Elastic Buckling of Aluminium (Al-2024-T3) Plate With Cut-out Using Numerical Approach

*P.V.Gopalkrishna, K.Kishore, Amit Yadav
Department of Mechanical Engineering, Vasavi College of Engineering, Hyderabad, India

Abstract

In aircraft structures, cutouts are commonly found as access ports for mechanical and electrical systems, to reduce weight of structures, maintenance activities. The thin-walled structure become unstable and starts to buckle if they subjected to a load greater than their buckling capability. Moreover, these discontinuities can effect on their stability. The main objective of the work is to study the instability in panels with and without circular cutout using finite element method under compression and shear loading for various end conditions and aspect ratio. Finite element studies aimed at computing the buckling coefficient and critical buckling load for central circular cut-out. Simply supported and clamped plates for shear loading and simply supported plate for compression loading have been studied. Plates with simply supported and clamped edges in the out-of-plane direction and subjected to uniaxial end compression in their longitudinal direction are considered. Plate aspect ratios, a/b=1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and 5, have been chosen to assess the effect of aspect ratio on the plate buckling load

Keywords: Aluminium Plate, aspect ratio, Buckling, Finite Element Method

1. Introduction

Thin-walled members are the elements of many engineering structures. They become unstable and start to buckle if they subjected under a compressive or shear loads greater than their ultimate buckling load. Moreover, some of these members usually have cut outs due to their applications and these discontinuities can effect on their stability. Many authors [8-15] have studied the problems associated with the elastic stability of thin flat plate elements subjected to loads consisting of in-plane forces or stresses. There are two types of failures associated with a structure namely material failure and form or configuration failure. In the former, the stresses exceed the permissible values which may result in the formation of cracks. In the later case, even though the stresses are within permissible range, the structure is unable to maintain its designed configuration under the external disturbances (or applied loads which could be tensile and/or compressive). The loss of stability due to tensile loads falls in the broad category of material instability, whereas the stability loss under compressive load is usually termed structural or geometrical instability commonly known as buckling. Buckling refers to the loss of stability of a component and is usually independent of material strength. The load at which buckling occurs depends on the stiffness of a component, not upon the strength of its materials.
Analysis and design of flight vehicle structure by E. F. Bruhn [1] presented the method of solving the buckling problem for the plates without cut-outs under shear and compression loading with boundary conditions. The buckling coefficient curve available in this book can be utilized to obtain the buckling load for the flat plate under shear and compressive boundary conditions.
ESDU 71005 [2] presents the method of predicting buckling stress for flat isotropic plates loaded in uniform shear. The elastic buckling stress coefficient curves are plotted against aspect ratio of plates (a/b) for various boundary conditions with combinations of simply-supported and clamped edges. ESDU 72012 [2] presents the method of predicting buckling stress of various edge conditions under uniaxial compression. The elastic buckling stress coefficient curves are plotted against aspect ratio of plates (a/b) for various boundary conditions with combinations of simply-supported and clamped edges.
Mahmoud Shariati [2], studied the buckling behaviour of steel rectangular plates with circular and square cut-outs in elasto-plastic range with various band using FEM. The results concluded that the buckling load of the specimen with circular cut outs is little more than the specimen with the square cut outs with the equal surface area. They conclude that when the buckling phenomenon occurs, the capacity of the load toleration is considerably decreased. The results show that, as loading band increases, the ultimate buckling load also increases.

2. Problem definition

The study of buckling analysis would be carried out in two phases:
The first phase validate the proposed results in comparison with available ESDU and Bruhn results. This covers the available ESDU and Bruhn curves for buckling behaviour of the metallic rectangular plates without cut out in elastic range for plates with different aspect ratios and boundary conditions. Also, this study would include the plates with cut out for general loading scenarios as available in the existing ESDU methods.
The same approach would be followed to establish the method for metallic rectangular plates with cut-outs under various loading and boundary conditions using the numerical technique (FE method) for the cases which are not available in ESDU. These curves will be used to study the effect of cut-outs for bucking analysis and will help to ensure the load carrying capability of structures with cut-outs. As the ESDU methods are based on empirical relation with certain number of configurations and boundary conditions however using FEM, it would be possible to include many configurations and boundary conditions.
An attempt has been made to study the buckling of plates with cut out for aerospace applications. As the accessibility of ESDU is limited, the methods and curves established using the proposed approach for the plate with cut out will be easily accessible for users to perform the buckling analysis. Also, the proposed approach would investigate the applicability of available elements to capture the behaviour of structure. The problem of elastic buckling of a simply supported rectangular plate with and without cutouts subjected to uniaxial compression and shear loads are considered. The plate has length (a), width (b), thickness (t) and a circular cutout with the diameter (d) as shown in Fig. 1

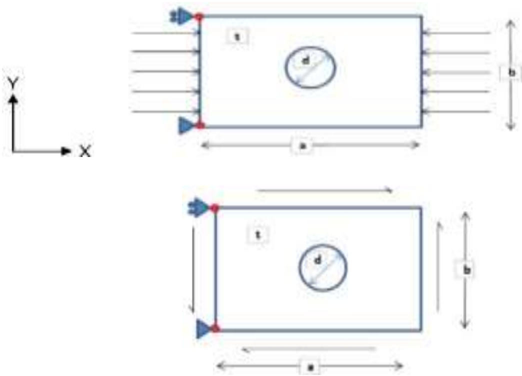


Fig. 1. Plate with circular cutout under compression and shear load

Material used for the plate is Aluminium alloy Al-2024-T3. 2024-T3 clad aluminium sheet is commonly used when a high strength to weight ratio is necessary, such as structural applications in aircraft industry and motor sports because it is nonmagnetic and heat treatable.Copper is the main alloying ingredient in 2024. It is very strong compared to most aluminium alloys and has average machinability. Al-2024-T3 material data used in FE analysis are shown in table 1.

Table 1. property of Al-2024-T3

Ultimate strength,	483 MPa
Yield strength,	345 MPa
Modulus of elasticity	73 GPa
Passion ratio,	0.34
Modulus of rigidity	28 GPa

3. FEA Analysis for plate without cutout

Fundamental equation to determine the compressive buckling stress for flat sheet defined in Bruhn is:

$$\sigma_{cr} = \frac{\pi^2 E k_c}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \quad (3.1)$$

Compressive buckling stress by ESDU is

$$\sigma'_{cr} = k_c E\left(\frac{t}{b}\right)^2 \quad (3.2)$$

Correcting σ'_{cr} for given value of Poisson's ratio give

$$\sigma_{cr} = \sigma'_{cr} \times \frac{0.91}{[1-\nu^2]} \quad (3.3)$$

The value of compressive buckling coefficient k_c for the plate at different edge condition and aspect ratio is taken from the Bruhn and ESDU curves. Critical buckling load at which the plate would buckle is observed from critical buckling stress as given below:

$$F_{cr} = \sigma_{cr} \times t \times b \quad (3.4)$$

Fundamental equation for shear buckling stress is similar as compressive buckling stress, the value of shear buckling coefficient is taken from curves available in Bruhn and ESDU. The equation for shear buckling stress is:

$$\tau_{cr} = \frac{\pi^2 k_s E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \quad (3.5)$$

Buckling coefficient and critical buckling load are found from the fundamental equations. These results are compared with Bruhn and ESDU for various condition for the plate without cutout.

Compressive buckling coefficient taken from Bruhn and ESDU for considered edge conditions and for different aspect ratio given in table 2 to 4.

Table 2. Compressive buckling coefficient for simply supported edge condition under compression load

Plate aspect ratio	**Compressive buckling coefficient from Bruhn**	**Compression buckling coefficient from ESDU**
1.0	4.00	3.58
1.5	4.34	3.97
2.0	4.00	3.59
2.5	4.07	3.82
3.0	4.00	3.60
3.5	4.08	3.81
4.0	4.00	3.63
4.5	4.03	3.63
5.0	4.00	3.63

Table 3. Shear buckling coefficient for simply supported edge condition under shear load

Plate aspect ratio	**Shear buckling coefficient from Bruhn**	**Shear buckling coefficient from ESDU**
1.0	9.60	8.41
1.5	7.10	6.42
2.0	6.38	5.92
2.5	6.00	5.49
3.0	5.80	5.30
3.5	5.75	5.19
4.0	5.72	5.10
4.5	5.67	5.05
5.0	5.50	5.00

Table 4. Shear buckling coefficient for clamped edge condition under shear load

Plate aspect ratio	**Shear buckling coefficient from Bruhn**	**Shear buckling coefficient from ESDU**
1.0	14.60	13.45
1.5	11.54	10.48
2.0	10.40	9.40
2.5	9.75	8.97
3.0	9.62	8.72
3.5	9.50	8.54
4.0	9.44	8.45
4.5	9.44	8.39
5.0	9.50	8.35

The FE analysis is carried out for plate without cut out with simply supported boundary conditions under shear load, and mesh size as stated in table 5 for aspect ratios (a/b=1). The error was calculated and compared with Brhun and ESDU with FEM these details are shown in table 6.

Table 5. Results from FEM method

Elements	Buckling load factor	Critical buckling load from FEM(N)
36	1.1180	330204.60
289	0.9617	285645.69
1600	0.9516	282631.14
2500	0.9497	282069.81
3600	0.9504	282289.59
6400	0.9488	281817.36

Table 6. % error between FEM, ESDU and Bruhn methods

Critical buckling load from Bruhn(N)	Critial buckling load from ESDU(N)	% error between Bruhn and FEM	% error between ESDU and FEM
296941.86	287818.74	11%	15%
296941.86	287818.74	4%	1%
296941.86	287818.74	5%	2%
296941.86	287818.74	5%	2%
296941.86	287818.74	5%	2%
296941.86	287818.74	5%	2%

Error convergence graph and load convergence graph for aspect ratio a/b=1 under shear load for simply supported edge condition are shown in Fig 2 and Fig 3 respectively.

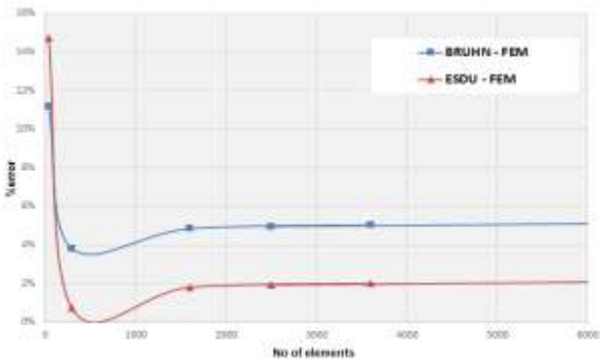


Fig 2: % Deviation between FEM and Bruhn or ESDU method for shear load, simply supported edge condition (a/b=1)

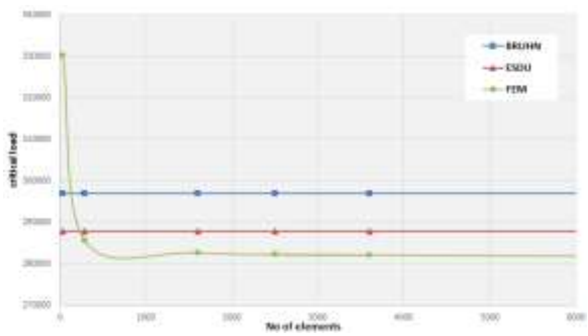
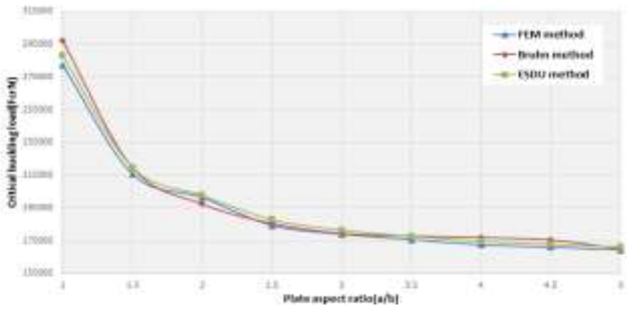


Fig 3: Critical buckling load comparisons FEM, Bruhn and ESDU methods for shear loading simply supported edge condition (a/b=1)

After finalising the mesh size for simply supported and clamped edge condition under shear load, and for simply supported edge condition for compression load, the value of critical load is calculate from the result obtained from FE analysis for aspect ratio a/b= 1 to 5.Validating the results for defined boundary condition under shear and compression load from Bruhn and ESDU method are plotted and are shown in figure 4 to 6.



. Fig 4: Critical buckling load comparison between FEM, ESDU and Bruhn method for simply supported plate under shear load

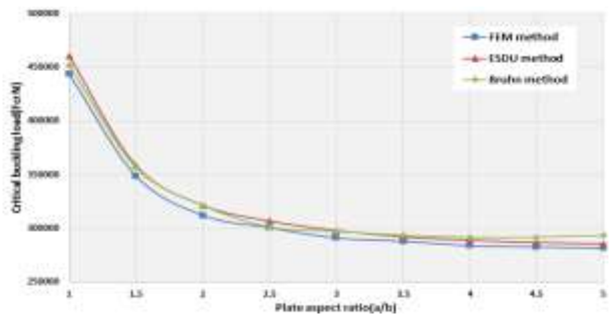


Fig 5: Critical buckling load comparison between FEM, ESDU and Bruhn method for clamped plate under shear load

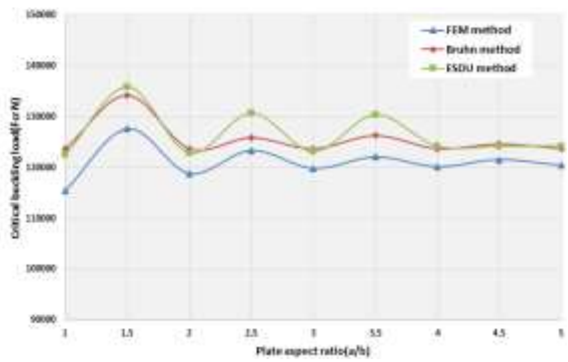


Fig 6: Critical buckling load comparison between FEM, ESDU and Bruhn method for simply supported plate under compression load

By observing the Fig 4 to 6 it can be observed that as the aspect ratio increases the value of critical buckling load for the plate under shear loading is decreasing continuously and there is a large variation in the critical buckling load whereas for the plate under compression loading the variation of critical buckling load is not so large that is the reason why validation graph showing more variation as compared to plate under shear loading.

It is noted from the comparison of ESDU, Bruhn and FE critical buckling loads the percentage of deviation is less than 7%. Hence it can be concluded that the FE results is in good agreement with ESDU and Bruhn critical buckling loads.

4. Results for plate with circular cutout

FE meshed modal of plate with circular cutout is shown in figure 7, a 20mm radius cutout is added on the plate. Critical buckling load obtained from FE analysis is presented in the table 7 to 9 and the variation of critical buckling load with change in aspect ratio are presented in the figure 8 to 10.

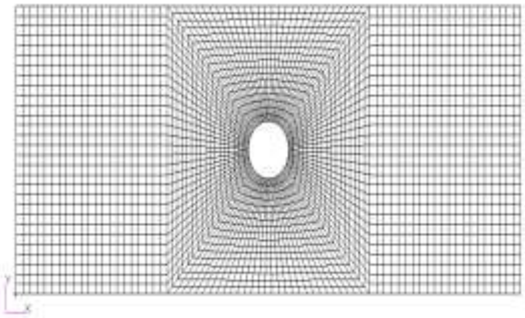


Fig 7: Plate with circular cut out at the centre

4.1 Compression loading

Table 7: FEM results for plate with circular cutout for simply supported plate

Plate aspect ratio	Critical buckling load from FEM for plate with circular cutout	Compressive buckling coefficient from FEM for plate with circular cutout(Bruhn method)	Compressive buckling coefficient from FEM for plate with circular cutout(ESDU method)
1.0	102933.01	3.32	3.00
1.5	129991.18	4.22	3.79
2.0	123519.61	3.99	3.60
2.5	117970.90	3.81	3.44
3.0	115186.73	3.72	3.36
3.5	118749.15	3.83	3.46
4.0	121813.44	3.93	3.55
4.5	118445.28	3.82	3.46
5.0	117544.93	3.80	3.43

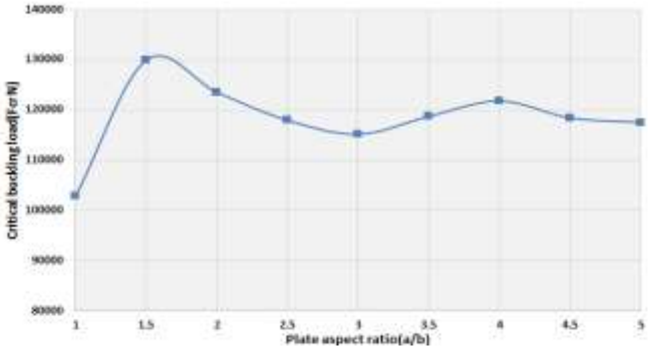


Figure 8: Critical buckling load for plate with circular cutout for simply supported plate.

4.2 Shear loading

Table 8: FEM results for plate with circular cutout for simply supported plate

Plate aspect ratio	Critical buckling load from FEM for plate with circular cutout	Shear buckling coefficient from FEM for plate with circular cutout(Bruhn method)	Shear buckling coefficient from FEM for plate with circular cutout(ESDU method)
1.0	206652.60	6.68	6.03
1.5	172115.89	5.56	5.02
2.0	165292.00	5.34	4.82
2.5	161408.60	5.21	4.71
3.0	157267.42	5.08	4.59
3.5	154348.77	5.00	4.51
4.0	152989.64	4.94	4.47
4.5	152565.69	4.93	4.45
5.0	152402.98	4.92	4.45

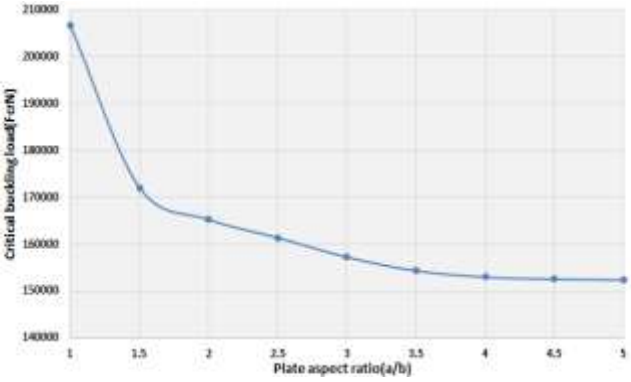


Figure 9: Critical buckling load for plate with circular cutout for simply supported plate

Table 9: FEM results for plate with circular cutout for clamped plate

Plate aspect ratio	Critical buckling load from FEM for plate with circular cutout	Shear buckling coefficient from FEM for plate with circular cutout(Bruhn method)	Shear buckling coefficient from FEM for plate with circular cutout(ESDU method)
1.0	337227.03	10.90	9.85
1.5	279284.03	9.02	8.16
2.0	265893.60	8.59	7.77
2.5	255560.58	8.26	7.47
3.0	251351.90	8.12	7.34
3.5	250417.26	8.09	7.32
4.0	249781.63	8.07	7.29
4.5	249261.89	8.05	7.28
5.0	249092.01	8.05	7.28

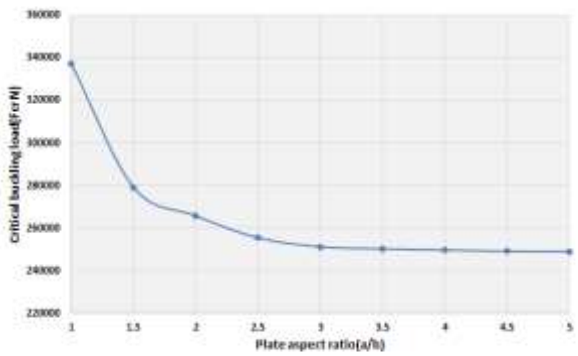


Fig 10: Critical buckling load for plate with circular cutout for clamped plate

5. Conclusions

The first buckling mode of critical elastic buckling from FE analyses for a plate without cut out is in good agreement with the critical load computed from analytical analysis. When the buckling phenomenon occurs, the capacity of the load toleration is considerably decreased. The buckling load of the plate with clamped support is about twice bigger than plates with simply supported. The fundamental FE mode for a plate without cut out is in agreement with analytical buckling mode shapes for various plate aspect ratios.

Minimum mesh density should be at least 4 elements for a square plate and to be used for an Eigen value of FE analysis to match results within 7% deviation with respect to analytical methods for compressive load case.

Minimum mesh density should be at least 16 elements for a square plate to be used for an Eigen value of FE analysis to match results within 7% deviation with respect to analytical methods for shear load case.

Plate tends to buckle out of plane along diagonal tension and perpendicular to the diagonal compression directions for shear load where compressive load case, plate tends to buckle out of plane across the compressive load directions.

The Buckling curve pattern derived from FE approach for a plate without cut out is in agreement with that of ESDU method

The percentage of deviation in ESDU method for a plate without cut out is less as compared to Bruhn method to simulate the simply supported boundary conditions.

Nomenclature

a	: Length of the plate
b	: Width of the plate
a/b	: Aspect ratio
t	: Thickness of the plate
E	: Young's modulus
ν	: Poisson ratio
σ_{cr}	: Compressive buckling stress
τ_{cr}	: Shear buckling stress
F_{cr}	: Critical buckling load

Acknowledgment

The authors are sincerely thankful to the Principal and Management of Vasavi College of Engineering, Hyderabad for extending their support in carrying out this work. The Cooperation extended by Mr. Rajasekhar, lab technician, CAD/CAM lab during analysis is highly appreciated.

References

[1] E.F Bruhn**,** Analysis and design of flight vehicle structures, published and distributed by Tri-State offset company. 1973

[2] Engineering Sciences Data Unit (ESDU) an engineering advisory organization. ESDU 71005, 1995, Buckling of flat plate in shear, ESDU 72012, 1978, Buckling stress of various edge conditions under uniaxial compression.

[3] Y. Faradjian, J. Taheri Kahnamouei, "Experimental and numerical investigation of buckling in rectangular steel plates with groove shaped cutouts", Journal of Zhejiang university (Applied Physics& Engineering), vol 13(6 2012), pp. 469-480.

[4] Mahmoud Shariati and Ali Dadrasi, 2012," Numerical and Experimental Investigation of loading band on buckling of perforated rectangular steel plates". Research journal of recent sciences vol.1 (10), pp.63-71.

[5] Christopher j. Brown and Alan L. Yettram, 1986,"The Elastic Stability of Square Perforated Plates under Combinations of Bending, Shear and Direct Load", Thin walled structures, vol 4, pp.239-246.

[6] Christopher j. Brown and Alan L. Yettram, 1986, "The elastic stability of square perforated plates under bi-axial loading", Computers and structures, vol 22(4), pp.589-594.

[7] Christopher j. Brown and Alan L. Yettram, 1985," The elastic stability of square perforated plates", Computers and structures, vol 21(6), pp. 1267-1272.

[8] Christopher j. Brown, 1990," Elastic buckling of perforated plates subjected to concentrated loads", Computers and structures, vol 36, pp.1103-1109.

[9] R. Narayanan and N.G.V. Der Avanessian, 1984,” Elastic buckling of perforated plates under shear”, thin walled structures, vol 2, pp. 51-73.

[10] El-Sawy, K.M., Nazmy, A.S., 2001. “Effect of aspect ratio on the elastic buckling of uniaxially loaded plates with eccentric holes”, Thin walled structure, vol 39, pp.983-998.

[11] El-Sawy, K.M., Nazmy, A.S., Martini, M.I., 2004, “Elasto plastic buckling of perforated plates under uniaxial compression”, Thin walled structure, vol 42(8), pp.1083-1101.

[12] N.E. Shanmugam, V. Thevendran, Y.H. Tan, 1999, “Design formula for axially compressed perforated plates”, Thin-Walled Structures, vol 34, pp.1–20

[13] Vincenzo Piscopo, 2012, “Classical and refined buckling analysis of plates under shear and uniaxial stresses” international journal of advanced computer science, vol 2, pp.131-138.

[14] Ali Reza Pouladkhan, JalilEmadi and Majid Safamehr, 2011,”Numerical study of buckling of thin plates”, Research gate (World Academy of Science, Engineering and Technology), vol 78, pp.152-157

[15] Muhammad Aslam Bhutto, Abdul Aziz Ansari, Noor Ahmed Memon, 2013, “Prediction of Behaviour of Steel Plate Subjected to Shear”, International Journal of Engineering and Advanced Technology (IJEAT) ISSN, Volume-3, pp.2249 – 8958

[16] AbinPaul, Subin p George, 2014, “ Buckling analysis of wing upper skin panels of a transport Aircraft”, International Journal of Science, Engineering and Technology Research (IJSETR), Vol 3, pp. 2868-2872

[17] Cristopher D.Moen, B.W.Schafer, 2008, “Elastic buckling of thin plates with holes in compression or bending”, Thin-Walled Structures, vol 47, pp.1597–1607

[18] Paik J. K., 2008, “Ultimate strength of perforated steel plates under combined biaxial compression and edge shear loads”, (Thin-Walled Structures), vol 46, pp.207–213,

[19] Christopher j. brown and Alan L. Yettram. , Burnett, M., 1987, “Stability of plates with rectangular holes”, Journals of structural engineering, vol 113(5), pp.1111-1116.

[20] George Gerard and Herbert Becker, NACA Tech note 3781, handbook of structural stability, part 1 buckling of flat plates, July 1957.

[21] Timoshenko, S.P. Gere, J.M., 1961, Theory of elastic stability, McGraw-Hill publication, second edition. New York.