Effect of Autocorrelation on Combined Monitoring Schemes

Dr.Annie Cherian
Department of Statistics
Baselius College,Kottayam, Kerala, India

Abstract- Advanced measurement and data collection technology leads to data correlation. Autocorrelation present in the process significantly affects both the process mean chart and the process variance chart which are designed by conventional methods. Positive autocorrelation result in an increased average false alarm rate for a location chart and decreased false alarm rate for a variance chart. In this paper, assuming an AR(1) model for the process data, effect of autocorrelation on combined monitoring schemes for mean and variance is studied. The performance evaluation is made based on ARLs obtained using simulation.

Keywords – Autocorrelation, ARL, Monitoring Schemes, EWMA

I. INTRODUCTION

In traditional application of control charts the observations from the manufacturing process are usually assumed to be independent and identically distributed. With development of advanced measurements and data collection technology, processes can be sampled at higher rates and the high frequency of sampling leads to data correlation. Also, in continuous flow processes like chemical processes, the data are correlated [1] Many authors have discussed the performance characteristics of standard control charts when applied to correlated observations. A basic conclusion that can be drawn from their studies is that correlation has a significant effect on the properties of the control charts that were investigated. When correlation is present in the data there are serious problems of not detecting the special causes that truly exist and giving false signals when there is no special cause.

For some processes, special causes can result in a simultaneous change in both the mean and the variance. In this case it is more reasonable to combine the mean and variance information on one scheme and look at their behavior jointly. Also, when the exact model of the process observations is not known then it is meaningful to consider model free schemes to monitor the process.

Autocorrelation significantly affects both the process mean chart and the process variance chart. Positive autocorrelation in observations result in negative bias in traditional estimators of the standard deviation. Due to the under estimation of process standard deviation, the control limits for standard control charts for process mean become much narrow than desired. Lu and Reynolds [2] shows that narrow control limits, combined with autocorrelation in the observations plotted, could result in an average false alarm rate much higher than expected. That is, when autocorrelation increase the in-control ARL become much lesser than expected. But in the case of variance charts the in – control ARL increase when there is positive correlation and become less sensitive to increase in process variability.

Performances of four simultaneous monitoring schemes applied to correlated observations are studied in the following session.

II. PERFORMANCE EVALUATION OF SIMULTANEOUS MONITORING SCHEMES

Performances of four simultaneous monitoring schemes are compared when they are applied directly to the autocorrelated observations. The first scheme is the combination of the traditional Individuals chart and Moving Range chart. The second is the combination scheme of EWMA for observations ($EWMA_y$) and EWMA for

squared deviations of the observations ($EWMA_s$) which is proposed by Reynolds and Stambous [3]. The third chart is the Generalized Likelihood Ratio (GLR) Chart proposed by Hawkins and Deng [4]
We assume that individual observations are sampled at each time point. Let X_t represents the observation taken at time point t. Let $\mu_0$and σ_0 denote the in-control values for the process mean and variance μ and σ respectively. In this study it is assumed that they are estimated without error.

2.1*The Shewhart Individual Chart and Moving Range chart–*

The Shewhart Individual Chart plots X_t at time t and a signal is generated if it falls outside the control limits,

$$\mu_0 \pm k\sigma_0$$

where k is determined to get a desired in- control ARL.
The Shewhart Moving Range chart is based on the statistic,

$$R_t = |X_t - X_{t-1}| \qquad t = 1, 2, \ldots\ldots \qquad (1)$$

R_t is plotted at time point t and a signal is given when it exceeds the upper control limit $h_R + \sigma_0$ where h_R is a constant determined to get a specified in- control ARL.

2.2 $EWMA_y$ and $EWMA_s$ Chart

The $EWMA_y$ chart for detecting changes in μ is based on the control statistic,

$$Y_t = (1-\lambda)Y_{t-1} + \lambda X_t \qquad t = 1, 2, \ldots\ldots \qquad (2)$$

whereλ is a smoothing constant satisfying $0 < \lambda \leq 1$. When λ is small EWMA is effective in detecting small shifts and for large values of λ EWMA is effective in detecting large shifts. It can be noted that when $\lambda = 1$ EWMA is equivalent to Shewhart Individuals statistic. A signal is given if the $EWMA_y$ statistic Y_t falls outside the control limits

$$\mu_0 \pm h_y\sigma_0\sqrt{\lambda/(2-\lambda)} \qquad (3)$$

where$\sigma_0\sqrt{\lambda/(2-\lambda)}$ is the in- control standard deviation of Y_t. $Y_t$is initialised as $Y_0 = 0$.

The one sided $EWMA_s$ chart for detecting an increase in σ is based on the control statistic,

$$S_t = (1-\lambda)\max\{S_{t-1}, {\sigma_0}^2\} + \lambda(X_t - \mu_0)^2 \qquad t = 1, 2, \ldots\ldots \qquad (4)$$

The chart is defined so that if S_{t-1} is below ${\sigma_0}^2$ then there is a reset back to ${\sigma_0}^2$ before computing S_t. A signal is given if S_t falls outside the upper control limit,

$${\sigma_0}^2 + h_s{\sigma_0}^2\sqrt{2\lambda/(2-\lambda)} \qquad (5)$$

When the process is in-control, $\sqrt{2}{\sigma_0}^2$ is the standard deviation of $(X_t - \mu_0)^2$. The smoothing constant λ is taken as 0.2 for both the EWMA statistics [5]. S_0 is initialized as $S_0 = {\sigma_0}^2$.

2.3 The GLR Chart

The GLR chart proposed by Hawkins and Deng plots the statistic,

$$G = \left(\frac{\sqrt{n}\,(\bar{x} - \mu_0)}{\sigma_0}\right)^2 + \frac{(n-1)S^2}{\sigma_0^2} - n\log\frac{(n-1)S^2}{\sigma_0^2} + n\log n - n \qquad (6)$$

where n is the size of the sample, $\bar{x}$ is the sample mean and S^2 is the sample variance. We consider a sample of size 2 at each time point t. The chart signals if the statistic G falls outside the upper control limit h_g. h_g is a constant to be fixed to get desired in control ARL.

III. TIME SERIES MODELS FOR THE AUTOCORRELATED DATA

The Autoregressive Integrated Moving Average (ARIMA) model of order (p,d,q) of Box and Jenkins [6] is generally used for modeling the autocorrelated data. Two ARIMA models which are often used in SPC for modeling the correlated data are (1) the autoregressive model of order 1 [AR(1) model] and (2) the integrated moving average model of order (1,1) [IMA(1,1) model]. In the present study AR(1) model assumed for the data.

AR(1) model

An AR(1) model is given by

$$X_t - \mu = \varphi_t(X_{t-1} - \mu) + \varepsilon_t \qquad (7)$$

where φ is the autoregressive parameter, |φ |<1 and ε_t is a sequence of independent and identically distributed normal random errors with mean 0 and variance $\sigma_\varepsilon{}^2$. This study considers only positive correlation and therefore, 0<φ ≤1.

$$E\,(X_t) = \mu \qquad (8)$$

$$Var\,(X_t) = \frac{\sigma_\varepsilon{}^2}{(1-\varphi^2)^2} \qquad (9)$$

and the lag k autocorrelation $\rho_k = \varphi^k$, k=1,2,3.... . When $\varphi = 0$ then $\{X_t\}$ is an i.i.d. sequence.

IV. DESCRIPTION OF THE SIMULATION

In this simulation study, the in-control ARL of all the three schemes were made equal to 185. The control chart parameters of all the charts are selected to get the desired in control ARL of 185 for the combination charting. Without loss of generality we assume that the process mean μ = 0. ε_t s are assumed to be Normal with mean zero and variance $\sigma_\varepsilon{}^2 = 1$. The autoregressive parameter φ is allowed to vary from 0 to .95. The process is allowed to be in control during the initial period and a shift is introduced at the 51st observation. We have considered shifts in both mean and variance. Process mean is allowed to shift from 0 to $0.5\sigma_\varepsilon{}^2$, $1\sigma_\varepsilon{}^2$, $1.5\sigma_\varepsilon{}^2$, $2\sigma_\varepsilon{}^2$ and $3\sigma_\varepsilon{}^2$. The variance of ε_t is allowed to vary from $1\sigma_\varepsilon{}^2$ to $3\sigma_\varepsilon{}^2$.

The control limits for the individuals chart are taken as ± $3\sigma_\varepsilon{}^2$ and the upper control limit of the one sided moving range chart is $4.01\sigma_\varepsilon$. For the EWMA chart, the parameter values are selected according to Reynolds and Stambous. The value of λ is chosen as 0.20 for both the EWMA charts and control limits are computed according (5.3) and (5.5). While selecting the control limits for the GLR chart, we followed the guide lines given by Hawkins and Deng . The control limit h_g for the GLR chart is obtained as 20.727.

Comparison of the performance of the charts under consideration is based on the simulation study conducted according to the following steps.

- Step1: N(0,1) random numbers ε_t are generated using R.
- Step2: The observations form an AR(1) model are obtained using equations (7) for φ ranging from 0 to 0.95
- Step3: The first 50 ε_ts are discarded to allow the time series to stabilize.
- Step4: The observations are monitored using the three monitoring schemes and the run length of each control schemes are recorded.
- Step5: Steps (1) to (4) are repeated 1,00,000 times and the ARLs are calculated.

The in-control ARLs are obtained in Zero state (by simulating the process from start-up) and the out-of-control ARLs are obtained in steady state. (Simulating a process that has been running in an in-control state for certain time).

V. OBSERVATIONS

The behavior of the three schemes applied to an AR (1) process with $\varphi = .25$ is shown in figure 1 . The process is allowed to be in control for the first 50 time points and then a step shift of size 2 $\sigma_\varepsilon{}^2$ is introduced in the 51st time point. It can be observed that all the three schemes are sensitive to the shift. In the Individuals and Moving Range combination moving range is not sensitive to shift in the process mean.

 ISSN No : 1006-7930

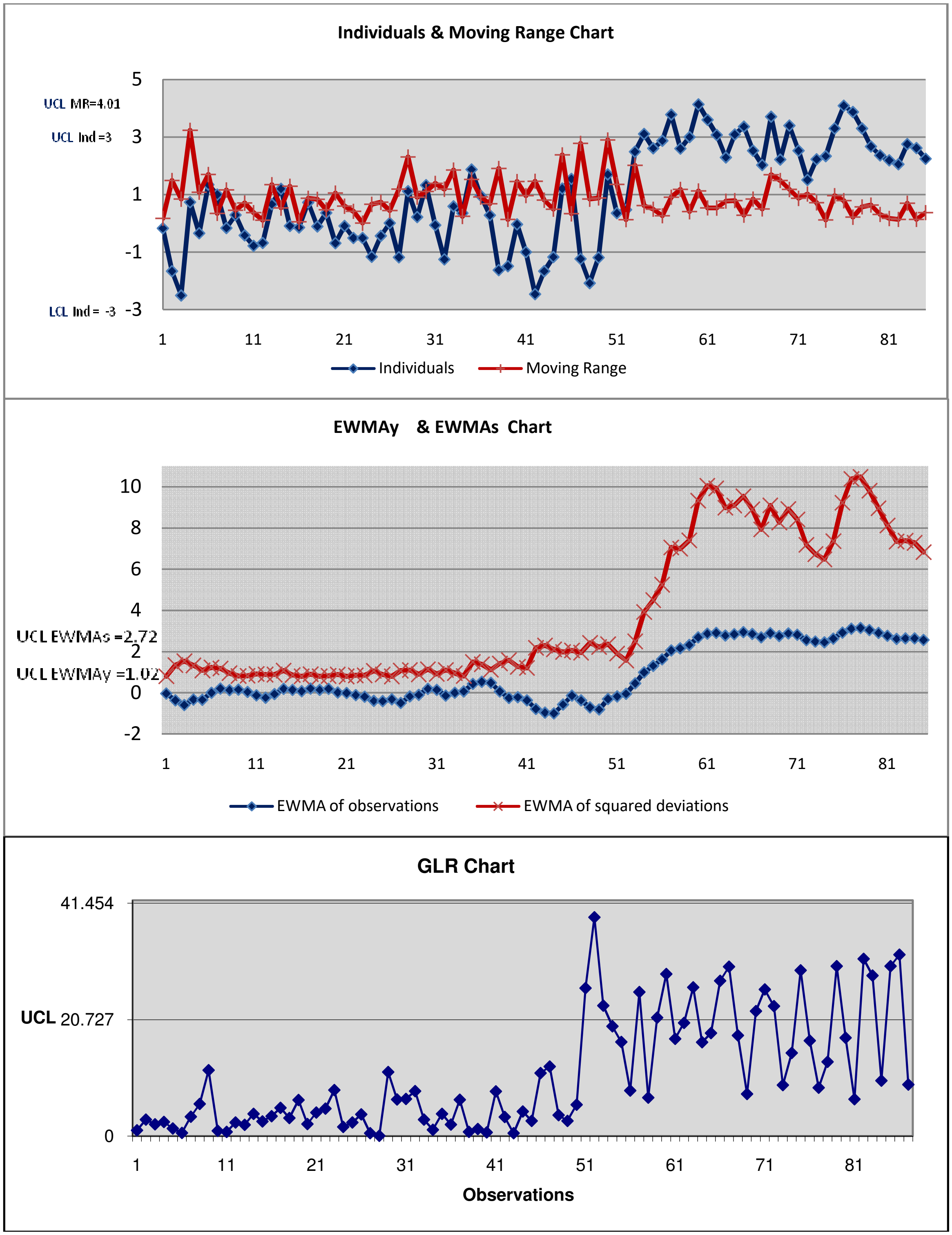

Figure 1 : Plots of the three control schemes when step shift of size $2\sigma^2$ in the mean is introduced at the 51st time point of an AR(1) process with $\varphi = 0.25$

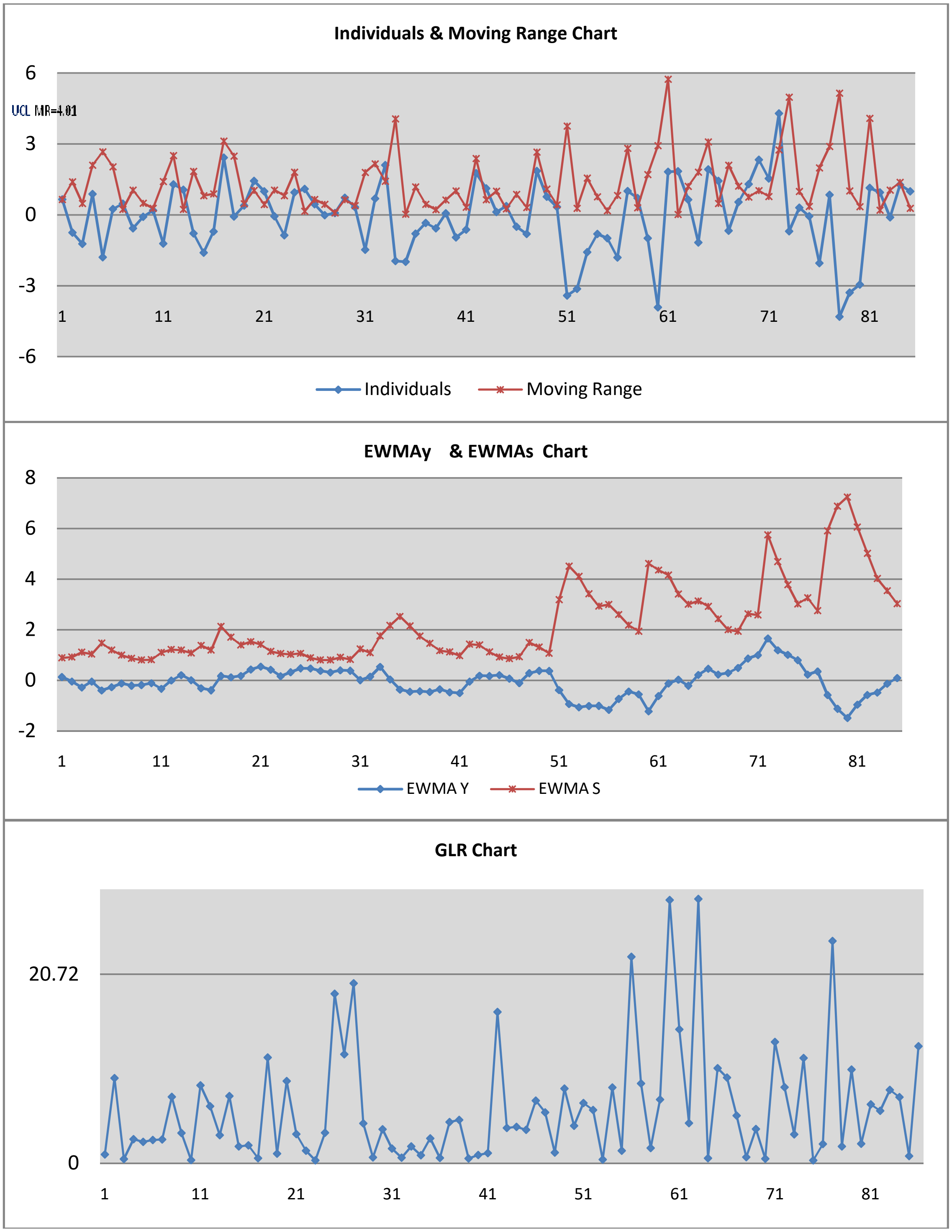

Figure 2 : Plots of the three control schemes when variance is increased to 2 σ^2 at the 51st time point of an AR(1) process with $\varphi = 0.25$

Figure 2 shows the behaviour of the schemes applied an AR (1) process with $\varphi = .25$ when the variance is increased. The process is in the state of control till 50^{th} time point and then the variance is doubled ($2\,\sigma_{\varepsilon}^{2}$) at the 51^{st} time point. All the charts are sensitive to an increase in variance.

			λ				
ϕ	Control Scheme	δ	1	1.25	1.5	1.75	2
0	Individuals & MR	0.0	185.40	37.45	15.10	8.80	6.02
	$EWMA_y$& $EWMA_s$		185.76	31.73	13.10	7.93	5.57
	GLR		183.72	154.57	85.15	37.45	18.17
	Individuals & MR	0.5	111.98	28.94	13.23	8.01	5.73
	$EWMA_y$& $EWMA_s$		39.45	18.14	10.49	7.01	5.19
	GLR		147.04	107.85	57.23	27.80	15.06
	Individuals & MR	1.0	42.10	16.28	9.12	6.30	4.86
	$EWMA_y$& $EWMA_s$		10.53	8.49	6.51	5.27	4.37
	GLR		70.37	44.83	25.20	14.89	9.72
	Individuals & MR	2.0	6.37	4.70	3.87	3.42	3.07
	$EWMA_y$& $EWMA_s$		3.31	3.16	3.04	2.94	2.77
	GLR		9.16	6.89	5.52	4.53	3.79
	Individuals & MR	3.0	2.00	1.99	1.99	2.02	1.97
	$EWMA_y$& $EWMA_s$		1.75	1.79	1.83	1.86	1.89
	GLR		1.98	2.03	2.04	1.97	1.94
0.25	Individuals & MR	0.0	221.45	40.66	15.80	9.00	6.08
	$EWMA_y$& $EWMA_s$		89.79	24.34	11.77	7.43	5.47
	GLR		134.93	90.47	44.74	22.30	12.98
	Individuals & MR	0.5	81.97	25.65	12.73	7.92	5.73
	$EWMA_y$& $EWMA_s$		20.79	13.35	8.98	6.55	5.06
	GLR		81.12	48.15	26.28	16.02	10.30
	Individuals & MR	1.0	21.50	11.60	7.74	5.79	4.61
	$EWMA_y$& $EWMA_s$		7.11	6.36	5.60	4.77	4.08
	GLR		24.54	16.10	11.22	8.25	6.46
	Individuals & MR	2.0	3.77	3.44	3.20	3.00	2.83
	$EWMA_y$& $EWMA_s$		2.75	2.74	2.72	2.66	2.59
	GLR		3.32	3.19	3.00	2.88	2.70
	Individuals & MR	3.0	1.69	1.75	1.80	1.85	1.85
	$EWMA_y$& $EWMA_s$		1.64	1.68	1.73	1.76	1.79
	GLR		1.39	1.47	1.53	1.56	1.61

Table 1: ARLs of the three schemes under comparison when there is an increase of δσ in the mean level and when the process variance increase to $\lambda\sigma^2$ for autoregressive parameter φ ranging from 0 to 0.95.

ϕ	Control Scheme	δ	λ = 1	1.25	1.5	1.75	2
0.5	Individuals & MR	0.0	116.90	29.71	13.59	8.27	5.81
	$EWMA_y$& $EWMA_s$		32.91	15.29	9.34	6.46	5.09
	GLR		74.37	38.68	19.72	11.63	7.84
	Individuals & MR	0.5	32.38	16.46	10.19	7.10	5.30
	$EWMA_y$& $EWMA_s$		11.64	9.43	7.26	5.71	4.68
	GLR		26.42	16.92	11.69	8.43	6.39
	Individuals & MR	1.0	9.39	7.16	6.00	5.04	4.32
	$EWMA_y$& $EWMA_s$		5.13	5.08	4.70	4.29	3.82
	GLR		7.06	6.17	5.41	4.80	4.21
	Individuals & MR	2.0	2.70	2.74	2.71	2.65	2.60
	$EWMA_y$& $EWMA_s$		2.41	2.43	2.46	2.49	2.47
	GLR		2.01	2.06	2.10	2.12	2.12
	Individuals & MR	3.0	1.58	1.64	1.70	1.73	1.78
	$EWMA_y$& $EWMA_s$		1.58	1.62	1.66	1.70	1.75
	GLR		1.24	1.29	1.33	1.38	1.41
0.75	Individuals & MR	0.0	37.19	16.69	9.79	6.85	5.16
	$EWMA_y$& $EWMA_s$		14.96	9.63	7.03	5.60	4.53
	GLR		23.74	13.29	8.67	6.32	4.96
	Individuals & MR	0.5	12.52	9.73	7.54	5.82	4.75
	$EWMA_y$& $EWMA_s$		7.58	6.80	5.79	4.88	4.18
	GLR		7.82	6.81	5.76	4.94	4.29
	Individuals & MR	1.0	5.18	5.00	4.71	4.26	3.85
	$EWMA_y$& $EWMA_s$		4.10	4.14	4.07	3.79	3.55
	GLR		3.31	3.34	3.34	3.25	3.04
	Individuals & MR	2.0	2.30	2.35	2.40	2.42	2.41
	$EWMA_y$& $EWMA_s$		2.23	2.26	2.29	2.33	2.32
	GLR		1.69	1.71	1.73	1.77	1.81
	Individuals & MR	3.0	1.53	1.57	1.62	1.67	1.74
	$EWMA_y$& $EWMA_s$		1.56	1.59	1.63	1.66	1.71
	GLR		1.17	1.21	1.24	1.28	1.32
0.95	Individuals & MR	0.0	15.61	10.28	7.30	5.69	4.59
	$EWMA_y$& $EWMA_s$		9.32	7.12	5.85	4.78	4.08
	GLR		8.95	6.66	5.25	4.32	3.68
	Individuals & MR	0.5	7.68	6.96	5.91	4.98	4.30
	$EWMA_y$& $EWMA_s$		6.05	5.57	4.94	4.35	3.82
	GLR		4.47	4.31	3.94	3.63	3.29
	Individuals & MR	1.0	3.98	4.08	4.01	3.80	3.57
	$EWMA_y$& $EWMA_s$		3.62	3.71	3.66	3.50	3.26
	GLR		2.55	2.64	2.70	2.68	2.59
	Individuals & MR	2.0	2.15	2.19	2.25	2.29	2.31
	$EWMA_y$& $EWMA_s$		2.13	2.15	2.21	2.25	2.24
	GLR		1.58	1.58	1.62	1.64	1.69
	Individuals & MR	3.0	1.52	1.55	1.59	1.63	1.68
	$EWMA_y$& $EWMA_s$		1.55	1.57	1.61	1.65	1.69
	GLR		1.12	1.17	1.20	1.24	1.29

Table 1 continued ….

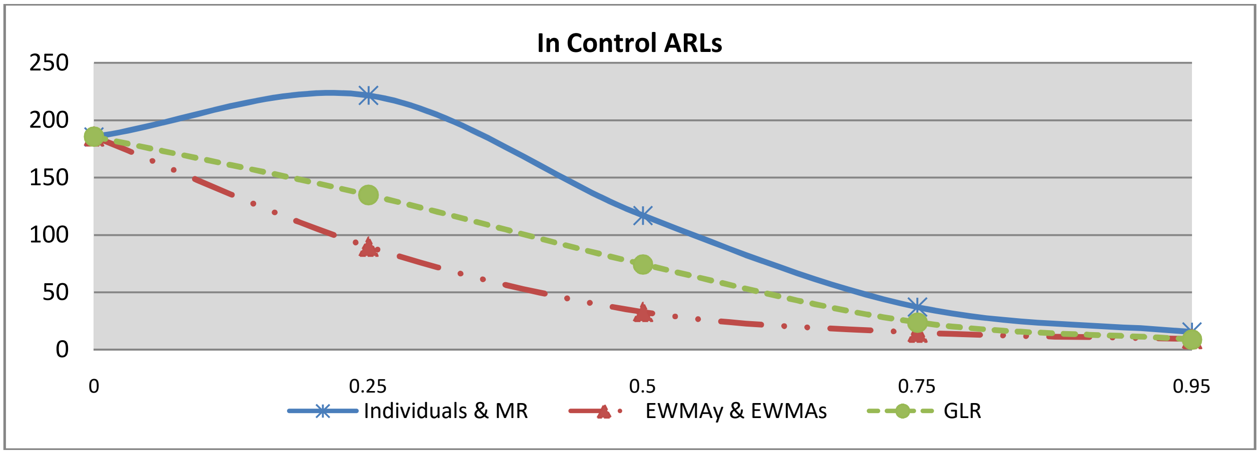

Figure 3: The ARL in the in-control state for the three control schemes as the autoregressive parameter φ moves from 0 to .95

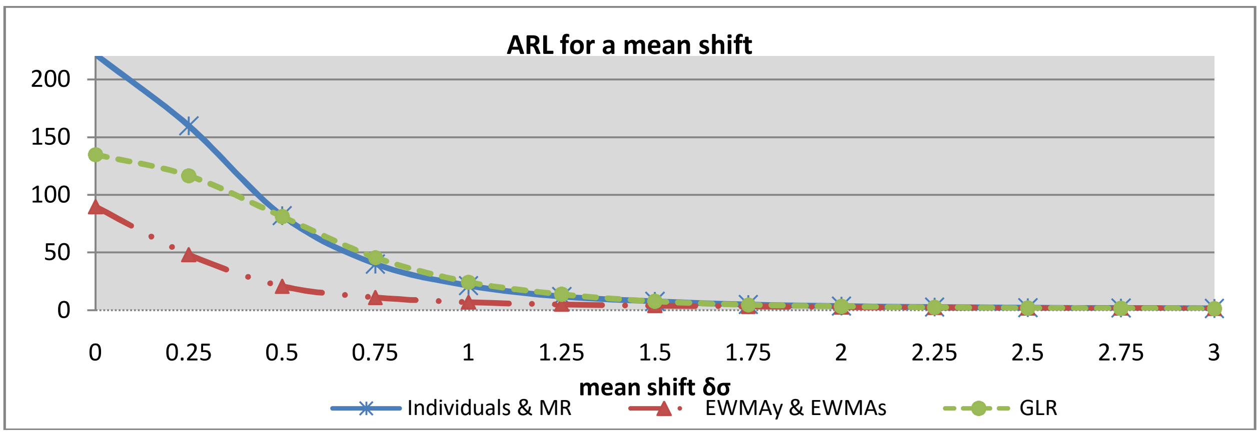

Figure 4: The ARL the three control schemes applied to an AR(1) process with $\varphi = .25$ when there is a shift of magnitude $\delta\sigma$ in the process mean level.

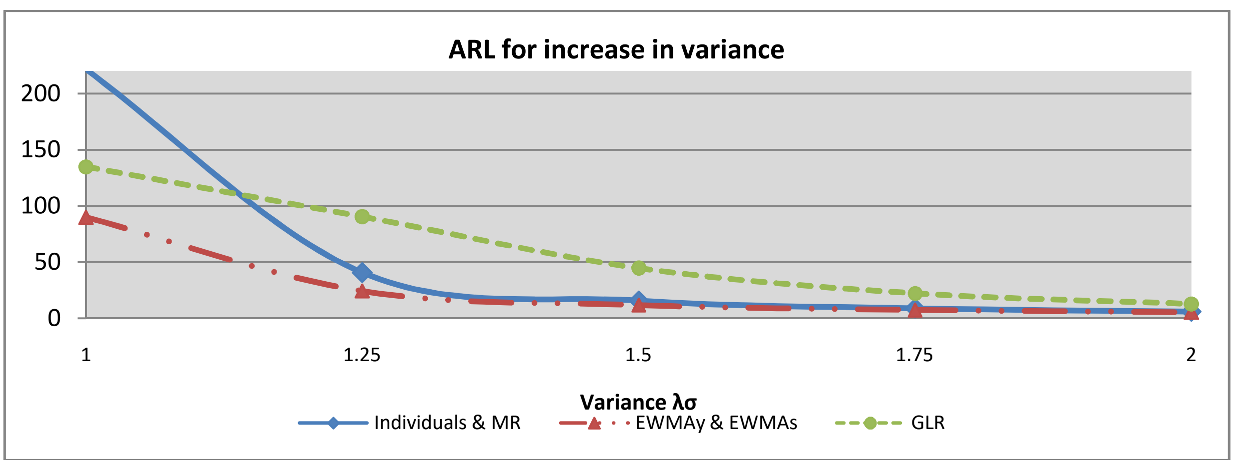

Figure 5: The ARL of the three control schemes applied to an AR(1) process with $\varphi = .25$ when process variance increase to $\lambda\sigma$.

VI. CONCLUSION

As in the case of process mean chart and process variance chart, combined control schemes are also very seriously affected by autocorrelation. Figure 3 shows the in control ARL values for different values of the autoregressive parameter φ. It can be observed that for all the three schemes ARL is much lesser than the desired value when there is strong autocorrelation. For Individuals chart & moving range chart combination in control ARL increase for $\varphi = .25$ and then decrease. This is due to the fact that for positive correlation ARL_0 (in-control ARL) reduce for individuals but ARL_0 increase for moving range chart and therefore their combination charting is giving slightly bigger ARL_0.

In Table 1 simulated ARL values for increase in mean and variance are given for $\varphi = 0, .25, .5, .75, and\ .95$. $\varphi = 0$ corresponds to i.i.d observations. From the table we see that the EWMA chart combination is much affected when there is correlation. In Individuals and Moving Range combination ARL_0 is high but ARL for 1σ or higher levels of shift in mean is equivalent or lesser than that of other schemes. GLR is performing better than EWMA combination with respect to in-control ARL. From figure 5 it can be noted that when there is an increase in variance Individuals & Moving Range combination is giving lesser ARL values than GLR .

The study reveals that all the three schemes shows lowered performance when autocorrelation is present. Therefore, when there is significant autocorrelation in the observations from the process, it is not advisable to apply traditional control charts directly without any modification.

REFERENCES

[1] English, J.R, Krishnamurthi, M and Sastri,T. ,"Quality Monitoring of Continuous Flow Processes." , *Computers Ind. Engg.,* 20, pp. 251-260,1991

[2] Lu, C.W. and Reynolds, M.R.,Jr. , "Control Charts for Monitoring the Mean and Variance of Autocorrelated Processes, " *Journal of Quality Technology*, 31(3), pp. 259-274, 1999

[3] Reynolds, M. R., Jr. and Stoumbos, Z. G. ," Should exponentially weighted moving average and cumulative sum charts be used with Shewhart limits? " , *Technometrics*, 47(4), pp 409-424, 2005

[4] Hawkins, D.M and Deng, Q " A non-parametric Change Point Control Chart " *Journal of Quality Technology*., 42(2), pp. 165-173, 2010

[5] Reynolds, M. R., Jr. and Stoumbos, Z. G. , " Individuals control schemes for monitoring the mean and variance of processes subject to drifts.", *Stochastic Analysis and Applications*, 19(5), pp. 863-892, 2001

[6] Box, G.E.P and Jenkins, G.M , "*Time Series Analysis : Forecasting and Control"*, San-Francisco, Holden- Day, 1976