Maximum sum numbering for some graphs

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Abstract

Labeling gives a new definition to a given graph and the graph becomes more useful than its unlabeled structure. A function ζ from the set of natural numbers N whose range is a subset of real numbers R is called a sequence. In this paper, we will concern with sequence in Rand discuss what we mean by the maximum sum numbering for an n-gon, trees and bipartite graph conjecture. We also show an example that the inverse transformation of maximum sum numbering is maximum sum numbering.

Key words: Graph labeling, n-gon, trees, bipartite graph, sequence.

1. Introduction

The two important parameters which represent a graph abstractly are the vertices and edges of a graph G= (V, E).Labeling of a graph G means to give labels to the vertices or edges or both under the certain conditions of human interest. Immense conditions are known for particular types of graph labeling but the labeling techniques are scanty. Labeling gives a new definition to an unlabeled graph and the graph becomes more powerful than its unlabeled structure [5]. Almost all types of graph labeling are considered from a paper of Rosa [7] and than by Golumb [3]. All types of graph labeling collections is written in a very unique book by Gallian[2] which is considered to be the most important book of this area.

All graphs considered in this paper are finite, simple, connected and undirected. On the basis of pre-defined graph labeling technique discussed in [2], we give a new labeling which will call maximum sum numbering, discussed in next section[1],[4],[6],[8].

2. Definitions

In this section, we develop a function f which gives labels to the vertices of a group $G=(V,E)$. Each term of sequence generated by the function f is in a particular manner. We call this function odd-even maximum function. The sequence generated by this function has three subsequence in which one of them converges to zero.

Definition 2.1: A function f in a variable ‘x’ is said to be odd-even maximum function if it is defined as:

$$f(x)=\begin{cases} x_i = a+2(i-1)d & \textit{if } i \textit{ is odd }, i \in N \\ x_i = 0 & \textit{if } i \textit{ is even }, i \in N \\ y_i = x_i + 1 & \textit{if } i \textit{ is odd }, i \in N \\ y_i = 0 & \textit{if } i \textit{ is even}, i \in N \end{cases}$$

Where $a,d \in N$ and the sequence $< x_i, y_i : i \in N >$ in two variables contain at least a null subsequence. □

Consider a sequence $< x_1, y_1, x_2, y_2, x_3, y_3, ... >$ generated by the function f and if we take a=2 and d=1 then we get a sequence $< \rho_n >$ such that $< \rho_n > = < 2,3,0,0,6,7,0,0,10,11,0,0,... >$.

By plotting the graph of the sequence $< \rho_n >$ in a two dimensional co-ordinate system generated by f, we see that it has an inverted cup shaped structure. From the sequence $< \rho_n >$ we can generate two new subsequence:

$$< \delta_p > = < 2,3,6,10,14,... > and < \delta_p^{'} > = < 2,3,7,11,15,... >$$

Let us suppose that m and M is the maximum value in $< \delta_p > and < \delta_p^{'} >$ respectively. Arrange at least first three numbers starting from 2 and 3 in clockwise or anti-clockwise direction such that these numbers represent vertex label of n-gon, trees and bipartite graph. The number of terms either in the sequence $< \delta_p > and < \delta_p^{'} >$ represents the number of edges of an n-gon, trees and bipartite graph. Let $G=(V,E)$ be the graph represent either of the sequence $<\delta_p> or <\delta_p^{'}>$. The vertex set V of G is either $V=\{2,3,6,10,14,...\}$ or $\{2,3,7,11,15,...\}$.

Definition 2.2: consider a graph G has a vertex labeling η and an induced edge labeling $\gamma_i; i \in N$. The vertex labeling η is said to be maximum sum numbering for G if it satisfy the following properties:

(i) η Is an injective mapping.

(ii) $\lambda_i = |\eta(u)-\eta(v)|, \forall u,v \in V, uv \in E, i \in N$.

(iii) $\gamma_i = k \in N$ For some $i \in N$.

On the basis of definition 2.1 and 2.2 we prove that all n-gon (C_n), trees and bipartite graphs admits maximum sum numbering.

3. Results

Observation 3.1: the maximum edge label in maximum sum numbering of n-gon is the sum of remaining edge labels. The table given below shows that the maximum edge labels in C_n is the sum of remaining (n-1) edges label for the two subsequences $<\delta_p> and <\delta_p^{'}>$.

N	3	4	5	6	7	8	9	10
$i_{\max}$ for x_i	3	5	7	9	11	13	15	17
$\max.edge label in \delta_p$	4	8	12	16	20	24	28	32

N	3	4	5	6	7	8	9	10
$i_{\max}$ for y_i	3	5	7	9	11	13	15	17
$\max.edge label in \delta_p^{'}$	5	9	13	17	21	25	29	33

Observation3.2: In the maximum sum numbering of n-gon, trees and bipartite graph, if we take the first subsequence $<\delta_p>$ then it should be noted that the sequence $<\lambda_i>$ generated by the above formula contains (n-3) times 4 whenever $n \geq 4$.

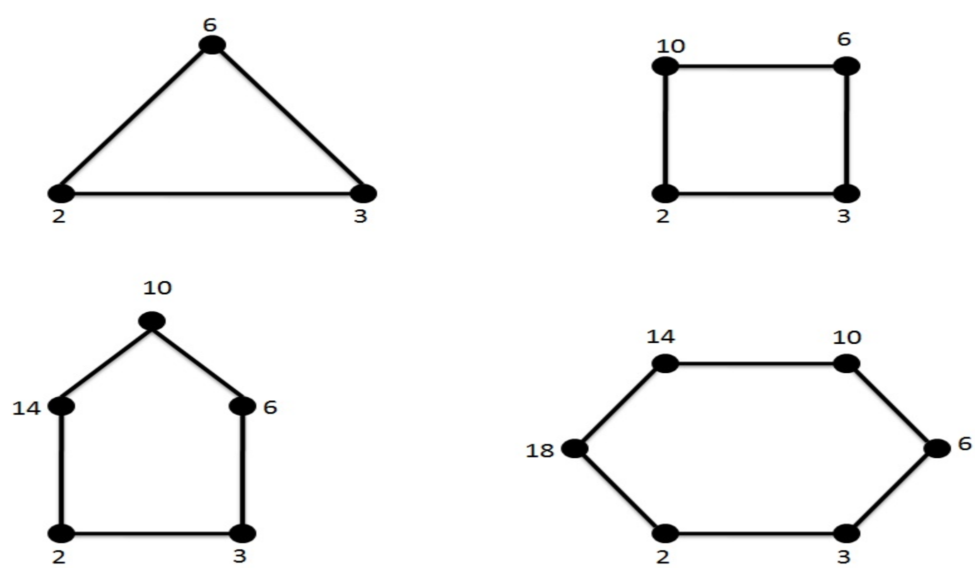
Observation3.3: In the maximum sum numbering of n-gon, trees and bipartite graph, if we take the second subsequence $<\delta_p^{'}>$ then it should be noted that the sequence $<\lambda_i>$ generated by the above formula contains (n-2) times 4 whenever $n \geq 3$.

Theorem 3.1: n-gon admits maximum sum numbering.

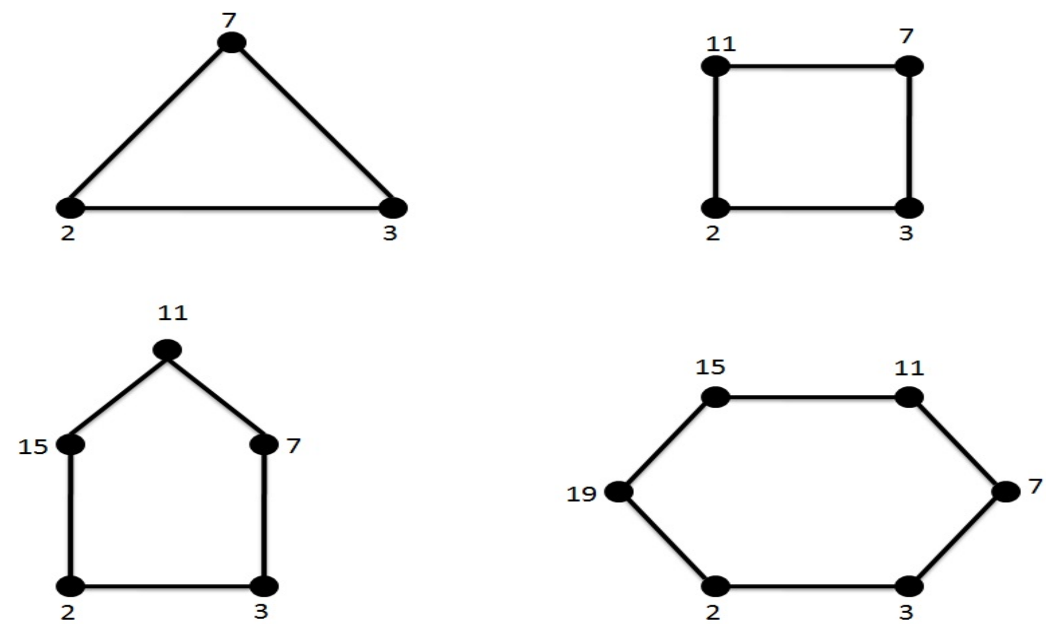
Proof: let us consider a sequence $<x_1, y_1, x_2, y_2, x_3, y_3, ...>$ generated by the function f (see definition 2.1) and each even term is zero. For the subsequence $<\delta_p>$, the vertex labels for n-gon $x_1 = 2, y_1 = 3, x_3 = 6, x_5 = 10$....... and the next term for each member of $<\delta_p>$ is increase by 4. It is also same for the subsequence $<\delta_p^{'}>$. Therefore the last edge label, either in clockwise or anticlockwise is the maximum edge label. □

Illustration: Graph given below illustrate the maximum sum numbering of n-gon for n=3, 4, 5, 6.

Case1: if $V = \{2, 3, 6, 10, 14, ...\}$ then graph for maximum sum numbering for n=3, 4, 5, 6 given below:



Case2: if $V = \{2, 3, 7, 11, 15, ...\}$ then graph of maximum sum numbering for n=3,4,5,6 given below:



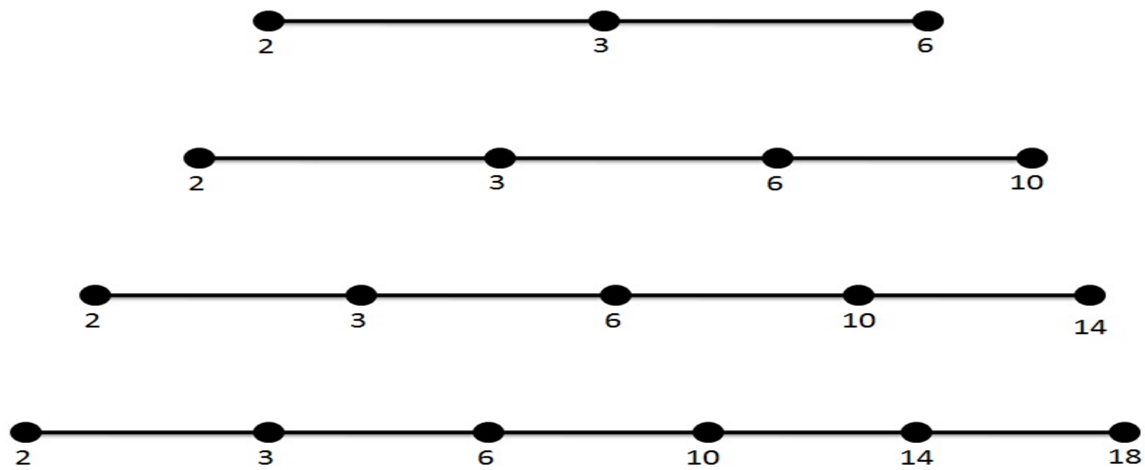
Maximum sum numbering of n-gon for n= 3, 4, 5, 6

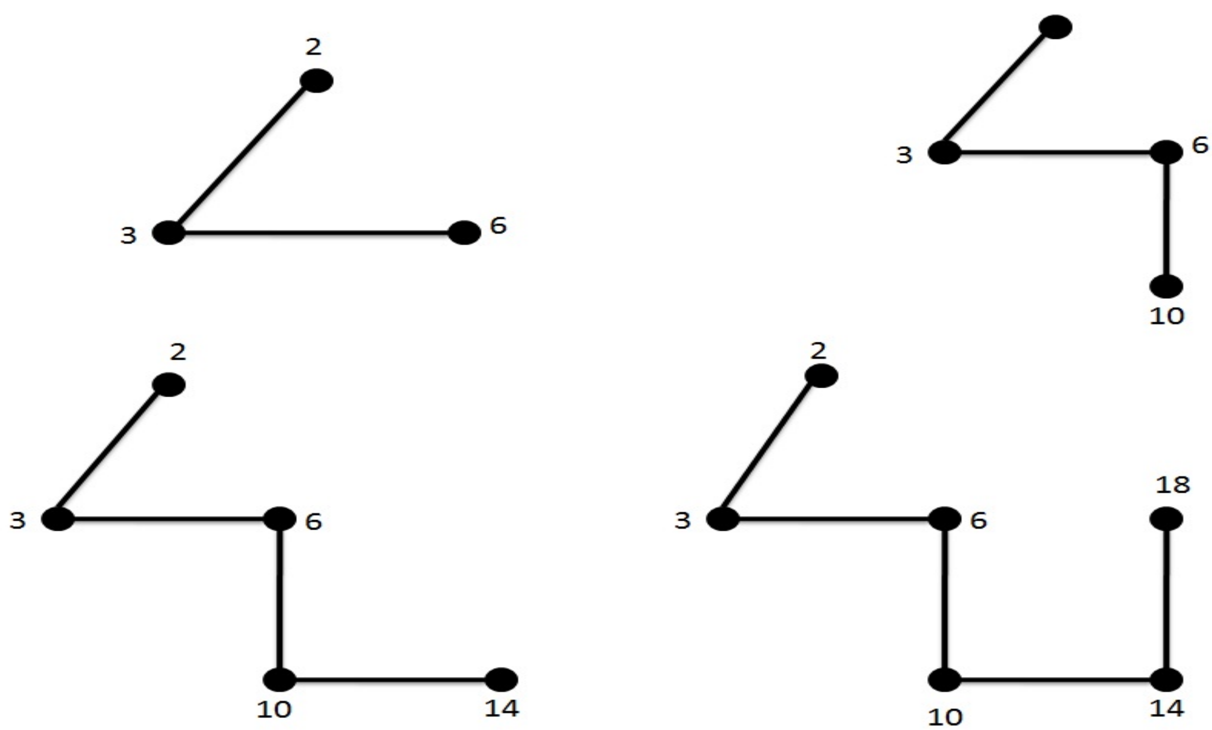
Theorem 3.2: Some trees admit maximum sum numbering.

Proof: let us consider a sequence $< x_1, y_1, x_2, y_2, x_3, y_3, ... >$ generated by the function f (see definition 2.1) and each even term is zero. For the subsequence $< \delta_p >$, the vertex labels for some trees $x_1 = 2, y_1 = 3, x_3 = 6, x_5 = 10$....... and the next term for each member of $< \delta_p >$ is increase by 4. It is also same for the subsequence $< \delta_p^{'} >$ .In case of tree (for zero distance tree and spanning tree), it is clear that the difference between first and last vertex is equal to the sum of the difference of all vertices. Hence the last edge label, either in clockwise or anticlockwise is the maximum edge label. □

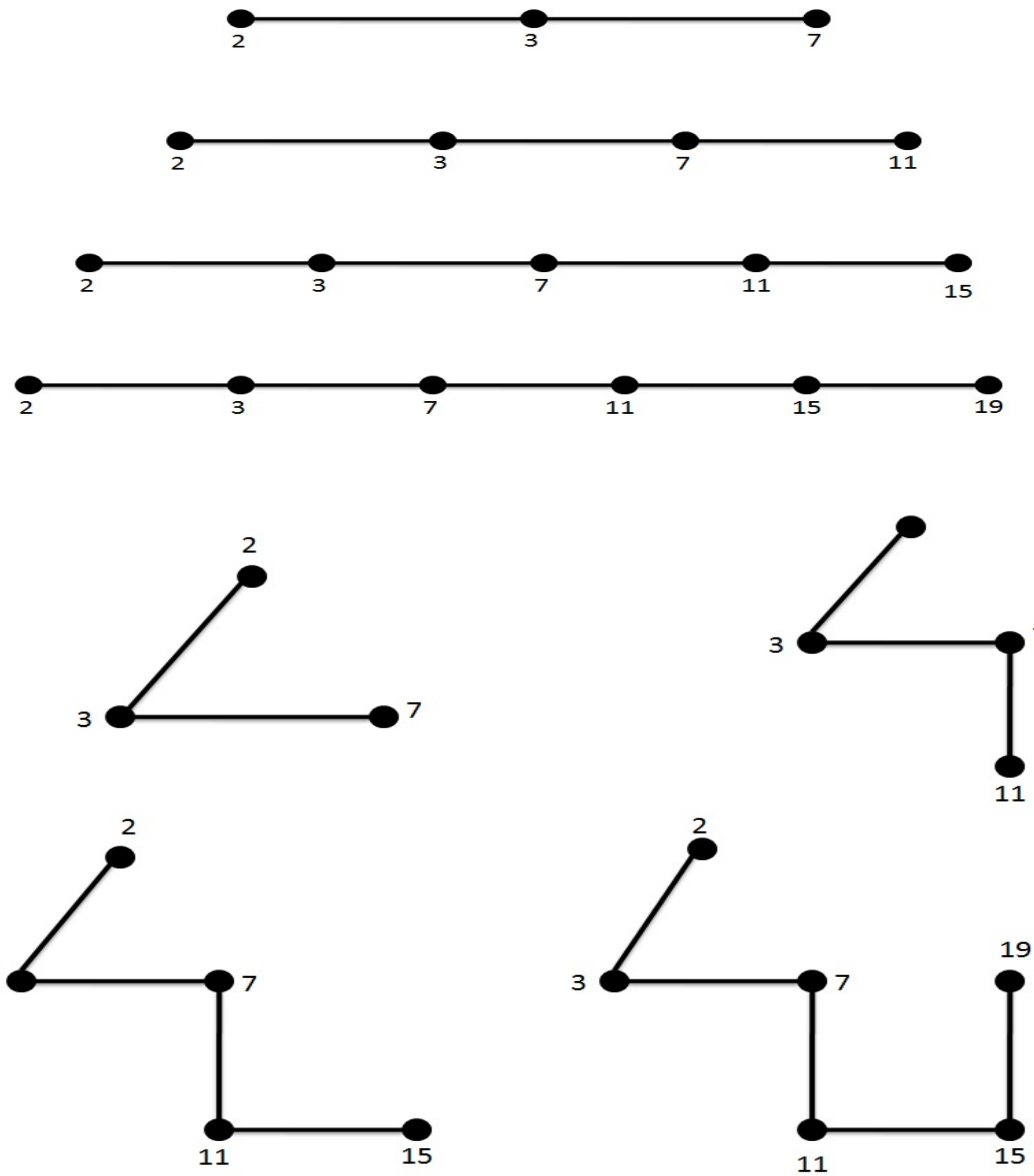
Illustration: Graph given below illustrate the maximum sum numbering of some trees conjecture i.e zero distance tree, spanning tree for n= 3, 4, 5, 6.

Case1: if $V = \{2, 3, 6, 10, 14, ...\}$ then the graph of maximum sum numbering for n=3, 4, 5, 6 given below:





Case2: if $V = \{2, 3, 7, 11, 15, ...\}$ then the graph of maximum sum numbering for n=3, 4, 5, 6 given below:



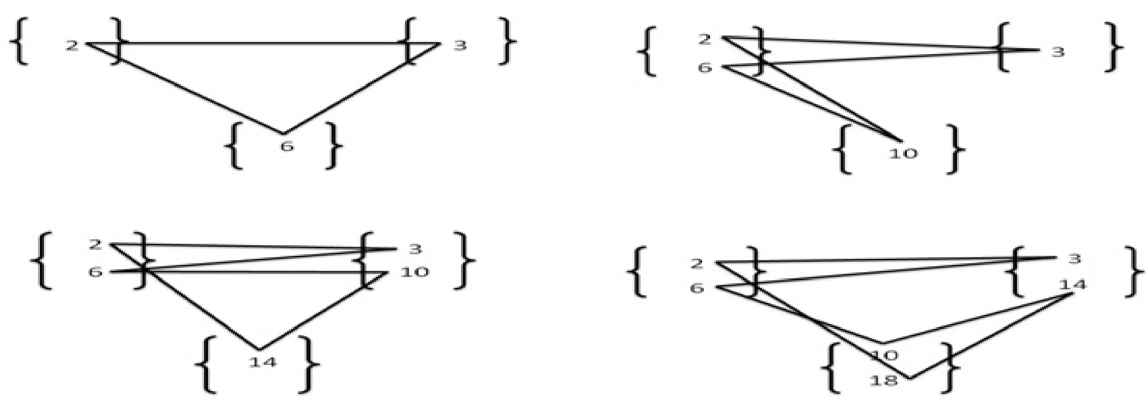
Maximum sum numbering of zero distance tree and spanning tree for n= 3, 4, 5, 6.

Theorem 3.3: all bipartite graphs admit maximum sum numbering.

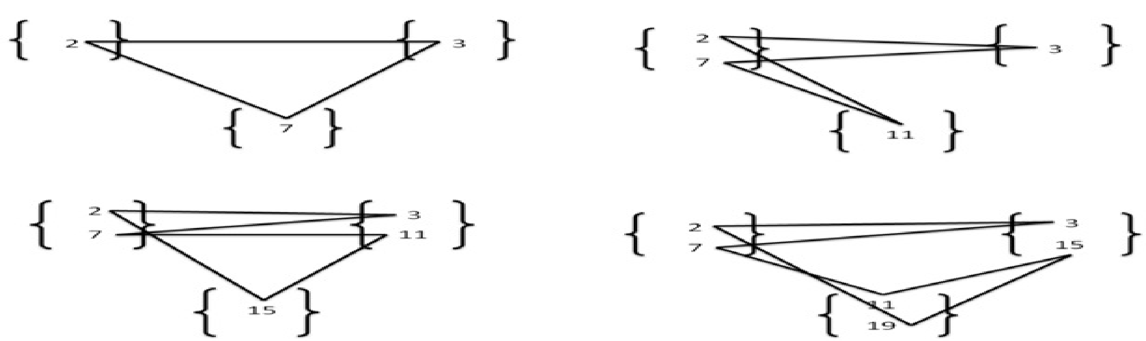
Proof: let us consider a sequence $< x_1, y_1, x_2, y_2, x_3, y_3, ... >$ generated by the function f (see definition 2.1) and each even term is zero. For the subsequence $< \delta_p >$, the vertex labels for bipartite graph $x_1 = 2, y_1 = 3, x_3 = 6, x_5 = 10$....... and the next term for each member of $< \delta_p >$ is increase by 4. It is also same for the subsequence $< \delta_p^{'} >$ .In case of bipartite graph, it is clear that the difference between first and last vertex is equal to the sum of the difference of all vertices. Hence the last edge label, either in clockwise or anticlockwise is the maximum edge label. □

Illustration: Graph given below illustrate the maximum sum numbering of bipartite graph for n= 3, 4, 5, 6.

Case1: if $V = \{2, 3, 6, 10, 14, ...\}$ then the graph of maximum sum numbering for n=3, 4, 5, 6given below:



Case2: if $V = \{2, 3, 7, 11, 15, ...\}$ then the graph of maximum sum numbering for n=3, 4, 5, 6 given below:



Maximum sum numbering of bipartite graph for n= 3, 4, 5, 6

Conclusion

In the above paper we observe that every n-gon, tree, bipartite graph have maximum sum numbering. Result of these graphs are divide into two subsequence in which one sequence represent the even number of term and other sequence represent the odd number of term. We also prove that the edge label is maximum for this particular graph i.e. the sum of the difference of n-1 vertices label is equal to the difference between first and last vertex of the graph.

Reference

1. Doe, N., "graph theory with application to engineering and computer science", Prentice Hall of India, (2005).
2. Gallian, J.A., "A dynamic survey of graph labeling", the electronic journal of Combinatorics, (2018), #DS6.
3. Golomb, S. W., "how to number a graph" in graph theory and computing, R. C. Read, ed., Academic press, New York, (1972) 23-37.
4. Harary, F., "graph theory", Nervosa publication house, (2001).
5. Kumar, A., Mishra, D., Verma, A., and Srivastava, V.K., "Alpha labeling of cyclic graph-1", ARS Combinatorial (to appear).
6. Murugesan, N., and Uma, R., "Graceful labeling of some graphs and their sub graph", Asian journal of current engineering and math, (2012).
7. Rosa, A., "on certain valuation of the vertices of graph", theory of graph (internat. Symposium, Rome July 1966), Gordon and Breach, N, Y, and Dunned Paris, (1967) 349-355.
8. Yvonne, M.M., "discrete math", (1980) 39-46.