Neutrosophic Vague Projection Based Models for Solving Neutrosophic Vague Multiple Attribute Decision Making Problems

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Abstract: The aim of this paper is to develop two new methods neutrosophic vague weighted projection method and neutrosophic vague angle cosine and projection method for solving multiple attribute decision making problem with neutrosophic vague sets.

Keywords: Neutrosophic vague sets, projection measure, weighted projection measure, neutrosophic vague angle cosine.

I. INTRODUCTION

Multiple criterion decision making (MCDM) refers to making decisions in the presence of multiple, usually conflicting criteria. The problems of MCDM can be classified into two categories: multiple attribute decision making (MADM) and multiple objective decision making (MODM), depending on whether the problem is a selection problem or a design problem. MODM methods have decision variable values that are determined in a continuous or integer domain, with either an infinitive or a large number of choices, the best of which should satisfy the decision maker's constraints and preference priorities. MADM methods, on the other hand, are generally discrete, with a limited number of predetermined alternatives. MADM is an approach employed to solve problems involving selection from among a finite number of alternatives. Multiple attribute decision making (MADM) is one of the most significant parts of modern decision making and it is a well known method for selecting the most desirable alternative from a set of all feasible alternatives with respect to some predefined attributes. However, the information about the attributes is generally incomplete, indeterminate and inconsistent in nature due to the complexity of real world problems.An MADM method specifies how attribute information is to be processed in order to arrive at a choice. MADM methods require both inter- and intra-attribute comparisons, and involve appropriate explicit tradeoffs. Each decision table (also called decision matrix) in MADM methods has four main parts, namely: (a) alternatives A_i (for i = 1, 2, ….. , N), (b) attributes B_j (for j = 1, 2, ….. , M), (c) weight or relative importance of each attribute w_j (for j=1, 2, ….., M), and (d) measures of performance of alternatives with respect to the attributes m_{ij} (for i= 1, 2, ….., N; j=1, 2, ….., M). Given the decision matrix information and a decision-making method, the task of the decision maker is to find the best alternative and/or to rank the entire set of alternatives. It may be added here that all the elements in the decision table must be normalized to the same units, so that all possible attributes in the decision problem can be considered.

The concept of fuzzy sets was introduced by Zadeh[9] in 1965. Using this fuzzy sets intuitionistic fuzzy sets was introduced by Atanassov[1] in 1986. The intuitionistic fuzzy multiple attribute decision making (IF-

MADM) was first proposed by Ye[8] in 2013. The theory of vague sets was first proposed and developed as an extension of fuzzy set theory by Gau and Buehrer[3]. Then, Smarandache[5] introduces the neutrosophic elements T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where]−0,1+[is that the non-standard unit interval in 1998. Neutrosophic MADM problems with unknown weight information was solved by Biswas et al.[2]. Projection measure is useful device for solving decision making problems because it takes into account the distance as well as the included angle between points was evaluated by Xu [6]. Xu and Hu [7] provided projection models for dealing with intuitionistic fuzzy MADM problems. Shawkat Alkhazaleh[4] in 2015 introduced and constructed the concept of neutrosophic vague set.

In this paper, we use the neutrosophic vague set and weighted projection to defined two methods neutrosophic vague weighted projection method and neutrosophic vague angle cosine and projection method. By using these methods we solve a neutrosophic MADM problem.

II. PRELIMINARIES

Definition 2.1:[4] A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $A_{NV} = \left\{\left\langle x; \hat{T}_{A_{NV}}(x); \hat{I}_{A_{NV}}(x); \hat{F}_{A_{NV}}(x)\right\rangle; x \in X\right\}$, whose truth membership, indeterminacy membership and false membership functions is defined as:

$$\hat{T}_{A_{NV}}(x) = \left[T^-, T^+\right], \hat{I}_{A_{NV}}(x) = \left[I^-, I^+\right], \hat{F}_{A_{NV}}(x) = \left[F^-, F^+\right]$$

where,

1) $T^+ = 1 - F^-$
2) $F^+ = 1 - T^-$ and
3) $^-0 \le T^- + I^- + F^- \le 2^+$.

Definition 2.2:[4] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, $\hat{T}_{A_{NV}}(u_i) \le \hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i) \ge \hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i) \ge \hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where $1 \le i \le n$.

Definition 2.3:[4] The complement of NVS A_{NV} is denoted by A_{NV}^c and is defined by

$$\hat{T}_{A_{NV}}^c(x) = \left[1 - T^+, 1 - T^-\right], \hat{I}_{A_{NV}}^c(x) = \left[1 - I^+, 1 - I^-\right], \hat{F}_{A_{NV}}^c(x) = \left[1 - F^+, 1 - F^-\right].$$

Definition 2.4:[4] Let A_{NV} be NVS of the universe U where $\forall u_i \in U$, $\hat{T}_{A_{NV}}(x) = [1,1]$; $\hat{I}_{A_{NV}}(x) = [0,0]$; $\hat{F}_{A_{NV}}(x) = [0,0]$. Then A_{NV} is called unit NVS(1_{NV} in short), where $1 \le i \le n$.

Definition 2.5:[4] Let A_{NV} be NVS of the universe U where $\forall u_i \in U$, $\hat{T}_{A_{NV}}(x) = [0,0]$; $\hat{I}_{A_{NV}}(x) = [1,1]$; $\hat{F}_{A_{NV}}(x) = [1,1]$. Then A_{NV} is called zero NVS(0_{NV} in short), where $1 \le i \le n$.

Definition 2.6:[6] Let $e = (e_1, e_2, ..., e_q)$ be a vector, then norm of e is defined by $\| e \| = \sqrt{\sum_{i=1}^{q} e_j^2}$

Definition 2.7:[6] Let $e = (e_1, e_2, ..., e_q)$ and $f = (f_1, f_2, ..., f_q)$ be two vectors, then angle cosine between e and f is defined as $Cos(e, f) = \frac{\sum_{j=1}^{q}(e_j, f_j)}{\sqrt{\sum_{j=1}^{q} e_j^2} \times \sqrt{\sum_{j=1}^{q} f_j^2}}$. Obviously, $0 \le Cos(e, f) \le 1$ and $Cos(e, f)$ denotes the closeness between e and f only in direction.

Definition 2.8:[6] Let $e = (e_1, e_2, ..., e_q)$ and $f = (f_1, f_2, ..., f_q)$ be two vectors, then the projection of vector e onto vector f can be defined as follows:

$$\Pr oj(e)_f = Cos(e, f) = \sqrt{\sum_{j=1}^{q} e_j^2} \times \frac{\sum_{j=1}^{q}(e_j, f_j)}{\sqrt{\sum_{j=1}^{q} e_j^2} \times \sqrt{\sum_{j=1}^{q} f_j^2}} = \frac{\sum_{j=1}^{q}(e_j, f_j)}{\sqrt{\sum_{j=1}^{q} f_j^2}}$$

III. SOLVING NEUTROSOPHIC VAGUE MADM PROBLEMS USING NEUTROSOPHIC VAGUE WEIGHTED PROJECTION METHOD

In this method we consider a MADM problem which is represented in linguistic variables, it has a discrete set of alternatives $G = \{G_1, G_2, ..., G_m\}, (m \ge 2)$ and $A = \{A_1, A_2, ..., A_n\}, (n \ge 2)$ the set of attributes. Then the linguistic variables are converted into neutrosophic vague sets and weights are assigned to each alternative. If an attribute has a small effect on the alternatives then the attribute value should be assigned with a small weight and the attribute which creates bigger deviation should be assigned with a bigger weight. If an attribute has very small or no effect on the alternatives then the weight of such attribute may be taken as zero. Assume $W = \{w_1, w_2, ..., w_n\}$ be the assigned weight vector of the attributes, where $0 \le w_j \le 1$ with $\sum_{j=1}^{n} w_j = 1$.

3.1. Determination of neutrosophic vague ideal solution:

We determine the neutrosophic vague ideal solution $Z^* = (Z_1^*, Z_2^*, ..., Z_n^*)$ as given below.

$$Z_j^* = \langle [1,1]; [0,0]; [0,0] \rangle, \ j = 1, 2, ..., n$$

The virtual neutrosophic vague ideal solution $Z^* = (\mu_1^*, \mu_2^*, ..., \mu_n^*)$ can be obtained by identifying the best of each attributes as given as $\mu_j^* = \langle \alpha_j, \beta_j, \gamma_j \rangle$ ------------------------ (3.1)

Where, $\alpha_j = [\alpha_j^-, \alpha_j^+] = [\underset{i}{Max} T_{ij}^-, \underset{i}{Max} T_{ij}^+]$; $\beta_j = [\beta_j^-, \beta_j^+] = [\underset{i}{Min} I_{ij}^-, \underset{i}{Min} I_{ij}^+]$;
$\gamma_j = [\gamma_j^-, \gamma_j^+] = [\underset{i}{Min} F_{ij}^-, \underset{i}{Min} F_{ij}^+]$.

3.2. Calculation of neutrosophic vague weighted projection:

The neutrosophic vague weighted projection of the alternatives $G_i\left(i=1,2,...,m\right)$ on the neutrosophic vague ideal solution Z^* is defined as follows.

$$\mathrm{Pr}oj_{\,w}(G_i)_{Z^*}=\frac{1}{\left\|Z^*\right\|_w}\sum_{j=1}^{n}w_j^2\left(T_{ij}^-\alpha_j^-+T_{ij}^+\alpha_j^++I_{ij}^-\beta_j^-+I_{ij}^+\beta_j^++F_{ij}^-\gamma_j^-+F_{ij}^+\gamma_j^+\right)$$

$$=\frac{\sum_{j=1}^{n}w_j^2\left(T_{ij}^-\alpha_j^-+T_{ij}^+\alpha_j^++I_{ij}^-\beta_j^-+I_{ij}^+\beta_j^++F_{ij}^-\gamma_j^-+F_{ij}^+\gamma_j^+\right)}{\sqrt{\sum_{j=1}^{n}w_j^2\left(\left(\alpha_j^-\right)^2+\left(\alpha_j^+\right)^2+\left(\beta_j^-\right)^2+\left(\beta_j^+\right)^2+\left(\gamma_j^-\right)^2+\left(\gamma_j^+\right)^2\right)}}\quad -----(3.2)$$

3.3. Ranking of the alternatives:

Rank the alternatives $G_i\left(i=1,2,...,m\right)$ according to the neutrosophic vague projection $\mathrm{Pr}oj_{\,w}(G_i)_{Z^*}$ and bigger value of $\mathrm{Pr}oj_{\,w}(G_i)_{Z^*}$ reflects the better alternative.

3.4. Algorithm 1:

An algorithm for MADM problems with neutrosophic vague information based on neutrosophic vague weighted projection method is provided in the following steps:

Step 1: Transform the linguistic decision matrix into neutrosophic vague decision matrix.

Step 2: We assume the weights $w_j=\left(j=1,2,...,n\right)$ to the attributes $A_j=\left(j=1,2,...,n\right)$ such that $\sum_{j=1}^{n}w_j=1, w_j\geq 0,\ j=1,2,...,n$.

Step 3: Calculate the virtual neutrosophic vague ideal solution using the equation 3.1.

Step 4: Determine the neutrosophic vague weighted projection $\mathrm{Pr}oj_{\,w}(G_i)_{Z^*}$ using the equation 3.2.

Step 5: Rank the alternatives $G_i\left(i=1,2,...,m\right)$ based on $\mathrm{Pr}oj_{\,w}(G_i)_{Z^*}$ and select the best one.

IV. SOLVING NEUTROSOPHIC VAGUE MADM PROBLEMS USING NEUTROSOPHIC VAGUE ANGLE COSINE AND PROJECTION METHOD

4.1. Calculation of neutrosophic vague angle cosine:

The neutrosophic vague angle cosine between the alternatives $G_i\left(i=1,2,...,m\right)$ and the neutrosophic vague ideal solution Z^* is defined as follows:

$$Cos\left(G_i, Z^*\right) = \frac{\sum_{j=1}^{n}\left(T_{ij}^-\alpha_j^- + T_{ij}^+\alpha_j^+ + I_{ij}^-\beta_j^- + I_{ij}^+\beta_j^+ + F_{ij}^-\gamma_j^- + F_{ij}^+\gamma_j^+\right)}{\sqrt{\sum_{j=1}^{n}\left(\left(T_{ij}^-\right)^2 + \left(T_{ij}^+\right)^2 + \left(I_{ij}^-\right)^2 + \left(I_{ij}^+\right)^2 + \left(F_{ij}^-\right)^2 + \left(F_{ij}^+\right)^2\right)}\sqrt{\sum_{j=1}^{n}\left(\left(\alpha_j^-\right)^2 + \left(\alpha_j^+\right)^2 + \left(\beta_j^-\right)^2 + \left(\beta_j^+\right)^2 + \left(\gamma_j^-\right)^2 + \left(\gamma_j^+\right)^2\right)}} \quad -- (4.1)$$

4.2. Calculation of relative closeness:

Now we propose the direction indicator $\xi\left(0 \le \xi \le 1\right)$ to convert the direction closeness and magnitude closeness into relative closeness $\rho_i \left(i = 1, 2, ...n\right)$. If the decision matrix gives more interest on direction, then we provide bigger value to ξ. Otherwise, smaller value to ξ is provided if the magnitude is more important to the decision matrix.

Therefore, the relative closeness for selecting the best alternative is given as follows:

$$\rho_i = \xi\, Cos\left(G_i, Z^*\right) + \left(1-\xi\right)\mathrm{Pr}oj\left(G_i\right)_{Z^*} \quad ------------------------ \quad (4.2)$$

Where $$\mathrm{Pr}oj\left(G_i\right)_{Z^*} = \frac{\sum_{j=1}^{n}\left(T_{ij}^-\alpha_j^- + T_{ij}^+\alpha_j^+ + I_{ij}^-\beta_j^- + I_{ij}^+\beta_j^+ + F_{ij}^-\gamma_j^- + F_{ij}^+\gamma_j^+\right)}{\sqrt{\sum_{j=1}^{n}\left(\left(\alpha_j^-\right)^2 + \left(\alpha_j^+\right)^2 + \left(\beta_j^-\right)^2 + \left(\beta_j^+\right)^2 + \left(\gamma_j^-\right)^2 + \left(\gamma_j^+\right)^2\right)}} \quad ---- (4.3)$$

The bigger value of $\rho_i \left(i = 1, 2, ...n\right)$ gives the better alternative.

4.3. Algorithm 2:

An algorithm for MADM problems with neutrosophic vague information based on neutrosophic vague angle cosine and weighted projection method is provided in the following steps:

Step 1: Transform the linguistic decision matrix into neutrosophic vague decision matrix.

Step 2: We assume the weights $w_j = \left(j = 1, 2, ..., n\right)$ to the attributes $A_j = \left(j = 1, 2, ..., n\right)$ such that $\sum_{j=1}^{n} w_j = 1, w_j \ge 0, j = 1, 2, ..., n$.

Step 3: Calculate the virtual neutrosophic vague ideal solution using the equation 3.1.

Step 4: Determine the neutrosophic vague angle cosine $Cos\left(G_i, Z^*\right)$ using the equation 4.1.

Step 5: Find the neutrosophic vague projection $\mathrm{Pr}oj\left(G_i\right)_{Z^*}$ using the equation 4.3.

Step 6: Calculate the relative closeness $\rho_i \left(i = 1, 2, ...n\right)$ by using the equation 4.2.

Step 7: Rank the alternatives according to the decreasing order of the relative closeness $\rho_i \left(i = 1, 2, ...n\right)$ and choose the most suitable alternative.

V. NUMERICAL EXAMPLE

In this section, we choose the problem of weaver selection on a textile company where the information about the attributes is expressed by linguistic variables. We consider a textile company who wants to recruit two most eminent weavers from a panel of three weavers G_1, G_2, G_3. Seven main attributes for weaver selection are: Skill(A_1), Previous experience(A_2), Honesty(A_3), Physical fitness(A_4), Locality of the weaver(A_5), Personality(A_6), Economic condition of the weaver(A_7). The textile company hires an expert to choose the eminent weaver based on the seven attributes. The expert gives the evaluation information in terms of linguistic variables which is given in table 1.

Table 1: Linguistic decision matrix

	A_1	A_2	A_3	A_4	A_5	A_6	A_7
G_1	VG	VG	G	VG	G	M	M
G_2	G	MG	VG	G	MG	G	ML
G_3	VG	G	MG	G	VG	M	MG

5.1. Conversion of Linguistic Variables into Neutrosophic Vague Set:

A variable whose values can be represented in terms of words or sentences in a natural language is said to be linguistic variable. The linguistic variables can be transformed into neutrosophic vague sets as given in the table 2.

Table 2: Linguistic Variables into Neutrosophic Vague Sets

Linguistic Variables	Neutrosophic Vague Sets
Extreme Good (EG)	$\langle [0.95,1];[0.15,0.2];[0,0.05] \rangle$
Very Good (VG)	$\langle [0.8,0.95];[0.05,0.15];[0.05,0.2] \rangle$
Good (G)	$\langle [0.7,0.8];[0.1,0.2];[0.2,0.3] \rangle$
Medium Good (MG)	$\langle [0.6,0.7];[0.05,0.1];[0.3,0.4] \rangle$
Medium(M)	$\langle [0.45,0.6];[0.1,0.25];[0.4,0.55] \rangle$
Medium Low (ML)	$\langle [0.3,0.45];[0.1,0.2];[0.55,0.7] \rangle$
Low (L)	$\langle [0.2,0.3];[0.1,0.2];[0.7,0.8] \rangle$
Very Low (VL)	$\langle [0.1,0.2];[0.3,0.4];[0.8,0.9] \rangle$
Extreme Low (EL)	$\langle [0,0.1];[0.15,0.25];[0.9,1] \rangle$

5.2. Method 1: Neutrosophic Vague Weighted Projection Method:

Here we use the neutrosophic vague weighted projection method, that is algorithm 1 to select the eminent weaver from the panel of three weavers.

Step 1: Transform the linguistic decision matrix into neutrosophic vague decision matrix. The resulted neutrosophic vague decision matrix is given below.

B=

$$\begin{bmatrix} \langle [0.8,0.95];[0.05,0.15];[0.05,0.2] \rangle & \langle [0.8,0.95];[0.05,0.15];[0.05,0.2] \rangle & \langle [0.7,0.8];[0.1,0.2];[0.2,0.3] \rangle & \langle [0.8,0.95];[0.05,0.15];[0.05,0.2] \rangle & \langle [0.7,0.8];[0.1,0.2];[0.2,0.3] \rangle & \langle [0.45,0.6];[0.1,0.25];[0.4,0.55] \rangle & \langle [0.45,0.6];[0.1,0.25];[0.4,0.55] \rangle \\ \langle [0.7,0.8];[0.1,0.2];[0.2,0.3] \rangle & \langle [0.6,0.7];[0.05,0.1];[0.3,0.4] \rangle & \langle [0.8,0.95];[0.05,0.15];[0.05,0.2] \rangle & \langle [0.7,0.8];[0.1,0.2];[0.2,0.3] \rangle & \langle [0.6,0.7];[0.05,0.1];[0.3,0.4] \rangle & \langle [0.45,0.6];[0.1,0.25];[0.4,0.55] \rangle & \langle [0.3,0.45];[0.1,0.2];[0.55,0.7] \rangle \\ \langle [0.8,0.95];[0.05,0.15];[0.05,0.2] \rangle & \langle [0.7,0.8];[0.1,0.2];[0.2,0.3] \rangle & \langle [0.6,0.7];[0.05,0.1];[0.3,0.4] \rangle & \langle [0.7,0.8];[0.1,0.2];[0.2,0.3] \rangle & \langle [0.8,0.95];[0.05,0.15];[0.05,0.2] \rangle & \langle [0.7,0.8];[0.1,0.2];[0.2,0.3] \rangle & \langle [0.6,0.7];[0.05,0.1];[0.3,0.4] \rangle \end{bmatrix}$$

Step 2: We assume the weights $w_j = (j = 1, 2, ..., n)$ to the attributes $A_j = (j = 1, 2, ..., n)$ such that $\sum_{j=1}^{n} w_j = 1, w_j \geq 0,\ j = 1, 2, ..., n$. The weights of the attributes are given below.

$w_1 = 0.225$, $w_2 = 0.225$, $w_3 = 0.165$, $w_4 = 0.165$, $w_5 = 0.088$, $w_6 = 0.061$, $w_7 = 0.071$

Step 3: We calculate the virtual neutrosophic vague ideal solution using the equation3.1, the values are given below.

$$\mu_1^* = \langle [0.8,0.95];[0.05,0.15];[0.05,0.2] \rangle$$

$$\mu_2^* = \langle [0.8,0.95];[0.05,0.1];[0.05,0.2] \rangle$$

$$\mu_3^* = \langle [0.8,0.95];[0.05,0.1];[0.05,0.2] \rangle$$

$$\mu_4^* = \langle [0.8,0.95];[0.05,0.15];[0.05,0.2] \rangle$$

$$\mu_5^* = \langle [0.8,0.95];[0.05,0.1];[0.05,0.2] \rangle$$

$$\mu_6^* = \langle [0.7,0.8];[0.1,0.2];[0.2,0.3] \rangle$$

$$\mu_7^* = \langle [0.6,0.7];[0.05,0.1];[0.3,0.4] \rangle$$

Step 4: The neutrosophic vague weighted projection $\mathrm{Pr}oj_{\,w}(G_i)_{Z^*}$ of the alternatives $G_i\,(i=1,2,3)$ are calculated by using the equation 3.2 are as follows:

$$\mathrm{Pr}oj_{\,w}(G_1)_{Z^*} = 0.5068;\ \mathrm{Pr}oj_{\,w}(G_2)_{Z^*} = 0.4547;\ \mathrm{Pr}oj_{\,w}(G_3)_{Z^*} = 0.4753\,.$$

Step 5: We rank the alternatives $G_i\,(i=1,2,3)$ that is weavers according to descending order based on $\mathrm{Pr}oj_{\,w}(G_i)_{Z^*}\,(i=1,2,3)$. Here, we observe that

$$\mathrm{Pr}oj_{\,w}(G_1)_{Z^*} > \mathrm{Pr}oj_{\,w}(G_3)_{Z^*} > \mathrm{Pr}oj_{\,w}(G_2)_{Z^*}$$

Hence, G_1, G_3 are the most eminent alternatives for the textile company.

5.3. Method 2: Neutrosophic Vague Angle Cosine and Projection Method:

Here we use the neutrosophic vague angle cosine and projection method, that is algorithm 2 to select the eminent weaver from the panel of three weavers.

Step 1: Same as Step 1 of Method 1.

Step 2: Same as Step 2 of Method 1.

Step 3: Same as Step 3 of Method 1.

Step 4: The neutrosophic vague angle cosine between the alternatives $G_i\,(i=1,2,3)$ and the ideal solution Z^* is calculated using equation 4.1 as follows.

$$Cos(G_1, Z^*) = 0.9676;\ Cos(G_2, Z^*) = 0.9580;\ Cos(G_3, Z^*) = 0.9733\ .$$

Step 5: The neutrosophic vague projection measure between the alternatives $G_i\,(i=1,2,3)$ and the ideal solution Z^* is calculated using equation 4.3 as follows.

$$\mathrm{Pr}oj\,(G_1)_{Z^*} = 3.0051;\ \mathrm{Pr}oj\,(G_2)_{Z^*} = 2.8580;\ \mathrm{Pr}oj\,(G_3)_{Z^*} = 2.9463\,.$$

Step 6: Combining neutrosophic vague angle cosine and projection measure with direction indicator $\xi = 0.5$, the relative closeness $\rho_i\,(i=1,2,3)$ is obtained using the equation 4.2.

$$\rho_1 = 1.9864;\ \rho_2 = 1.9080;\ \rho_3 = 1.9598.$$

Step 7: Rank the alternatives $G_i\left(i=1,2,3\right)$, that is weavers according to descending order based on $\rho_i\left(i=1,2,3\right)$ we have,

$$\rho_1 > \rho_3 > \rho_2$$

Hence, G_1, G_3 are the most eminent alternatives for the textile company.

Also, if we take different values for direction indicators, the ranking order of the alternatives are obtained as shown in the table 3.

Table 3: Ranking based on different direction indicators

	$\xi=0$		$\xi=0.25$		$\xi=0.50$		$\xi=0.75$	
	ρ_i	**Ranking**	ρ_i	**Ranking**	ρ_i	**Ranking**	ρ_i	**Ranking**
G_1	3.0051	1	2.4957	1	1.9864	1	1.4770	1
G_2	2.8580	3	2.3830	3	1.9080	3	1.4330	3
G_3	2.9463	2	2.4531	2	1.9598	2	1.4666	2

VI. CONCLUSION

Thus, in this paper we have developed two different methods such as neutrosophic vague weighted projection method and neutrosophic vague angle cosine and projection method to solve neutrosophic vague MADM problem. By using this in a numerical example in which a textile company which wants to select two eminent weavers among a panel of three weavers by considering seven attributes. So in both the methods we conclude the G_1, G_3 are the most eminent alternatives for the textile company.

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