \Omega_1) \rightarrow (X_2, \Omega_2)$ is called as **somewhat-β-continuous** if for every $V \in \Omega_2$ and $f^{-1}(V) \neq \phi$ there exists $U \in \beta O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$.

Remark 3.3 It is a direct consequence of Definition 2.6 and Definition 3.2 that each somewhat continuous function is somewhat-β-continuous although the converse may not be valid always. The non validation of the converse part has been assured by the following example.

Example 3.4 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with its topology $\Omega_1 = \{\phi, X_1, \{a_1\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}\}$. The collection of closed sets is $F_1 = \{\phi, X_1, \{a_2, a_3, a_4\}, \{a_1, a_4\}, \{a_4\}\}$. By applying Definition 2.5, we calculate, $\beta O(X_1) = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_4\}, \{a_2, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}, \{a_4, a_1,$

$a_2\}\}$. Next, we consider $X_2 = \{b_1, b_2, b_3, b_4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{b_1\}, \{b_3\}, \{b_1, b_3\}\}$. Referring Definition 2.5, we compute, $RO(X_2) = \{\phi, X_2, \{b_1\}, \{b_3\}\}$. The function $f: (X_1, \Omega_1) \to (X_2, \Omega_2)$ is defined by $f(a_1) = b_3, f(a_2) = b_1, f(a_3) = b_4, f(a_4) = b_2$. It can be easily verified that for $V = \{b_1\} \in \Omega_2$, $f^{-1}(\{b_1\}) = \{a_2\} \neq \phi$ and there exists $U = \{a_2\} \in \beta O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, it has been verified that for the remaining open sets V of X_2, $\exists\ U \in \beta O(X_1)$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the mapping f is somewhat-β-continuous**. Next, we show that the mapping f is not somewhat continuous function. For $V = \{b_1\} \in \Omega_2$, $f^{-1}(V) = \{a_2\}$ but there **does not exist** any $U \in \Omega_1$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. **Therefore, the mapping f is not somewhat continuous.**

Remark 3.4 In view of Definition 3.1 and Definition 3.2, it is quite clear that somewhat-r-continuous function and somewhat-β-continuous function are independent of each other. In support of our assertion, we have constructed the following examples.

Example 3.5 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_2\}, \{a_3\}, \{a_2, a_3\}\}$. Applying Definition 2.5, we calculate, $\beta O(X_1) = \{\phi, X_1, \{a_2\}, \{a_3\}, \{a_2, a_3\}, \{a_1, a_2\}, \{a_3, a_4\}, \{a_2, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_1, a_3, a_4\}, \{a_4, a_1, a_2\}\}$. Let $X_2 = \{b_1, b_2, b_3, b_4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{b_1\}, \{b_2\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}\}$. The collection of closed sets corresponding to Ω_2 is $F_2 = \{\phi, X_2, \{b_2, b_3, b_4\}, \{b_1, b_3, b_4\}, \{b_3, b_4\}, \{b_1, b_4\}, \{b_4\}\}$. Again referring Definition 2.5, we compute, $RO(X_2) = \{\phi, X_2, \{b_1\}, \{b_2, b_3\}\}$. The function $f: (X_1, \Omega_1) \to (X_2, \Omega_2)$ is defined by $f(a_1) = b_2, f(a_2) = b_1, f(a_3) = b_3, f(a_4) = b_4$. Now, we show that the mapping f is somewhat-r-continuous. Let $V = \{b_1\} \in RO(X_2)$, $f^{-1}(\{b_1\}) = \{a_2\} \neq \phi$ and there exists $U = \{a_2\} \in \Omega_1$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, it has been verified that the condition $U \subset f^{-1}(V)\ \forall\ V \in RO(X_2)$ and for some $U \in \Omega_1$ holds. **Hence, the mapping f is somewhat-r-continuous.** Next, we show that the mapping f is not somewhat-β-continuous function. For $V = \{b_2\} \in \Omega_2$, $f^{-1}(V) = \{a_1\}$ but there **does not exist** any $U \in \beta O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. **Therefore, the mapping f is not somewhat-β-continuous.**

Example 3.6 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_1\}, \{a_2, a_3\}, \{a_1, a_2, a_3\},\}$. By using Definition 2.5, we compute, $\beta O(X_1) = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_4\}, \{a_2, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_1, a_3, a_4\}, \{a_4, a_1, a_2\}\}$. Let $X_2 = \{b_1, b_2, b_3, b_4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{b_1\}, \{b_3\}, \{b_1, b_3\}\}$. Applying Definition 2.5, we calculate, $RO(X_2) = \{\phi, X_2, \{b_1\}, \{b_3\}\}$. The function $f: (X_1, \Omega_1) \to (X_2, \Omega_2)$ is defined by $f(a_1) = b_3, f(a_2) = b_1, f(a_3) = b_4, f(a_4) = b_2$. Now, we show that the function f is somewhat-β-continuous. Let $V = \{b_3\} \in \Omega_2$, $f^{-1}(\{b_3\}) = \{a_1\} \neq \phi$ and there exists $U = \{a_1\} \in \beta O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, for the remaining open sets of X_2, there exists $U(\neq \phi) \in \beta O(X_1)$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the function f is somewhat-β-continuous**. Next, we show that the function f is not somewhat-r-continuous function. For $V = \{b_1\} \in RO(X_2)$, $f^{-1}(V) = \{a_2\}$ but there **does not exist** any $U \in \Omega_1$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. **Therefore, the function f is not somewhat-r-continuous.**

We are now set to define a more generalized form of continuous function, which is somewhat-βr-continuous function and it is a generalization of somewhat-r-continuous functions and somewhat-β-continuous functions (cf. Definition 3.1 and Definition 3.2).

• **Somewhat-βr-continuous functions**

Definition 3.3 A function $f: (X_1, \Omega_1) \to (X_2, \Omega_2)$ is known as **somewhat-βr-continuous** if for $V \in RO(X_2)$ and $f^{-1}(V) \neq \phi$, $\exists\ U \in \beta O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$.

Remark 3.5 Each somewhat-r-continuous function is somewhat-βr-continuous when we refer to Definition 3.1 and Definition 3.3 but it is not necessary that the converse be true and its validation has been assured by the following counter example.

Example 3.7 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with its topology $\Omega_1 = \{\phi, X_1, \{a_1\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}\}$. The collection of closed sets is $F_1 = \{\phi, X_1, \{a_2, a_3, a_4\}, \{a_1, a_4\}, \{a_4\}\}$. By referring Definition 2.5, we calculate, $\beta O(X_1) = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_4\}, \{a_2, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}, \{a_4, a_1, a_2\}\}$. Next, we consider $X_2 = \{b_1, b_2, b_3, b_4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{b_1\}, \{b_3\}, \{b_1, b_3\}\}$. Considering

Definition 2.5, we compute, $RO(X_2) = \{\phi, X_2, \{b_1\}, \{b_3\}\}$. The mapping $f: (X_1, \Omega_1) \rightarrow (X_1, \Omega_1)$ is defined by $f(a_1) = b_2, f(a_2) = b_3, f(a_3) = b_1, f(a_4) = b_4$. Now, we show that the function f is somewhat-βr-continuous. For $V = \{b_3\} \in RO(X_2), f^{-1}(\{b_3\}) = \{a_2\} \neq \phi$ and there exists $U = \{a_2\} \in \beta O(X_1)$ such that $U \subset f^{-1}(V)$. Similarly for all other regular open sets V of X_2, $\exists\ U \in \beta O(X_1)$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the mapping f is somewhat-βr-continuous.** But for $V = \{b_3\} \in RO(X_2), f^{-1}(\{b_3\}) = \{a_2\}$ and there **does not exist** any $U \in \Omega_1$ such that $U \subset f^{-1}(V)$. **Therefore, the mapping f is not somewhat-r-continuous.**

Remark 3.6 It is a right way consequence of Definition 3.2 and Definition 3.3 that every somewhat-β-continuous function is somewhat-βr-continuous but the converse might not be essentially true. So as to support this statement, the subsequent example could be referred.

Example 3.8 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_1\}, \{a_3\}, \{a_1, a_3\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}\}$. The collection of closed sets is $F_1 = \{\phi, X_1, \{a_2, a_3, a_4\}, \{a_2, a_4\}, \{a_4\}, \{a_1, a_4\}, \{a_1, a_2, a_4\}\}$. Applying Definition 2.5, we obtain, $\beta O(X_1) = \{\phi, X_1, \{a_1\}, \{a_3\}, \{a_1, a_3\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_4\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}\}$. Next, we consider $X_2 = \{b_1, b_2, b_3, b_4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{b_2\}, \{b_3\}, \{b_2, b_3\}, \{b_1, b_3\}, \{b_1, b_2, b_3\}\}$. Considering Definition 2.5, we calculate, $RO(X_2) = \{\phi, X_2, \{b_2\}, \{b_1, b_3\}\}$. The function $f: (X_1, \Omega_1) \rightarrow (X_2, \Omega_2)$ is defined by $f(a_1) = b_2, f(a_2) = b_4, f(a_3) = b_1, f(a_4) = b_3$. Now, we show that the function f somewhat-βr-continuous function. For $V = \{b_2\} \in RO(X_2)$, $f^{-1}(\{b_2\}) = \{a_1\} \neq \phi$ and there exists $U = \{a_1\} \in \beta O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, the condition $U \subset f^{-1}(V)\ \forall\ V \in RO(X_2)$ and for some $U \in \beta O(X_1)$ holds. **Hence, the function f is somewhat-βr-continuous function.** Next, we show that the function f is not somewhat-β-continuous function. For $V = \{b_3\} \in \Omega_2, f^{-1}(V) = \{a_4\}$ but there **exists no** $\mathbf{U(\neq \phi) \in \beta O(X_1)}$ satisfy the condition $U \subset f^{-1}(V)$. **Therefore, the function f is not somewhat-β-continuous.**

Next, we define somewhat- $\delta\beta_I$ -continuous function, which is the generalization of somewhat-β-continuous function.

• **Somewhat-$\delta\beta_I$-continuous functions**

Definition 3.4 A function $f: (X_1, \Omega_1, I_1) \rightarrow (X_2, \Omega_2)$ is known as **somewhat-$\delta\beta_I$-continuous** if for each $V \in \Omega_2$ and $f^{-1}(V) \neq \phi$ there exists $U \in \delta\beta_I O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$.

Remark 3.7 Noticing Definition 3.2 and Definition 3.4, it is clear that each somewhat-β-continuous function is somewhat-$\delta\beta_I$-continuous but it is not essential that the converse holds. The following example has been encouraged this assertion.

Example 3.9 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_1\}, \{a_3\}, \{a_1, a_3\}, \{a_1, a_2\}, \{a_1, a_2, a_3\}, \{a_3, a_4, a_1\}\}$. The collection of closed sets is $F_1 = \{\phi, X_1, \{a_2\}, \{a_4\}, \{a_3, a_4\}, \{a_2, a_4\}, \{a_1, a_2, a_4\}, \{a_2, a_3, a_4\}\}$. Considering $I_1 = \{\phi, \{a_2\}\}$ and using Definition 2.5, we calculate, $\beta O(X_1) = \{\phi, X_1, \{a_1\}, \{a_3\}, \{a_1, a_2\}, \{a_3, a_4\}, \{a_1, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_3, a_4, a_1\}, \{a_4, a_1, a_2\}\}$ and $\delta\beta_I O(X_1) = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_3\}, \{a_1, a_4\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_1, a_3, a_4\}, \{a_4, a_1, a_2\}\}$.Let $X_2 = \{b_1, b_2, b_3, b_4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{b_2\}, \{b_4\}, \{b_2, b_4\}, \{b_2, b_3\}, \{b_2, b_3, b_4\}, \{b_4, b_1, b_2\}\}$. The function $f: (X_1, \Omega_1) \rightarrow (X_2, \Omega_2)$ is defined by $f(a_1) = b_3, f(a_2) = b_4, f(a_3) = b_2, f(a_4) = b_1$. Now, we show that the function f is somewhat-$\delta\beta_I$-continuous. Let $V = \{b_4\} \in \Omega_2$ and $f^{-1}(\{b_4\}) = \{a_2\} \neq \phi$ and there exists $U = \{a_2\} \in \delta\beta_I O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, for the remaining open sets of X_2, $\exists\, U(\neq \phi) \in \delta\beta_I O(X_1)$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the function f is somewhat-$\delta\beta_I$-continuous.** Next, we show that the function f is not somewhat-β-continuous. For $V = \{b_4\} \in \Omega_2$, $f^{-1}(\{b_4\}) = \{a_2\}$ but there **does not exist** any $U \in \beta O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. **Therefore, the function f is not somewhat-β-continuous.**

Remark 3.8 In view of Definition 3.3 and Definition 3.4, it may be noted that the concepts of somewhat-$\delta\beta_I$-continuous function and somewhat-βr-continuous function are independent of one another. For this assertion the following examples may be referred.

Example 3.10 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be the non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_3\}, \{a_4\}, \{a_3, a_4\}, \{a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_4, a_1, a_3\}\}$. Considering $I_1 = \{\phi, \{a_2\}\}$ and referring Definition 2.5, we obtain, $\beta O(X_1) = \{\phi, X_1, \{a_3\}, \{a_4\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_3\}, \{a_1, a_4\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}\}$ and $\delta\beta_I O(X_1) = \{\phi, X_1, \{a_2\}, \{a_3\}, \{a_4\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_4\}, \{a_2, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_4, a_1, a_2\}, \{a_3, a_4, a_1\}\}$. Next, consider $X_2 = \{1,2,3,4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{2\}, \{3\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$. Applying Definition 2.5, we compute, $RO(X_2) = \{X_2, \phi, \{2\}, \{1,3\}\}$. The mapping $f: (X_1, \Omega_1, I_1) \to (X_2, \Omega_2)$ is defined by $f(a_1) = 3, f(a_2) = 4, f(a_3) = 2, f(a_4) = 1$. Now, we show that the function f is somewhat-βr-continuous. For $V = \{2\} \in RO(X_2), f^{-1}(\{2\}) = \{a_3\} \neq \phi$ and there exists $U = \{a_3\}(\neq \phi) \in \beta O(X_1)$ such that $U \subset f^{-1}(V)$. Similarly, for the other regular open sets V of X_2, $\exists\, U(\neq \phi) \in \beta O(X_1)$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the function f is somewhat-βr-continuous.** But for $V = \{3\} \in \Omega_2, f^{-1}(\{3\}) = \{a_1\} \neq \phi$ and $U(\neq \phi) \not\subset f^{-1}(V)$ for all $U \in \delta\beta_I O(X_1)$. **Therefore, the function f is not somewhat-$\delta\beta_I$-continuous.**

Example 3.11 In this example, we work on the same (X_1, Ω_1, I_1) and (X_2, Ω_2) as considered in Example 3.9. Now, we have, $\beta O(X_1) = \{\phi, X_1, \{a_1\}, \{a_3\}, \{a_1, a_2\}, \{a_3, a_4\}, \{a_1, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}, \{a_4, a_1, a_2\}\}$, $\delta\beta_I O(X_1) = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_3\}, \{a_1, a_4\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}, \{a_4, a_1, a_2\}\}$ and $RO(X_2) = \{\phi, X_2, \{b_4\}, \{b_2, b_3\}\}$. The mapping $f: (X_1, \Omega_1, I_1) \to (X_2, \Omega_2)$ is defined by $f(a_1) = b_2, f(a_2) = b_4, f(a_3) = b_1, f(a_4) = b_3$. Now, we show that the function f is somewhat-$\delta\beta_I$-continuous. For $V = \{b_2\} \in \Omega_2$, we have $f^{-1}(\{b_2\}) = \{a_1\} \neq \phi$ and there exists $U = \{a_1\} \in \delta\beta_I O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, for the remaining open sets of X_2, $\exists\, U(\neq \phi) \in \delta\beta_I O(X_1)$ satisfy the condition $U \subset f^{-1}(V)$. **Hence, the function f is somewhat-$\delta\beta_I$-continuous.** But for $V = \{b_4\} \in RO(X_2)$, $f^{-1}(V) = \{a_2\}$, there **does not exist** any $U \in \beta O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. **Therefore, the function f is not somewhat-βr-continuous.**

We now generalize somewhat e-I-continuous function in the context of regular open set (cf. Definition 2.7).

• **Somewhat-e_Ir-continuous functions**

Definition 3.5 A function $f: (X_1, \Omega_1, I_1) \to (X_2, \Omega_2)$ is said to be **somewhat-e_Ir-continuous** if for each $V \in RO(X_2)$ and $f^{-1}(V) \neq \phi$ there exists $U \in e_I O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$.

Remark 3.9 Each somewhat e-I-continuous function is somewhat-e_Ir-continuous when we refer to Definition 2.7 and Definition 3.5 but it is not necessary for the converse to be true and it is validated by the following example.

Example 3.12 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_1\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}\}$. With an ideal $I_1 = \{\phi, \{a_1\}, \{a_2\}, \{a_1, a_2\}\}$ and using Definition 2.5, we calculate, $e_I O(X_1) = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_1, a_4\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}, \{a_4, a_1, a_2\}\}$. Consider $X_2 = \{b_1, b_2, b_3, b_4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{b_1\}, \{b_2\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}, b_4\}\}$. Applying Definition 2.5, we compute, $RO(X_2) = \{\phi, X_2, \{b_1\}, \{b_2, b_3\}\}$. The function $f: (X_1, \Omega_1, I_1) \to (X_2, \Omega_2)$ is defined by $f(a_4) = b_2, f(a_2) = b_4, f(a_1) = b_3, f(a_3) = b_1$. Now, we show that the function f is somewhat-e_Ir-continuous. Let $V = \{b_2, b_3\} \in RO(X_2)$, we have $f^{-1}(\{b_2, b_3\}) = \{a_4, a_1\} \neq \phi$ and there exists $U = \{a_1\} \in e_I O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, for the other regular open sets V of X_2, there exists $U(\neq \phi) \in e_I O(X_1)$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the mapping f is somewhat-e_Ir-continuous.** But for $V = \{b_2\} \in \Omega_2$, $f^{-1}(\{b_2\}) = \{a_4\}$ and $\exists$ **no** $U(\neq \phi) e_I O(X_1)$ satisfying the condition $U \subset f^{-1}(V)$. **Therefore, the mapping f is not somewhat e-I-continuous.**

Remark 3.10 It is right way consequence of Definition 3.1 and Definition 3.5 that each somewhat-r-continuous function is somewhat-e_Ir-continuous but the converse might not be essentially true. So as to support this statement, the subsequent example could be referred.

Example 3.13 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be a non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_2\}, \{a_4\}, \{a_2, a_3\}, \{a_2, a_4\}, \{a_2, a_3, a_4\}\}$. Consider an ideal $I_1 = \{\phi, \{a_1\}\}$ and using Definition 2.5, we calculate, $e_I O(X_1) = \{\phi, X_1, \{a_4\}, \{a_2\}, \{a_3\}, \{a_3, a_4\}, \{a_2, a_3\}, \{a_1, a_4\}, \{a_2, a_4\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}, \{a_4, a_1, a_2\}\}$. Consider

$X_2 = \{b_1, b_2, b_3, b_4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{b_1\}, \{b_2\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}\}$. Applying Definition 2.5, we compute, $RO(X_2) = \{\phi, X_2, \{b_1\}, \{b_2, b_3\}\}$. The function $f: (X_1, \Omega_1) \to (X_2, \Omega_2)$ is defined by $f(a_4) = b_2, f(a_2) = b_4, f(a_1) = b_3, f(a_3) = b_1$. Now, we show that the function f is somewhat-e_Ir-continuous. Let $V = \{b_2, b_3\} \in RO(X_2)$ we have $f^{-1}(\{b_2, b_3\}) = \{a_4, a_1\} \neq \phi$ and there exists $U = \{a_4\} \in e_IO(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, for the other regular open sets V of X_2, there exists $U(\neq \phi) \in e_IO(X_1)$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the mapping f is somewhat-e_Ir-continuous.** But for $V = \{b_1\} \in RO(X_2)$, $f^{-1}(\{b_1\}) = \{a_3\}$ and there **exists no** $U(\neq \phi) \in \Omega_1$ satisfying the condition $U \subset f^{-1}(V)$. **Therefore, the mapping f is not somewhat-r-continuous.**

Next, we further generalize the concept of somewhat-$\delta\beta_I$-continuous function with reference to regular-open sets and define somewhat-$\delta\beta_I$r-continuous function, which is also a generalization of somewhat-βr-continuous function (cf. Definition 3.3 and Definition 3.4).

• **Somewhat-$\delta\beta_I$r-continuous functions**

Definition 3.6 A function $f: (X_1, \Omega_1, I_1) \to (X_2, \Omega_2)$ is said to be **somewhat-$\delta\beta_I$r-continuous** if for each $V \in RO(X_2)$ and $f^{-1}(V) \neq \phi$ there exists $U \in \delta\beta_IO(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$.

Remark 3.11 In view of Definition 3.4 and Definition 3.6, it is clear that every somewhat-$\delta\beta_I$-continuous function is somewhat-$\delta\beta_I$r-continuous but the converse may not hold necessarily. This assertion has been justified by Example 3.14.

It is a direct consequence of Definition 3.3 and Definition 3.6 that every somewhat-βr-continuous function is somewhat-$\delta\beta_I$r-continuous although the converse may not be valid always. The non validation of the converse part has been assured by Example 3.14.

Example 3.14 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be the non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_3\}, \{a_4\}, \{a_3, a_4\}, \{a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_4, a_1, a_3\}\}$. Now, considering $I_1 = \{\phi, \{a_2\}\}$ and referring Definition 2.5, we obtain, $\beta O(X_1) = \{\phi, X_1, \{a_3\}, \{a_4\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_3\}, \{a_1, a_4\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}\}$ and $\delta\beta_IO(X_1)\{\phi, X_1, \{a_2\}, \{a_3\}, \{a_4\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_4\}, \{a_2, a_4\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_4, a_1, a_2\}, \{a_3, a_4, a_1\}\}$. Next, consider $X_2 = \{1,2,3,4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{2\}, \{3\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$. Referring Definition 2.5, we compute, $RO(X_2) = \{X_2, \phi, \{2\}, \{1,3\}\}$. The mapping $f: (X_1, \Omega_1, I_1) \to (X_2, \Omega_2)$ is defined by $f(a_2) = 2, f(a_4) = 4, f(a_1) = 3, f(a_3) = 1$. Now, we show that the mapping f is somewhat- $\delta\beta_I$r -continuous. For this, $V = \{2\} \in RO(X_2)$, $f^{-1}(V) = \{a_2\} \neq \phi$ and there exists $U = \{a_2\} \in \delta\beta_IO(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. Similarly, for the other regular open sets of X_2, $\exists\, U(\neq \phi) \in \delta\beta_IO(X_1)$ such that the condition $U \subset f^{-1}(V)$ holds. **Hence, the function f is somewhat-$\delta\beta_I$r-continuous.** But for $V = \{3\} \in \Omega_2$, $f^{-1}(\{3\}) = \{a_1\} \neq \phi$ and there **does not exist** any $U \in \delta\beta_IO(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. **Therefore, the function f is not somewhat-$\delta\beta_I$-continuous.** It can also be verified that the function f is somewhat-$\delta\beta_I$r-continuous but not somewhat-βr-continuous, by considering the same example.

Finally, we set to define most generalized form of somewhat continuous functions, which is Almost-$\delta\beta_I$-somewhat continuous function.

• **Almost-$\delta\beta_I$r-somewhat continuous functions**

Definition 3.7 Let $f: (X_1, \Omega_1, I_1) \to (X_2, \Omega_2)$ be a function such that $f^{-1}(X_2) = X_1$, $f^{-1}(\phi) = \phi$ then the function f is known as **Almost-$\delta\beta_I$r-somewhat continuous** if for each $V \in RO(X_2)$ and $f^{-1}(V) \neq \phi$, $\exists\ U \in \delta\beta_I(X_1)$ such that $U \neq \phi(\neq X_1)$ and $U \cap f^{-1}(V) \neq \phi$.

Remark 3.12 Referring Definition 3.5 and Definition 3.7, it is clear that every somewhat e_Ir-continuous function is Almost-$\delta\beta_I$r-somewhat continuous although the converse may not hold essentially.

Noticing Definition 3.6 and Definition 3.7, it is clear that each somewhat-$\delta\beta_I$r-continuous function is Almost-$\delta\beta_I$r-somewhat continuous but the converse may not hold. Both these conclusions have been justified by Example 3.12.

Example 3.15 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be the non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_2\}, \{a_4\}, \{a_2, a_4\}, \{a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_4, a_1, a_2\}\}$. Considering $I_1 = \{\phi, \{a_3\}\}$ and applying Definition 2.5, we calculate, $e_I O(X_1) = \{\phi, X_1, \{a_2\}, \{a_3\}, \{a_4\}, \{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_4\}, \{a_2, a_4\}\{a_2, a_3, a_4\}, \{a_4, a_1, a_2\}, \{a_3, a_4, a_1\}\}$ and $\delta\beta_I O(X_1)\{\phi, X_1, \{a_2\}, \{a_3\}, \{a_4\}, \{a_1, a_2\}\{a_2, a_3\}, \{a_3, a_4\}, \{a_1, a_4\}, \{a_1, a_3\}, \{a_2, a_4\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_4, a_1, a_2\}, \{a_3, a_4, a_1\}\}$. Let $X_2 = \{1,2,3,4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{1\}, \{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$. By using Definition 2.5, we obtain, $RO(X_2) = \{X_2, \phi, \{1\}, \{2,3\}\}$. The mapping $f: (X_1, \Omega_1, I_1) \rightarrow (X_2, \Omega_2)$ is defined by $f(a_1) = 1, f(a_2) = 2, f(a_3) = 4, f(a_4) = 3$. Now, we show that function f is Almost-$\delta\beta_I$r-somewhat continuous. For $V = \{1\} \in RO(X_2), f^{-1}(\{1\}) = \{a_1\} \neq \phi$ and there exists $U = \{a_4, a_1\} \in \delta\beta_I O(X_1)$ such that $U \neq \phi (\neq X_1)$ and $U \cap f^{-1}(V) = \{a_1\} \neq \phi$. Similarly, for the remaining regular open sets V of $X_2, \exists\ U(\neq \phi$ and $X_1) \in \delta\beta_I O(X_1)$ such that the condition, $U \cap f^{-1}(V) \neq \phi$ holds. **Hence, the function f is Almost-$\delta\beta_I$r-somewhat continuous.** But for $V = \{1\} \in RO(X_2)$, $f^{-1}(\{1\}) = \{a_1\}$, there **does not exist** any $U \in e_I O(X_1)$ such that $U \neq \phi$ and $U \subset f^{-1}(V)$. **Therefore, the function f is not somewhat-e_Ir-continuous.** In addition, it has been also verified in this example that the function f is Almost-$\delta\beta_I$r-somewhat continuous but not somewhat-$\delta\beta_I$r-continuous.

Remark 3.13 The inter-continuity connections of somewhat functions have been displayed in figure 1

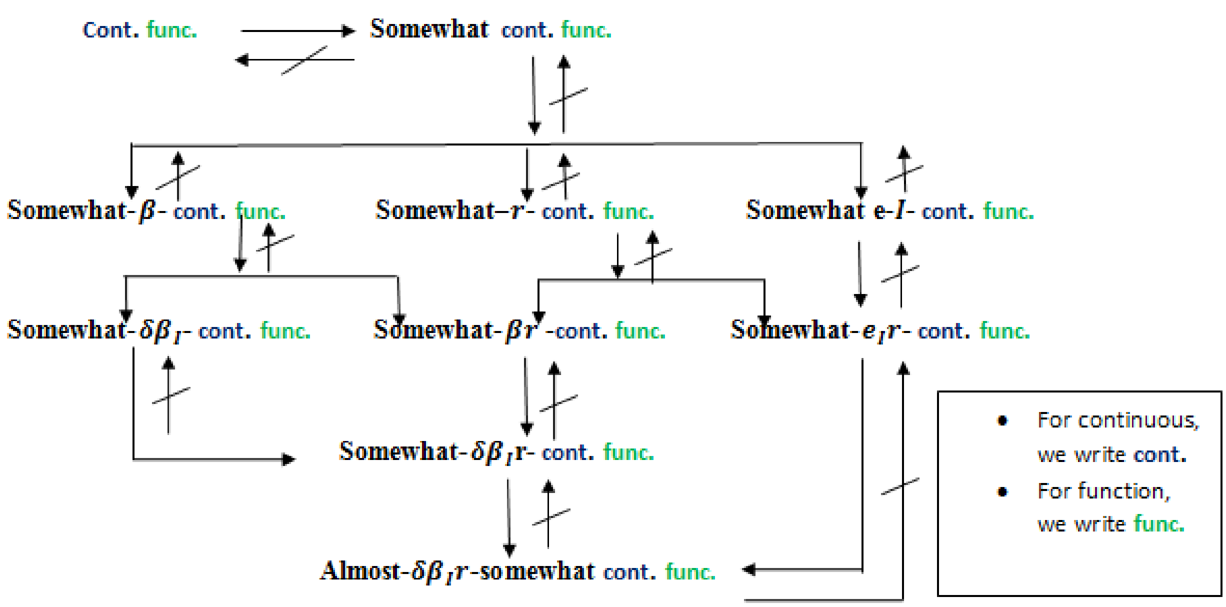

Figure 1: **Relationship amongst somewhat continuous functions and its generalizations.**

IV SOME IMPORTANT RESULTS

In this section, we have proved that for all x belonging to X, there exists a set B which is a member of $\beta O(X)$ such that x belongs to B. Certain composite maps of various somewhat functions have been also discussed.

Theorem 4.1 Let (X, Ω) be the topological space and $\beta O(X)$ be the collection of β-open sets. Then for each x belonging to X, there exists at least one β-open set containing it.

Proof. Let X be a nonempty set with the topology $\Omega(X)$. The collection of open sets is $\{U_\alpha\}_{\alpha \in J}$ and the collection F of closed sets is $\{V_\delta\}_{\delta \in k}$.

Now, we consider the following cases for the desired result.

Case I For each $x \in X$, $\exists$ an index $\beta \in J$ such that $U_\beta \in \Omega(X)$ and $x \in U_\beta$.

We know that

$$\Omega(X) \subset \beta O(X).$$

Hence, for each $x \in X$, $\exists$ one β-open set containing it.
Case II Let $x \in X$ and $x \notin U_\alpha(\neq X)$ for each $\alpha \in J$
Now, we construct $U_\alpha \cup \{x\} = W$ (say) (4.1)
We show that $W(\neq X) \in \beta O(X)$
For $V_\delta \in F$, if $W \subseteq X(V_\delta = X$, where X is the smallest closed set containing $W)$, then trivially $W \in \beta O(X)$. Next, we consider $W \subseteq V_\delta \in F$ (since, $V_\delta \subset X$).
In view of Definition 2.5, it is enough if we show the following:

$$W \subseteq cl(Int(cl(W))) \quad (4.2)$$

Now, by construction, we have,

$$cl(W) = V_\delta$$

By taking interior on both the sides, we have,

$$Int(cl(W)) = Int(V_\delta) = U_\alpha \quad \text{(cf. relation (4.1))}$$

Now, by taking closure on both the sides, we get,

$$cl(Int(cl(W))) = cl(U_\alpha) = V_\delta.$$

Since, $W \subseteq V_\delta$, hence, relation (4.2) follows directly,
Finally, we conclude that W is a β-open set containing $x \in X$.

Theorem 4.2 Let $g: (X_1, \Omega_1) \to (X_2, \Omega_2)$ and $h: (X_2, \Omega_2) \to (X_3, \Omega_3)$ be two functions, where g be surjective. Then the following hold.
(1). If g is somewhat-β-continuous and h is somewhat continuous then $hog: (X_1, \Omega_1) \to (X_3, \Omega_3)$ is somewhat-β-continuous function,
(2). If g is somewhat-β-continuous and h is continuous then $hog: (X_1, \Omega_1) \to (X_3, \Omega_3)$ is somewhat-β-continuous function,
(3). If g is somewhat continuous and h is somewhat continuous then $hog: (X_1, \Omega_1) \to (X_3, \Omega_3)$ is somewhat continuous function,
(4). If g is somewhat continuous and h is continuous then $hog: (X_1, \Omega_1) \to (X_3, \Omega_3)$ is somewhat continuous function.

Proof. (1). Suppose G belongs to Ω_3 and in view of Definition 3.2, for $(hog)^{-1}(G) \neq \phi$, we show that there exists $U \in \beta O(X_1)$ such that $U \neq \phi$ and $U \subset (hog)^{-1}(G)$.
Now, consider $g^{-1}(h^{-1}(G)) \neq \phi$ then $h^{-1}(G) \neq \phi$. Since, the function h is somewhat continuous, there exists $V \in \Omega_2$, such that

$$V \neq \phi \text{ and } V \subset h^{-1}(G) \text{ (cf.Definition 2.6)} \quad (4.3)$$

By hypothesis, the function g is surjective, this implies

$g^{-1}(V) \neq \phi$ and from relation (4.3), we have,

$$g^{-1}(V) \subset g^{-1}(h^{-1}(G)) \quad (4.4)$$

Since, the function g is somewhat-β-continuous. Therefore, there exists $U \in \beta O(X_1)$ such that

$$U \neq \phi \text{ and } U \subset g^{-1}(V).$$

Now, considering relation (4.4), we conclude that

$$U \subset g^{-1}(h^{-1}(G)) = (hog)^{-1}(G) \; .$$

Finally, it has been concluded that $hog: (X_1, \Omega_1) \to (X_3, \Omega_3)$ is somewhat-β-continuous.
Similarly, we can prove the remaining parts of the Theorem.

Remark 4.1 If functions $g: (X_1, \Omega_1) \to (X_2, \Omega_2)$ and $h: (X_2, \Omega_2) \to (X_3, \Omega_3)$ are somewhat-β-continuous then it is not necessarily true that hog is somewhat-β-continuous function.

Example 4.1 Let $X_1 = \{a_1, a_2, a_3, a_4\}$ be the non-empty set with the topology $\Omega_1 = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}\}$. Applying Definition 2.5, we calculate, $\beta O(X_1) = \{\phi, X_1, \{a_1\}, \{a_2\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_1, a_4\}, \{a_2, a_4\}, \{a_2, a_3, a_4\}, \{a_4, a_1, a_2\}, \{a_1, a_2, a_3\}\}$. Let $X_2 = \{1,2,3,4\}$ with the topology $\Omega_2 = \{\phi, X_2, \{1\}, \{2,3\}, \{1,2,3\}\}$. Again using Definition 2.5, we obtain, $\beta O(X_2) = \{\phi, X_2, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,4\}, \{1,4\}, \{2,4\}, \{1,3\}, \{a_1, a_2, a_3\}, \{a_2, a_3, a_4\}, \{a_3, a_4, a_1\}, \{a_4, a_1, a_2\}\}$. Suppose $X_3 = \{a_1, a_2, a_3, a_4\}$ and $\Omega_3 = \{\phi, X_3, \{a_1\}, \{a_2\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}\}$. The mapping $g: (X_1, \Omega_1) \to (X_2, \Omega_2)$ is defined by $g(a_1) = 1, g(a_2) = 2, g(a_3) = 3, g(a_4) = 4$. It can be verified that for each $V \in \tau_2$, $g^{-1}(V) \neq \phi$, there exists $U(\neq \phi) \in \beta O(X_1)$ such that $U \subset f^{-1}(V)$. Hence, the function g is somewhat-β-continuous function.

Another mapping $h:(X_2,\Omega_2)\rightarrow(X_3,\Omega_3)$ is defined by $h(1)=a_3, h(2)=a_2, h(3)=a_1, h(4)=a_4$. It can also verified that the mapping h is somewhat-β-continuous function. But for $G=\{a_1\}\in\Omega_3$, $(hog)^{-1}(\{a_1\})=\{a_3\}\neq\phi$, there **does not exists** any $U\in\beta O(X_1)$ such that $U\subset(hog)^{-1}(\{a_1\})$. **Hence, the hog is not somewhat-β-continuous function.**

Theorem 4.3 Let $g:(X_1,\Omega_1,I_1)\rightarrow(X_2,\Omega_2)$ and $h:(X_2,\Omega_2)\rightarrow(X_3,\Omega_3)$ be two mappings, with g surjective. Then the following hold.
(1). If g is somewhat-$\delta\beta_I$-continuous and h is somewhat continuous then $hog:(X_1,\Omega_1,I_1)\rightarrow(X_3,\Omega_3)$ is somewhat-$\delta\beta_I$-continuous,
(2). If g is somewhat-$\delta\beta_I$-continuous and h is continuous then $hog:(X_1,\Omega_1,I_1)\rightarrow(X_3,\Omega_3)$ is somewhat-$\delta\beta_I$-continuous.

Proof. (1). Suppose W belongs to Ω_3 and for $(hog)^{-1}(W)\neq\phi$, we show that there exists $U\in\delta\beta_I O(X_1)$ such that $U\neq\phi$ and $U\subset(hog)^{-1}(W)$ (cf. Definition 3.4).
Consider $g^{-1}(h^{-1}(W))\neq\phi$ then $h^{-1}(W)\neq\phi$. Since, the function h is somewhat continuous, there exists $V\in\Omega_2$, such that

$$V\neq\phi \text{ and } V\subset h^{-1}(W) \text{ (cf. Definition 2.6)} \quad (4.5)$$

By hypothesis, the function g is surjective, this implies

$g^{-1}(V)\neq\phi$ and from relation (4.5), we have,

$$g^{-1}(V)\subset g^{-1}(h^{-1}(W)) \quad (4.6)$$

Since, the function g is somewhat-$\delta\beta_I$-continuous. Therefore, there exists $U\in\delta\beta_I O(X_1)$ such that

$$U\neq\phi \text{ and } U\subset g^{-1}(V).$$

Next, considering relation (4.6), we conclude that

$$U\subset g^{-1}(h^{-1}(W))=(hog)^{-1}(W).$$

In view of Definition 3.4, it has been concluded that $hog:(X_1,\Omega_1,I_1)\rightarrow(X_3,\Omega_3)$ is somewhat-$\delta\beta_I$-continuous.
(2). Following the technique of part (1), the proof is direct.

Remark 4.2 If functions $g:(X_1,\Omega_1,I_1)\rightarrow(X_2,\Omega_2,I_2)$ and $h:(X_2,\Omega_2,I_2)\rightarrow(X_3,\Omega_3)$ are somewhat-$\delta\beta_I$-continuous then it is not essentially true that the hog is somewhat-$\delta\beta_I$-continuous function.

V CONCLUSION

The concept of somewhat continuous function has been generalized in the form of somewhat-r-continuous functions, somewhat-β-continuous functions and Almost somewhat continuous functions. These somewhat functions have been further generalized in the context of regular open, β-open sets and $\delta\beta_I$-open sets. Somewhat e-I-continuous function is also generalized to somewhat-$e_I r$-continuous function. Finally, we have noted that Almost-$\delta\beta_I r$-somewhat continuous function is the most generalized version of somewhat functions. It has been also proved that for all x belonging to X, there exists a set B, which is a member of $\beta O(X)$ such that x belongs to B. Moreover, different composite maps of somewhat functions have been discussed. The concept of somewhat e-I-continuous functions may be extended to define somewhat e-I-homeomorphism to obtain somewhat e-I-homeomorphic image of the topological space X.

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