Application of Spherical Fuzzy Rough Set and α-Level Cut of Spherical Fuzzy Relation

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Abstract- Spherical fuzzy rough set is a pair of lower and upper approximations of a spherical fuzzy set. In this paper we give the application based on spherical fuzzy rough set and also we introduce the concept of α-level cut of spherical fuzzy relation in two universes. Then we give some operations and properties on α-level cut of spherical fuzzy relation. Finally we give a decision making problem in medical diagnosis.

Keywords- Spherical fuzzy relation, Spherical fuzzy approximation, Spherical fuzzy rough set, Decision making problem.

I. INTRODUCTION

Rough set theory is an extension of set theory. It is a pair of crisp sets which are not equal. It was proposed by Pawlak[6]. Later Dubois and Prade[3] introduced the concept of fuzzy rough set theory in one universe. It is a pair of lower and upper approximations of fuzzy sets such that both sets should not be equal. Fuzzy rough set become an easy tool to deal with computing, data mining, pattern recognition etc. Moris and yokout[4] provide the fuzzy set model. There are many models based on fuzzy rough set worked by many authors. Sun and Ma[7] gave an easy model based on fuzzy rough set in their paper[8].

Ashraf et al.[1] introduced spherical fuzzy set to deal with uncertainty. He applied the theory in decision making problem. It is considered to be an extended notion of picture fuzzy set. Cuong and Kreinovich [2] initiated the picture fuzzy theory. In this set the element defined by three degrees such as positive(μ^+) , neutral (μ') and negative degree (μ^-) with the condition that $0 \leq \mu^+ + \mu' + \mu^- \leq 1$. In this Spherical fuzzy set, opinion is not limited to yes or no but it gave an option to some abstain or refusal. The sum of the squares of μ^+, μ' and μ^- should be less than or equals to 1.

II PRELIMINARIES

This section recalls the definitions of spherical fuzzy set ($\mathbb{S}FS$), spherical fuzzy relation ($\mathbb{S}FR_\rho$) and spherical fuzzy approximation space.

2.1. Definition-

Let U be a non-empty universe set. A spherical fuzzy set $\mathbb{S}$ in U is defined by

$\mathbb{S} = \{(s/(\mu^{+}(s), \mu'(s), \mu^{-}(s))): (\mu^{+}(s)^2 + \mu'(s)^2 + \mu^{-}(s)^2) \leq 1 \forall\, s \in U\}$

where μ^{+}, μ', μ^{-} denotes the positive, neutral and negative degree of the element s.

The three membership functions are the mapping from the universal set U to the unit interval [0,1]. The collection of all spherical fuzzy sets is denoted by $\mathbb{S}FS$. The degree of refusal $s \in U$ is defined by $r(s) = \sqrt{1-(\mu^{+}(s)^2 + \mu'(s)^2 + \mu^{-}(s)^2)}$.

2.2. Definition-

Let U_1 and U_2 be two universe sets . A spherical fuzzy relation ρ in $U_1 \times U_2$ is a spherical fuzzy subset of $U_1 \times U_2$ and is given by

$$\rho = \{\big((r,s)/(\mu_{\rho}^{+}(r,s), \mu_{\rho}'(r,s), \mu_{\rho}^{-}(r,s))\big)/\mu_{\rho}^{+}(r,s)^2 + \mu_{\rho}'(r,s)^2 + \mu_{\rho}^{-}(r,s)^2\big) \leq 1$$

$$\forall\, r \in U_1, s \in U_2\}$$

Where $\mu_{\rho}^{+}, \mu_{\rho}', \mu_{\rho}^{-}: U \to [0,1]$.

2.3. Definition-

Let ρ be a spherical fuzzy relation on two universes $U_1 \times U_2$ then we call the space (U_1, U_1, ρ) is a spherical fuzzy approximation space.

2.4. Definition-

Let (U_1, U_1, ρ) be a spherical fuzzy approximation space. For any spherical fuzzy set $\mathbb{S} \in U_2$, the lower and upper approximations of $\mathbb{S}$ with respect to (U_1, U_2, ρ) is defined by

$$\underline{\rho}(\mathbb{S}) = \left\{\left(s/(\mu_{\underline{\rho}(s)}^{+}(s), \mu_{\underline{\rho}(s)}'(s), \mu_{\underline{\rho}(s)}^{-}(s))\right): s \in U_1\right\}$$

$$\bar{\rho}(\mathbb{S}) = \left\{\left(s/(\mu_{\bar{\rho}(s)}^{+}(s), \mu_{\bar{\rho}(s)}'(s), \mu_{\bar{\rho}(s)}^{-}(s))\right): s \in U_1\right\}$$

Where

$$\mu_{\underline{\rho}(s)}^{+}(s) = \bigwedge_{s \in U_2} \{\mu_{\rho}^{+}(r,s) \wedge \mu_{\mathbb{S}}^{+}(s)\}$$

$$\mu_{\underline{\rho}(s)}'(s) = \bigwedge_{s \in U_2} \{\mu_{\rho}'(r,s) \wedge \mu_{\mathbb{S}}'(s)\}$$

$$\mu_{\underline{\rho}(s)}^{-}(s) = \bigvee_{s \in U_2} \{\mu_{\rho}^{-}(r,s) \vee \mu_{\mathbb{S}}^{-}(s)\}$$

$$\mu_{\bar{\rho}(s)}^{+}(s) = \bigvee_{s \in U_2} \{\mu_{\rho}^{+}(r,s) \vee \mu_{\mathbb{S}}^{+}(s)\}$$

$$\mu_{\bar{\rho}(s)}'(s) = \bigwedge_{s \in U_2} \{\mu_{\rho}'(r,s) \wedge \mu_{\mathbb{S}}'(s)\}$$

$$\mu^{-}_{\bar{\rho}(s)}(s) = \bigwedge_{s \in U_2} \{\mu^{-}_{\rho}(r,s) \wedge \mu^{-}_{\mathbb{S}}(s)\}$$

Then the pair of spherical fuzzy sets is said to be a spherical fuzzy rough set ($\mathbb{S}FRS$) such that

$\underline{\rho}(\mathbb{S}) \neq \bar{\rho}(\mathbb{S})$. The collection of all $\mathbb{S}FRS$ is denoted by $\mathfrak{F}(\mathbb{S}FRS)$.

III. OPERATIONS AND PROPERTIES OF SPHERICAL FUZZY ROUGH SET

In this section we study the operations and properties on spherical fuzzy rough sets.

3.1. Operations-

Suppose $\rho(\mathbb{S})$ and $\rho(\mathbb{T})$ are two $\mathbb{S}FRSs$ in (U_1, U_2, ρ) then the following operations are defined $\forall\ a \in U_1$

(i) $\rho(\mathbb{S}) \cap \rho(\mathbb{T}) = (\underline{\rho}(\mathbb{S}) \cap \underline{\rho}(\mathbb{T}), \bar{\rho}(\mathbb{S}) \cap \bar{\rho}(\mathbb{T}))$ where

$$\mu_{\underline{\rho}(\mathbb{S}) \cap \underline{\rho}(\mathbb{T})}(a) = (\mu^{+}_{\underline{\rho}(\mathbb{S})}(a) \wedge \mu^{+}_{\underline{\rho}(\mathbb{T})}(a), \mu'_{\underline{\rho}(\mathbb{S})}(a) \wedge \mu'_{\underline{\rho}(\mathbb{T})}(a), \mu^{-}_{\underline{\rho}(\mathbb{S})}(a) \vee \mu^{-}_{\underline{\rho}(\mathbb{T})}(a))$$
$$\mu_{\bar{\rho}(\mathbb{S}) \cap \bar{\rho}(\mathbb{T})}(a) = (\mu^{+}_{\bar{\rho}(\mathbb{S})}(a) \wedge \mu^{+}_{\bar{\rho}(\mathbb{T})}(a), \mu'_{\bar{\rho}(\mathbb{S})}(a) \wedge \mu'_{\bar{\rho}(\mathbb{S})}(a), \mu^{-}_{\bar{\rho}(\mathbb{S})}(a) \vee \mu^{-}_{\bar{\rho}(\mathbb{T})}(a))$$

(ii) $\rho(\mathbb{S}) \cup \rho(\mathbb{T}) = (\underline{\rho}(\mathbb{S}) \cup \underline{\rho}(\mathbb{T}), \bar{\rho}(\mathbb{S}) \cup \bar{\rho}(\mathbb{T}))$ where

$$\mu_{\underline{\rho}(\mathbb{S}) \cup \underline{\rho}(\mathbb{T})}(a) = (\mu^{+}_{\underline{\rho}(\mathbb{S})}(a) \vee \mu^{+}_{\underline{\rho}(\mathbb{T})}(a), \mu'_{\underline{\rho}(\mathbb{S})}(a) \wedge \mu'_{\underline{\rho}(\mathbb{T})}(a), \mu^{-}_{\underline{\rho}(\mathbb{S})}(a) \wedge \mu^{-}_{\underline{\rho}(\mathbb{T})}(a))$$
$$\mu_{\bar{\rho}(\mathbb{S}) \cup \bar{\rho}(\mathbb{T})}(a) = (\mu^{+}_{\bar{\rho}(\mathbb{S})}(a) \vee \mu^{+}_{\bar{\rho}(\mathbb{T})}(a), \mu'_{\bar{\rho}(\mathbb{S})}(a) \wedge \mu'_{\bar{\rho}(\mathbb{T})}(a), \mu^{-}_{\bar{\rho}(\mathbb{S})}(a) \wedge \mu^{-}_{\bar{\rho}(\mathbb{T})}(a))$$

(iii) $\rho(\mathbb{S}) \subseteq \rho(\mathbb{T})$ if and only if

$$\mu^{+}_{\underline{\rho}(\mathbb{S})}(a) \leq \mu^{+}_{\underline{\rho}(\mathbb{T})}(a) \ and\ \mu^{+}_{\bar{\rho}(\mathbb{S})}(a) \leq \mu^{+}_{\bar{\rho}(\mathbb{T})}(a)$$
$$\mu'_{\underline{\rho}(\mathbb{S})}(a) \leq \mu'_{\underline{\rho}(\mathbb{T})}(a) \ and\ \mu'_{\bar{\rho}(\mathbb{S})}(a) \leq \mu'_{\bar{\rho}(\mathbb{T})}(a)$$
$$\mu^{-}_{\underline{\rho}(\mathbb{S})}(a) \geq \mu^{-}_{\underline{\rho}(\mathbb{T})}(a) \ and\ \mu^{-}_{\bar{\rho}(\mathbb{S})}(a) \geq \mu^{-}_{\bar{\rho}(\mathbb{T})}(a)$$

(iv) $\rho(\mathbb{S}) = \rho(\mathbb{T})$ if and only if

$$\mu^{+}_{\underline{\rho}(\mathbb{S})}(a) = \mu^{+}_{\underline{\rho}(\mathbb{T})}(a) \ and\ \mu^{+}_{\bar{\rho}(\mathbb{S})}(a) = \mu^{+}_{\bar{\rho}(\mathbb{T})}(a)$$
$$\mu'_{\underline{\rho}(\mathbb{S})}(a) = \mu'_{\underline{\rho}(\mathbb{T})}(a) \ and\ \mu'_{\bar{\rho}(\mathbb{S})}(a) = \mu'_{\bar{\rho}(\mathbb{T})}(a)$$
$$\mu^{-}_{\underline{\rho}(\mathbb{S})}(a) = \mu^{-}_{\underline{\rho}(\mathbb{T})}(a) \ and\ \mu^{-}_{\bar{\rho}(\mathbb{S})}(a) = \mu^{-}_{\bar{\rho}(\mathbb{T})}(a)$$

(v) The complement of a $\mathbb{S}FRS$ $\rho(\mathbb{S})$ is denoted by $(\rho(\mathbb{S}))^c = (\underline{\rho}(\mathbb{S})^c, \bar{\rho}(\mathbb{S})^c)$ if and only if

$$\underline{\rho}(\mathbb{S})^c = \{(a/(\mu^{-}_{\underline{\rho}(\mathbb{S})}(a), \mu'_{\underline{\rho}(\mathbb{S})}(a), \mu^{+}_{\underline{\rho}(\mathbb{S})}(a))) : a \in U_2\}$$
$$\bar{\rho}(\mathbb{S})^c = \{(a/(\mu^{-}_{\bar{\rho}(\mathbb{S})}(a), \mu'_{\bar{\rho}(\mathbb{S})}(a), \mu^{+}_{\bar{\rho}(\mathbb{S})}(a))) : a \in U_2\ \}$$

3.2. Proposition-

If $\rho(\mathbb{S})$ and $\rho(\mathbb{T})$ are two $\mathbb{S}FRSs$ of (U_1, U_2, ρ) then the following properties hold.

(i) $\underline{\rho}(\mathbb{S} \cap \mathbb{T}) = \underline{\rho}(\mathbb{S}) \cap \underline{\rho}(\mathbb{T})$
(ii) $\bar{\rho}(\mathbb{S} \cap \mathbb{T}) \subseteq \bar{\rho}(\mathbb{S}) \cap \bar{\rho}(\mathbb{T})$
(iii) $\underline{\rho}(\mathbb{S} \cup \mathbb{T}) \supseteq \underline{\rho}(\mathbb{S}) \cap \underline{\rho}(\mathbb{T})$
(iv) $\bar{\rho}(\mathbb{S} \cup \mathbb{T}) = \bar{\rho}(\mathbb{S}) \cup \bar{\rho}(\mathbb{T})$.

Proof: (i) Let $a \in U_1$. Consider

$$\mu^{+}_{\underline{\rho}(\mathbb{S} \cap \mathbb{T})}(a) = \bigwedge_{b \in U_2} \{\mu^{+}_{\rho}(a,b) \wedge \mu^{+}_{\mathbb{S} \cap \mathbb{T}}(b)\}$$

$$= \bigwedge_{b\in U_2} \{\mu_\rho^+(a,b) \wedge (\mu_{\mathbb{S}}^+(b) \wedge \mu_{\mathbb{T}}^+(b))\}$$

$$= \bigwedge_{b\in U_2} \{(\mu_\rho^+(a,b) \wedge \mu_{\mathbb{S}}^+(b)) \wedge (\mu_\rho^+(a,b) \wedge \mu_{\mathbb{T}}^+(b))\}$$

$$= \bigwedge_{b\in U_2} \{(\mu_\rho^+(a,b) \wedge \mu_{\mathbb{S}}^+(b))\} \wedge \bigwedge_{b\in U_2} \{(\mu_\rho^+(a,b) \wedge \mu_{\mathbb{T}}^+(b))\}$$

$$= \mu_{\underline{\rho}(\mathbb{S})}^+(a) \wedge \mu_{\underline{\rho}(\mathbb{T})}^+(a),$$

$$\mu'_{\underline{\rho}(\mathbb{S}\cap\mathbb{T})}(a) = \wedge_{b\in U_2}\{\mu'_\rho(a,b) \wedge \mu'_{\mathbb{S}\cap\mathbb{T}}(b)\}$$

$$= \wedge_{b\in U_2}\{\mu'_\rho(a,b) \wedge (\mu'_{\mathbb{S}}(b) \wedge \mu'_{\mathbb{T}}(b))\}$$

$$= \wedge_{b\in U_2}\{(\mu'_\rho(a,b) \wedge \mu'_{\mathbb{S}}(b)) \wedge (\mu'_\rho(a,b) \wedge \mu'_{\mathbb{T}}(b))\}$$

$$= \bigwedge_{b\in U_2} \{\mu'_\rho(a,b) \wedge \mu'_{\mathbb{S}}(b)\} \wedge \bigwedge_{b\in U_2} \{\mu'_\rho(a,b) \wedge \mu'_{\mathbb{T}}(b)\}$$

$$= \mu'_{\underline{\rho}(\mathbb{S})}(a) \wedge \mu'_{\underline{\rho}(\mathbb{T})}(a)$$

and

$$\mu^-_{\underline{\rho}(\mathbb{S}\cap\mathbb{T})}(a) = \bigvee_{b\in U_2} \{\mu_\rho^-(a,b) \vee \mu^-_{\mathbb{S}\cap\mathbb{T}}(b)\}$$

$$= \bigvee_{b\in U_2} \{\mu_\rho^-(a,b) \vee (\mu_{\mathbb{S}}^-(b) \wedge \mu_{\mathbb{T}}^-(b))\}$$

$$= \bigvee_{b\in U_2} \{(\mu_\rho^-(a,b) \vee \mu_{\mathbb{S}}^-(b)) \wedge (\mu_\rho^-(a,b) \vee \mu_{\mathbb{T}}^-(b))\}$$

$$= \bigvee_{b\in U_2} \{\mu_\rho^-(a,b) \vee \mu_{\mathbb{S}}^-(b)\} \wedge \bigvee_{b\in U_2} \{\mu_\rho^-(a,b) \vee \mu_{\mathbb{T}}^-(b)\}$$

$$= \mu^-_{\underline{\rho}(\mathbb{S})}(a) \wedge \mu^-_{\underline{\rho}(\mathbb{T})}(a)$$

Therefore $\underline{\rho}(\mathbb{S} \cap \mathbb{T}) = \underline{\rho}(\mathbb{S}) \cap \underline{\rho}(\mathbb{T})$. Similarly we can prove (ii)

(iii) Let $a \in U_1$. Consider

$$\mu^+_{\underline{\rho}(\mathbb{S}\cup\mathbb{T})}(a) = \bigwedge_{b\in U_2} \{\mu_\rho^+(a,b) \wedge \mu^+_{\mathbb{S}\cup\mathbb{T}}(b)\}$$

$$= \bigwedge_{b\in U_2} \{\mu_\rho^+(a,b) \wedge (\mu_{\mathbb{S}}^+(b) \vee \mu_{\mathbb{T}}^+(b))\}$$

$$= \bigwedge_{b\in U_2} \{(\mu_\rho^+(a,b) \wedge \mu_{\mathbb{S}}^+(b)) \vee (\mu_\rho^+(a,b) \wedge \mu_{\mathbb{T}}^+(b))\}$$

$$\geq \bigwedge_{b\in U_2}\{\mu_\rho^+(a,b)\wedge\mu_{\mathbb{S}}^+(b)\}\vee\bigwedge_{b\in U_2}\{\mu_\rho^+(a,b)\wedge\mu_{\mathbb{T}}^+(b)\}$$

$$\geq \mu^+_{\underline{\rho}(\mathbb{S})}(a)\vee\mu^+_{\underline{\rho}(\mathbb{T})}(a)\ ,$$

$$\mu'_{\underline{\rho}(\mathbb{S}\cup\mathbb{T})}(a)=\bigwedge_{b\in U_2}\{\mu'_\rho(a,b)\wedge\mu'_{\mathbb{S}\cup\mathbb{T}}(b)\}$$

$$=\bigwedge_{b\in U_2}\{\mu'_\rho(a,b)\wedge(\mu'_{\mathbb{S}}(b)\vee\mu'_{\mathbb{T}}(b))\}$$

$$=\bigwedge_{b\in U_2}\{(\mu'_\rho(a,b)\wedge\mu'_{\mathbb{S}}(b))\vee(\mu'_\rho(a,b)\wedge\mu'_{\mathbb{T}}(b))\}$$

$$\geq \bigwedge_{b\in U_2}\{\mu'_\rho(a,b)\wedge\mu'_{\mathbb{S}}(b)\}\vee\bigwedge_{b\in U_2}\{\mu'_\rho(a,b)\wedge\mu'_{\mathbb{S}}(b)\}$$

$$\geq \mu'_{\underline{\rho}(\mathbb{S})}(a)\vee\mu'_{\underline{\rho}(\mathbb{T})}(a)$$

and

$$\mu^-_{\underline{\rho}(\mathbb{S}\cup\mathbb{T})}(a)=\bigvee\nolimits_{b\in U_2}\{\mu_\rho^-(a,b)\vee\mu^-_{\mathbb{S}\cup\mathbb{T}}(b)\}$$

$$=\bigvee_{b\in U_2}\{\mu_\rho^-(a,b)\vee(\mu_{\mathbb{S}}^-(b)\vee\mu_{\mathbb{T}}^-(b))\}$$

$$=\bigvee_{b\in U_2}\{(\mu_\rho^-(a,b)\vee\mu_{\mathbb{S}}^-(b))\vee(\mu_\rho^-(a,b)\vee\mu_{\mathbb{T}}^-(b))\}$$

$$=\bigvee_{b\in U_2}\{\mu_\rho^-(a,b)\vee\mu_{\mathbb{S}}^-(b)\}\vee\bigvee_{b\in U_2}\{\mu_\rho^-(a,b)\vee\mu_{\mathbb{T}}^-(b)\}$$

$$=\mu^-_{\underline{\rho}(\mathbb{S})}(a)\vee\mu^-_{\underline{\rho}(\mathbb{T})}(a)$$

Therefore $\underline{\rho}(\mathbb{S}\cup\mathbb{T})\supseteq\underline{\rho}(\mathbb{S})\cup\underline{\rho}(\mathbb{T})$. Similarly we can prove (iv).

IV. APPLICATION OF SPHERICAL FUZZY ROUGH SETS ON TWO UNIVERSES

Following Ashraf [1] we define the score value of each element in $\mathbb{S}FRS$ as follows:

$$\underline{sv}(\mathbb{S})(r)=\frac{1}{3}(2+\mu^+_{\underline{\rho}(\mathbb{S})}(r)-\mu'_{\underline{\rho}(\mathbb{S})}(r)-\mu^-_{\underline{\rho}(\mathbb{S})}(r))$$

$$\overline{sv}(\mathbb{S})(r)=\frac{1}{3}(2+\mu^+_{\overline{\rho}(\mathbb{S})}(r)-\mu'_{\overline{\rho}(\mathbb{S})}(r)-\mu^-_{\overline{\rho}(\mathbb{S})}(r))\ \ \forall\ r\in U_1$$

In this section, to obtain the optimal solution in multi criteria decision making problem, a new method has been propounded. In our real life situations MCDM plays an essential role to solve the complicated and uncertain decisions under some experts. The proposed method for MCDM has been explained below.

Let $U_1=\{cp_1,cp_2,cp_3,\ldots\ldots,cp_i\}$ be any set of $'i'$ alternatives and $U_2=\{att_1,att_2,att_3,\ldots\ldots,att_j\}$ be the finite set of $'j'$ attributes. If all the attributes are of same type, then there is no need to normalize.

The relation between the alternatives and attributes is a spherical fuzzy relation($\mathbb{S}FR_{\rho}$) denoted by ρ. Now we obtain a spherical fuzzy approximation space (U_1, U_2, ρ). Decision maker should give the membership grades of the alternatives $cp_i(i = 1,2, \dots . n)$ corresponding to the set of attributes $att_j(j = 1,2, \dots n)$. Let

${\mu_{ij}}^{+}$ be the positive grade of cp_i with respect to att_j,

${\mu_{ij}}'$ be the neutral grade of cp_i with respect to att_j,

${\mu_{ij}}^{-}$ be the negative grade of cp_i with respect to att_j.

Define a $\mathbb{S}FS, \mathbb{S}$ in U_2. Thus we obtain the spherical approximation space as $(U_1, U_2, \rho, \mathbb{S})$.

We define

$$\mathbb{S}_{\alpha} = \{a \in U_1 : \max_{a \in U_1}(\underline{sv}(a))\} \qquad (1)$$

$$\mathbb{S}_{\beta} = \{a \in U_1 : \max_{a \in U_1}(\overline{sv}(a))\} \qquad (2)$$

$$\mathbb{S}_{\gamma} = \{a \in U_1 : \max_{a \in U_1}(\underline{sv}(a) + \overline{sv}(a))\} \qquad (3)$$

Where,

$\mathbb{S}_{\alpha}$ is the maximum of lower approximation criterion of decision making,

$\mathbb{S}_{\beta}$ is the maximum of upper approximation criterion of decision making and

$\mathbb{S}_{\gamma}$ is the total weight of lower and upper approximation criterion of decision making,

Then $R_1 = \mathbb{S}_{\alpha} \cap \mathbb{S}_{\beta} \cap \mathbb{S}_{\gamma}$ is the solution of the prescribed model. We have the following 2 rules to obtain the optimal solution.

Rule 1: If $R_1 = \min\{\mathbb{S}_{\alpha}, \mathbb{S}_{\beta}, \mathbb{S}_{\gamma}\} \neq \emptyset$ then R_1 is consider to be the best solution.

Rule 2: If $R_1 = \mathbb{S}_{\alpha} \cap \mathbb{S}_{\beta} \cap \mathbb{S}_{\gamma} = \emptyset$ then we may arise the following two cases:

Case (i) $R_1 = \mathbb{S}_{\alpha} \cap \mathbb{S}_{\beta} \cap \mathbb{S}_{\gamma} \neq \emptyset$ then the elements obtained in $\mathbb{S}_{\alpha} \cap \mathbb{S}_{\beta} = R_2(say)$ is said to be the solution.

Case (ii) $R_2 = \mathbb{S}_{\alpha} \cap \mathbb{S}_{\beta} = \emptyset$ then the set of all elements in $R_3(= \mathbb{S}_{\gamma})$ is the best solution(alternatives).

4.1. Procedure-

The following steps explain the procedure to obtain the optimal solution.

Step 1: Calculate the lower and upper approximations of the given spherical fuzzy rough set using the spherical fuzzy set and spherical fuzzy relation.

Step 2: Evaluate the score values of each element in the spherical fuzzy rough set.

Step 3: Compute $\mathbb{S}_{\alpha}, \mathbb{S}_{\beta}$ and $\mathbb{S}_{\gamma}$.

Step 4: Determine R_1 , then we follow the Rules (1) or (2) according to the values obtained.

4.2. Example-

A school in a village has decided to set up a smart classes in each classroom. Learning through the audio-visual mode make the learning more easier and understandable. For this the school has issued a notice in a daily newspaper. One experienced company takes the responsibility to choose the best company out of the following five alternatives

$U_1 = \{Bharathi\ solutions(cp_1), Southern\ enterprises(cp_2), City\ pvt.ltd(cp_3),$
$Rose\ computers\ company(cp_4), and\ SS\ softwares(cp_5)\}$ according to the factors

$U_2 = \{quality(att_1), price(att_2), user - friendly(att_3)\ and\ \ longevity(att_4)\}$

The weight vector for the four attributes are $w_1 = 0.40, w_2 = 0.28,\ w_3 = 0.12, w_4 = 0.20.$

According to the weight vector, the experienced company has evaluated each company and give their opinions in terms of $\mathbb{S}FR_\rho$.

$$\rho = \begin{matrix} & att_1 & att_2 & att_3 & att_4 \\ cp_1 & (0.8,0.2,0.5) & (0.7,0.5,0.3) & (0.6,0.2,0.7) & (0.8,0.2,0.4) \\ cp_2 & (0.6,0.2,0.1) & (0.7,0.2,0.4) & (0.8,0.2,0.5) & (0.9,0.3,0.1) \\ cp_3 & (0.3,0.4,0.1) & (0.5,0.3,0.7) & (0.9,0.4,0.3) & (0.9,0.1,0.2) \\ cp_4 & (0.7,0.3,0.2) & (0.5,0.3,0.4) & (0.3,0.4,0.5) & (0.6,0.5,0.2) \\ cp_5 & (0.3,0.2,0.1) & (0.7,0.3,0.5) & (0.6,0.3,0.5) & (0.8,0.1,0.5) \end{matrix}$$

The experienced company presents the right weight for each attributes in U_2 is defined by $\mathbb{S}FS \subseteq U_2$. i.e.,

$$\mathbb{S} = \{(att_1/(0.9,0.1,0.3)), (att_2/(0.8,0.4,0.3)), (att_3/(0.5,0.1,0.4)), (att_4/(0.5,0.4,0.6))\}$$

Using Definition 2.4 we find the lower and upper approximations of $\mathbb{S}$ with respect to $(U_1, U_2, \rho, \mathbb{S})$.

$$\underline{\rho}(\mathbb{S}) = \{(cp_1/(0.5,0.1,0.7)), (cp_2/(0.5,0.1,0.6)), (cp_3/(0.3,0.1,0.7)), (cp_4/(0.3,0.1,0.6)),$$

$$(cp_5/\ (0.3,0.1,0.6))\}.$$

$$\overline{\rho}(\mathbb{S}) = \{(cp_1/(0.9,0.1,0.3)), (cp_2/(0.9,0.1,0.1)), (cp_3/(0.9,0.1,0.1)), (cp_4/(0.9,0.1,0.2)),$$

$$(cp_5/(0.9,0.1,0.1))\}.$$

Now we calculate the score values for each element in $\mathbb{S}FRS$

$$\underline{sv}(cp_1) = \frac{1}{3}(2 + 0.5 - 0.1 - 0.7) = 0.567$$

$$\underline{sv}(cp_2) = \frac{1}{3}(2 + 0.5 - 0.1 - 0.6) = 0.6$$

$$\underline{sv}(cp_3) = \frac{1}{3}(2 + 0.3 - 0.1 - 0.7) = 0.5$$

$$\underline{sv}(cp_4) = \frac{1}{3}(2 + 0.3 - 0.1 - 0.6) = 0.533$$

$$\underline{sv}(cp_5) = \frac{1}{3}(2 + 0.3 - 0.1 - 0.6) = 0.533$$

we have $\mathbb{S}_\alpha = \{cp_2\}$

Now

$\overline{sv}(cp_1) = \frac{1}{3}(2 + 0.9 - 0.1 - 0.3) = 0.833$

$\overline{sv}(cp_2) = \frac{1}{3}(2 + 0.9 - 0.1 - 0.1) = 0.9$

$\overline{sv}(cp_3) = \frac{1}{3}(2 + 0.9 - 0.1 - 0.1) = 0.9$

$\overline{sv}(cp_4) = \frac{1}{3}(2 + 0.9 - 0.1 - 0.2) = 0.86$

$\overline{sv}(cp_5) = \frac{1}{3}(2 + 0.9 - 0.1 - 0.1) = 0.9$

we have $\mathbb{S}_\beta = \{cp_2, cp_3, cp_5\}$

Then

$\underline{sv}(cp_1) + \overline{sv}(cp_1) = 0.7333 + 0.9 = 1.6333$

$\underline{sv}(cp_2) + \overline{sv}(cp_2) = 0.6667 + 0.8667 = 1.5334$

$\underline{sv}(cp_3) + \overline{sv}(cp_3) = 0.6667 + 0.9 = 1.5667$

$\underline{sv}(cp_4) + \overline{sv}(cp_4) = 0.6 + 0.8667 = 1.4667$

$\underline{sv}(cp_5) + \overline{sv}(cp_5) = 0.533 + 0.9 = 1.433$

we have $\mathbb{S}_\gamma = \{cp_2\}$

Therefore $R_1 = \{cp_2\}$

Southern enterprises is the best company according to the attributes.

V. α −LEVEL CUT OF SPHERICAL FUZZY RELATION

In this section we introduce α-level cut of spherical fuzzy relation. We give some operations and properties based on α-level cut of spherical fuzzy relation

5.1 Definition-

Let U_1 , U_2 be two universe sets and ρ_α be the α-level cut of $\mathbb{S}FR_\rho$. For any $\alpha = (\alpha_1, \alpha_2, \alpha_3) \in (0,1]^3$, we define the $\alpha - level\ cut$ of $\mathbb{S}FR_\rho$ by

$$\rho_\alpha(\mathbb{S}) = \{s \in U_2 / \mu_\rho(r, s) \geq \alpha\ \forall\ \alpha \in (0,1]\ and\ r \in U_1\}.$$

5.2 Definition-

The membership value of an element $'s'$ in $\mathbb{S}FS$ is defined by

$s = \prec \underline{s} \succ, \prec \overline{s} \succ$ where

$\underline{s} = (\mu^+_{\underline{\rho}(s)}(s), \mu'_{\underline{\rho}(s)}(s), \mu^-_{\underline{\rho}(s)}(s))$ and $\overline{s} = (\mu^+_{\overline{\rho}(s)}(s), \mu'_{\overline{\rho}(s)}(s), \mu^-_{\overline{\rho}(s)}(s))$

5.3 Properties-

Let U_1 *and* U_2 be two universe sets and ρ_α be the $\alpha - level\ cut$ of $\mathbb{S}FR_\rho$ of $U_1 \times U_2$. For any $\mathbb{S}FR_\rho, \mathbb{S} \subseteq U_2$, then the lower and upper approximations of $\mathbb{S}FR_\rho$ $\underline{\rho}_\alpha(\mathbb{S})$ and the upper approximation of $\mathbb{S}FR_\rho$ $\overline{\rho}_\alpha(\mathbb{S})$ have the following properties:

1) $\underline{\rho}_\alpha(\mathbb{S}) \subseteq \mathbb{S} \subseteq \overline{\rho}_\alpha(\mathbb{S})$
2) $\underline{\rho}_\alpha(\mathbb{S} \cap \mathbb{T}) = \underline{\rho}_\alpha(\mathbb{S}) \cap \underline{\rho}_\alpha(\mathbb{T})$
3) $\underline{\rho}_\alpha(\mathbb{S} \cap \mathbb{T}) = \underline{\rho}_\alpha(\mathbb{S}) \cap \underline{\rho}_\alpha(\mathbb{T}), \overline{\rho}_\alpha(\mathbb{S} \cup \mathbb{T}) = \overline{\rho}_\alpha(\mathbb{S}) \cup \overline{\rho}_\alpha(\mathbb{T})$
4) If $\mathbb{S} \subseteq \mathbb{T}$ then $\underline{\rho}_\alpha(\mathbb{S}) \subseteq \underline{\rho}_\alpha(\mathbb{T})$ and $\overline{\rho}_\alpha(\mathbb{S}) \subseteq \overline{\rho}_\alpha(\mathbb{S})$
5) $\underline{\rho}_\alpha(\mathbb{S}) = (\overline{\rho}_\alpha(\mathbb{S}^c))^c,\ \overline{\rho}_\alpha(\mathbb{S}) = (\underline{\rho}_\alpha(\mathbb{S}^c))^c$.

Proof: The proof follows from the Definition of lower and upper approximation of α-level cut of $\mathbb{S}FR_\rho$..

The following definition is extended by Pawlak's rough set model.

5.4 Definition-

Let U_1, U_2 be two universe sets and ρ_α be the $\alpha - level\ cut$ of $\mathbb{S}FS$. For any $\mathbb{S} \in U_2$, the lower and upper approximations of ρ_α is defined by

$$\underline{\rho}_\alpha(\mathbb{S}) = \{r \in U_1 / \rho_\alpha(r) \subseteq \mathbb{S}\}$$

$$\overline{\rho}_\alpha(\mathbb{S}) = \{r \in U_1 / \rho_\alpha(r) \cap \mathbb{S} \neq \emptyset\}.$$

The sets $\underline{\rho}_\alpha(\mathbb{S})$ and $\overline{\rho}_\alpha(\mathbb{S})$ are not equal then it is said to be undefinable set.

The positive region, negative region and boundary region of any $\mathbb{S} \subseteq U_2$ about ρ_α is defined as follows:

(i) $P(\rho_\alpha(\mathbb{S})) = \underline{\rho}_\alpha(\mathbb{S})$
(ii) $N(\rho_\alpha(\mathbb{S})) = U - \overline{\rho}_\alpha(\mathbb{S})$
(iii) $B(\rho_\alpha(\mathbb{S})) = \overline{\rho}_\alpha(\mathbb{S}) - \underline{\rho}_\alpha(\mathbb{S})$.

5.5 Definition-

Let U_1 *and* U_2 be two universe sets. The relation ρ_α is from U_1 *to* U_2. Then for any $\mathbb{S}FS, \mathbb{S}$ the approximate precision $\gamma_\alpha(\mathbb{S})$ about ρ_α is defined by

$$\gamma_{\rho_\alpha}(\mathbb{S}) = \frac{\left|\underline{\rho}_\alpha(\mathbb{S})\right|}{\left|\overline{\rho}_\alpha(\mathbb{S})\right|}$$

Where $\left|\underline{\rho}_\alpha(\mathbb{S})\right|$ is the cardinality of $\underline{\rho}_\alpha(\mathbb{S})$ and $\left|\overline{\rho}_\alpha(\mathbb{S})\right|$ is the cardinality of $\overline{\rho}_\alpha(\mathbb{S})$.

$R_{\rho_\alpha}(\mathbb{S}) = 1 - \gamma_{\rho_\alpha}(\mathbb{S})$ is called the rough degree of $\mathbb{S}$ about ρ_α.

5.6 Application-

In this section we give a simple model based on $\mathbb{S}FR_\rho$. There are some of the applications in $\mathbb{S}FRS$s on two universes. Here we are applying the model proposed by B. Sun and W. Ma[8] in *α-level cut* of $\mathbb{S}FR_\rho$. It will be more reliable and scientific. There are many applications in medical diagnosis. This is one of the model based on $\mathbb{S}FR_\rho$ between two universes. The α-level cut of $\mathbb{S}FR_\rho$ is a relation between the alternatives and attributes. The positive region of ρ_α is the needed alternative, the boundary region of ρ_α is the unwanted alternatives and the negative region of ρ_α is on consideration.

5.7 Numerical example-

In a family of four members are suffered from some of the health issues. They have been afraid of whether having the disease "*chickenpox(D)*''. They decided to conceal their family doctor. The doctor analyzing the patients one by one. Then he give the relation between the patients and the issues. The relation is as follows:

$$\rho = \begin{matrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{matrix} \overset{\begin{matrix} fever & rashes & headache \end{matrix}}{\begin{pmatrix} (0.5,0.4,0.4) & (0.9,0.2,0.3) & (0.3,0.6,0.4) \\ (0.8,0.1,0.2) & (0.6,0.3,0.4) & (0.5,0.5,0.2) \\ (0.7,0.6,0.1) & (0.7,0.1,0.3) & (0.8,0.1,0.3) \\ (0.8,0.2,0.2) & (0.6,0.4,0.2) & (0.9,0.2,0.2) \end{pmatrix}}$$

The health issues $\{fever, rashes\} \subseteq U_2$ are the symptoms of the disease chicken pox. Let it be denoted by D. The weight vector of the attributes (health issues) is given by $\alpha = \prec 0.4,0.2,0.4 \succ$

Step 1: Find ρ_α

$\rho_\alpha(m_1) = \{fever, rashes\}$
$\rho_\alpha(m_2) = \{rashes, headache\}$

$\rho_\alpha(m_3) = \{fever\}$

$\rho_\alpha(m_4) = \{fever, rashes, headache\}$

Step 2: Calculate $\underline{\rho}_\alpha(D)$ *and* $\overline{\rho}_\alpha(D)$.

$\underline{\rho}_\alpha(D) = \{m_1, m_2\}$

$\overline{\rho}_\alpha(D) = \{m_1, m_2, m_3, m_4\}$

Step 3: Calculate $P\big(\rho_\alpha(\mathbb{S})\big), N\big(\rho_\alpha(\mathbb{S})\big)$ and $B\big(\rho_\alpha(\mathbb{S})\big)$.

$P\big(\rho_\alpha(\mathbb{S})\big) = \{m_1, m_2\}$

$B\big(\rho_\alpha(\mathbb{S})\big) = \{m_2, m_4\}$

$N\big(\rho_\alpha(\mathbb{S})\big) = \emptyset.$

Step 4: Calculate the approximate precision and rough degree of $\mathbb{S}$.

$\gamma_{\rho\alpha}(\mathbb{S}) = \frac{2}{4} = 0.5$

$R_{\rho\alpha} = 1 - 0.5 = 0.5$

Here we are concluding that

(i) The family members m_1 and m_2 have the disease Chicken pox(D). They must need proper treatment to get rid off.
(ii) The remaining two is on second thought, i.e., he/she may or may not have chicken pox(D).
(iii) No one in the list to be diagnose health.

This is one of the easiest model to deal with uncertainty.

VI. CONCLUSION

In this paper, we studied the properties of spherical fuzzy rough set. A model based on spherical fuzzy rough set is studied and explained with an example. In the last section we introduced $\alpha -$level set of spherical fuzzy relation. We presented a model based on spherical fuzzy relation and applied it in a medical diagnosis problem. Thus in all, spherical fuzzy rough set seems to be promising new concept, paving the way to numerous possibilities for future research.

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