INTUITIONISTIC FUZZY PRIME IMPLICATIVE FILTERS OF LATTICE WAJSBERG ALGEBRAS

A. Ibrahim* and D. Saravanan**

***** *P.G. and Research Department of Mathematics, H. H. The Rajah's College, Pudukkottai, Affiliated to Bharathidasan University, Tiruchirappalli, Tamilnadu, India.*

*** Department of Mathematics, Chikkanna Government Arts College, Tirupur, Research Scholar, P.G and Research Department of Mathematics, H. H. The Rajah's College, Pudukkotai, Affiliated to Bharathidasan University, Tamilnadu, India.*

Abstract- In this paper, we introduce the notion of an intuitionistic fuzzy prime implicative filter(IFPIF) of lattice Wajsberg algebra. Also, we investigate some properties with illustrations. Further, we obtain the relation between an IFPIF and fuzzy prime implicative filter(FPIF) in lattice Wajsberg algebra. Finally, we establish the equivalent conditions of an IFPIF.

Keywords- Wajsberg algebra; Lattice Wajsberg algebra; Implicative filter; Fuzzy implicative filter; Prime implicative filter; Fuzzy prime implicative filter.

Mathematical Subject classification 2010: 03G25, 03E72, 06F35.

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [10].The idea of intuitionistic fuzzy subset was introduced by Atanassov [1, 2], as generalization of the notion of fuzzy set. The concept of Wajsberg algebras was introduced by Mordchaj Wajsbreg[9] in 1935, and studied by Font, Rodriguez and Torrens[5]. They[5], defined lattice structure of Wajsberg algebras and the notion of an implicative filter of lattice Wajsberg algebras and discussed some of their properties. Basheer Ahamed and Ibrahim [3, 4] introduced the definitions of fuzzy implicative filter, an anti fuzzy implicative filter of lattice Wajsberg algebras and obtained some properties with illustrations. The authors [6, 7, 8] introduced the notions of strong implicative, fuzzy strong implicative, an anti fuzzy strong implicative, an intuitionistic fuzzy implicative and an intuitionistic anti fuzzy implicative filters of lattice Wajsberg algebra, and investigated some properties of them. We considered some basic definitions and properties of Wajsberg Algebra, fuzzy subset and an intuitionistic fuzzy set in [5, 6, 7].

In this paper, we introduce the notion of an IFPIF of lattice Wajsberg algebra. We investigate properties of an IFPIF with examples. We establish the relation between an IFPIF and FPIF.

2. PRELIMINARIES

In this section, we recall some basic definitions of implicative filter, fuzzy implicative filter, intuitionistic fuzzy implicative filter, which are helpful to our main results.

Definition 2.1.[5] Let $\mathcal{W}$ be a lattice Wajsberg algebra. A subset F of $\mathcal{W}$ is called an implicative filter of $\mathcal{W}$ if it satisfies the following axioms for all $x, y \in \mathcal{W}$.

(i) $1 \in F$

(ii) $x \in F$ and $x \to y \in F$ imply $y \in F$.

Definition 2.2.[3] Let $\mathcal{W}$ be a lattice Wajsberg algebra. A fuzzy subset μ of $\mathcal{W}$ is called a fuzzy implicative filter of $\mathcal{W}$ if it satisfies the following axioms for all $x, y, z \in \mathcal{W}$.

(i) $\mu(1) \geq \mu(x)$

(ii) $\mu(z) \geq min\{\mu(y), \mu(y \to z)\}$.

Definition 2.3.[1] An intuitionistic fuzzy subset S of a non-empty set $\mathcal{W}$ is an object having the form $S = \{(x, \mu_S(x), \gamma_S(x)) \,/\, x \in \mathcal{W}\} = (\mu_S, \gamma_S)$ where the functions $\mu_S: \mathcal{W} \to [0,1]$ and $\gamma_S: \mathcal{W} \to [0, 1]$ denotes the degree of membership and denotes the non-membership respectively, and $0 \leq \mu_S(x) + \gamma_S(x) \leq 1$ for any $x \in \mathcal{W}$.

Definition 2.4.[7] Let $\mathcal{W}$ be a lattice Wajsberg algebra. An intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ of $\mathcal{W}$ is called an intuitionistic fuzzy implicative filter of $\mathcal{W}$ if it satisfies the following inequalities for any $x, y \in \mathcal{W}$,

(i) $\mu_S(1) \geq \mu_S(x)$ and $\gamma_S(1) \leq \gamma_S(x)$

(ii) $\mu_S(y) \geq min\{\mu_S(x), \mu_S(x \to y)\}$

(iii) $\gamma_S(y) \leq max\{\gamma_S(x), \gamma_S(x \to y)\}$.

Proposition 2.5.[7] Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy implicative filter of a lattice Wajsberg algebra $\mathcal{W}$ if and only if (μ_S, μ_S^c) and (γ_S^c, γ_S) are intuitionistic fuzzy implicative filters of $\mathcal{W}$.

Definition 2.6.[4] A non-constant fuzzy implicative filter μ of a lattice Wajsberg algebra $\mathcal{W}$ is said to be a prime if $\mu(x \vee y) \leq max\{\mu(x), \mu(y)\}$ for all $x, y \in \mathcal{W}$.

Proposition 2.7.[7] Let $\mathcal{W}$ be a lattice Wajsberg algebra, V a non-empty subset of [0, 1] and $J_t = \{x/J(x) \geq t,\ x \in \mathcal{W}\}$ such that $t \in V$ and J is a fuzzy implicative filter of $\mathcal{W}$ satisfies the following:

(i) $\mathcal{W} = \cup_{t \in V} J_t$

(ii) $r > t$ if and only if $J_r \subseteq J_t$ for all $r, t \in V$

(iii) Let $\mu_P(x) = Sup\{t \in V/x \in J_t\}$ and $\gamma_P(x) = Inf\{t \in V/x \in J_t\}$ for all $x \in \mathcal{W}$, then $P = (\mu_P, \gamma_P)$ is an intuitionistic fuzzy implicative filter of $\mathcal{W}$.

3. PROPERTIES OF IFPIF

In this section, we introduce the notion of an IFPIF. Further, we obtain some properties of an IFPIF.

Definition 3.1. Let $\mathcal{W}$ be a lattice Wajsberg algebra. An intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ of $\mathcal{W}$ is called an IFPIF of $\mathcal{W}$ if it satisfies the following

(i) $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter of $\mathcal{W}$

(ii) $\mu_S(x \vee y) \leq max\{\mu_S(x), \mu_S(y)\}$

(iii) $\gamma_S(x \vee y) \geq min\{\gamma_S(x), \gamma_S(y)\}$ for all $x, y \in \mathcal{W}$.

Example 3.2. Let $\mathcal{W} = \{0, g, h, i, j, 1\}$ be a set with Figure (1) as partial ordering. Define a unary operation complement " $*$" and a binary operation implication "→" on $\mathcal{W}$ as in the Table (1) and Table (2).

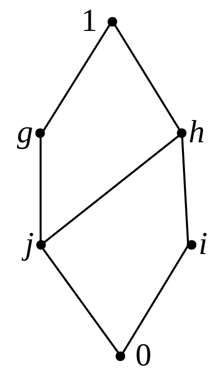
Table: 3.1 Complement

x	x^*
0	1
g	i
h	j
i	g
j	h
1	0

Table: 3.2 Implication

→	0	g	h	i	j	1
0	1	1	1	1	1	1
g	i	1	h	i	h	1
h	j	g	1	h	g	1
i	g	g	1	1	g	1
j	h	1	1	h	1	1
1	0	g	h	i	j	1

Figure: 3.1



Consider a intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ on $\mathcal{W}$ as
$\mu_S(x) = \begin{cases} 0.7 & \text{if } x \in \{h, i, 1\} \\ 0.3 & \text{if } x \in \{0, g, j\} \end{cases}$; $\gamma_S(x) = \begin{cases} 0.2 & \text{if } x \in \{h, i, 1\} \\ 0.7 & \text{if } x \in \{0, g, j\} \end{cases}$ is an IFPIF of $\mathcal{W}$.

In the same Example 3.2, consider a intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ on $\mathcal{W}$ as
$\mu_S(x) = \begin{cases} 0.6 & \text{if } x \in \{g, h, 1\} \\ 0.2 & \text{if } x \in \{0, i, j\} \end{cases}$; $\gamma_S(x) = \begin{cases} 0.3 & \text{if } x \in \{g, h, 1\} \\ 0.5 & \text{if } x \in \{0, i, j\} \end{cases}$ is not an IFPIF of $\mathcal{W}$.
Since $\mu_S(i \vee j) = \mu_S((i \to j) \to j) = \mu_S(h) = 0.6$; $\max\{\mu_S(i), \mu_S(j)\} = 0.2$.
Thus $\mu_S(i \vee j) \nleq max\{\mu_S(i), \mu_S(j)\}$ and similarly, $\gamma_S(i \vee j) \ngeq min\{\gamma_S(i), \gamma_S(j)\}$.

Proposition 3.3. Every IFPIF of lattice Wajsberg algebra $\mathcal{W}$ is an intuitionistic fuzzy implicative filter of $\mathcal{W}$.

Proof. From (i) of the Definition 3.1, we have IFPIF of $\mathcal{W}$ is an intuitionistic fuzzy implicative filter of $\mathcal{W}$.

Proposition 3.4. An intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ of a lattice Wajsberg algebra $\mathcal{W}$ is an intuitionistic fuzzy implicative filter of $\mathcal{W}$ if and only if for each m, $n \in [0, 1]$, the sets ${\mu_S}_n$ and ${\gamma_S}^m$ is either empty or fuzzy implicative filters of $\mathcal{W}$.

***Proof*.** Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy implicative filter of $\mathcal{W}$ and ${\mu_S}_n \neq \emptyset$, ${\gamma_S}^m \neq \emptyset$ for all m, $n \in [0, 1]$. From the (i) of Definition 3.1, we have $1 \in {\mu_S}_n$ and $1 \in {\gamma_S}^m$.
Let $x, y \in \mathcal{W}$ be such that $x \in {\mu_S}_n$ and $x \to y \in {\mu_S}_n$.
Then, we have $\mu_S(x) \geq n$ and $\mu_S(x \to y) \geq n$.
Thus, we get $\mu_S(y) \geq min\{\mu_S(x), \mu_S(x \to y)\} \geq n$, that is, $y \in {\mu_S}_n$.

Thus, μ_{S_n} is a fuzzy implicative filter of $\mathcal{W}$.
Similarly, let $x, y \in \mathcal{W}$ be such that $x \in \gamma_S{}^m$ and $x \to y \in \gamma_S{}^m$.
Then, we have $\gamma_S(x) \leq m$ and $\gamma_S(x \to y) \leq m$.
Thus, we get $\gamma_S(y) \leq max\{\gamma_S(x), \gamma_S(x \to y)\} \leq m$, that is, $y \in \gamma_S{}^m$.
Thus $\gamma_S{}^m$ is a fuzzy implicative filter of $\mathcal{W}$.
Conversely, for each $m, n \in [0, 1]$, the sets μ_{S_n} and $\gamma_S{}^m$ is either empty or fuzzy implicative filters of $\mathcal{W}$.
For any $x \in \mathcal{W}$, let $\mu_S(x) = n$ and $\gamma_S(x) = m$.Then, $x \in \mu_{S_n} \cap \gamma_S{}^m$ and so that $\mu_{S_n} \neq \phi \neq \gamma_S{}^m$.
Since μ_{S_n} and $\gamma_S{}^m$ are implicative filters of $\mathcal{W}$, therefore $1 \in \mu_{S_n}$ and $1 \in \gamma_S{}^m$.
Hence $\mu_S(1) \geq n = \mu_S(x)$ and $\gamma_S(1) \leq m = \gamma_S(x)$.
To prove: (i). $\mu_S(y) \geq min\{\mu_S(x), \mu_S(x \to y)\}$ and **(ii).** $\gamma_S(y) \leq max\{\gamma_S(x), \gamma_S(x \to y)\}$.

(i) If not, then there exists $p, q \in A$ such that $\mu_S(p) < min\{\mu_S(q), \mu_S(q \to p)\}$.
Taking $n_0 = \frac{1}{2}[\mu_S(p) + min\{\mu_S(q), \mu_S(q \to p)\}]$,
we have $\mu_S(p) < n_0 < min\{\mu_S(q), \mu_S(q \to p)\}$.

Therefore, $q \in \mu_{S_{n_0}}$ and $q \to p \in \mu_{S_{n_0}}$ but $p \notin \mu_{S_{n_0}}$. Thus, $\mu_{S_{n_0}}$ is not an implicative filter of $\mathcal{W}$, which is a contradiction. Therefore, $\mu_S(y) \geq min\{\mu_S(x), \mu_S(x \to y)\}$.

(ii) If not, then there exists $u, v \in A$ such that $\gamma_S(u) > max\{\gamma_S(v), \gamma_S(v \to u)\}$.
Taking $m_0 = \frac{1}{2}[\gamma_S(u) + max\{\gamma_S(v), \gamma_S(v \to u)\}]$,
we have $max\{\gamma_S(v), \gamma_S(v \to u)\} < m_0 < \gamma_S(u)$.

Therefore, $v \in \gamma_S{}^{m_0}$ and $v \to u \in \gamma_S{}^{m_0}$ but $u \notin \gamma_S{}^{m_0}$. Thus, $\gamma_S{}^{m_0}$ is not an implicative filter of $\mathcal{W}$, which is a contradiction. Therefore, $\gamma_S(y) \leq max\{\gamma_S(x), \gamma_S(x \to y)\}$.
Hence, $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter of $\mathcal{W}$. □

Theorem 3.5. If $S = (\mu_S, \gamma_S)$ be an IFPIF of lattice Wajsberg algebra $\mathcal{W}$ if and only if the sets $I_\mu = \{x \in \mathcal{W} / \mu_S(x) = 1\}$ and $I_\gamma = \{x \in \mathcal{W} / \gamma_S(x) = 1\}$ are either empty or prime implicative filters of $\mathcal{W}$.

Proof. Let $S = (\mu_S, \gamma_S)$ be an IFPIF of lattice Wajsberg algebra $\mathcal{W}$.
To prove: The sets I_μ and I_γ are prime implicative filters of $\mathcal{W}$.
Let $\mu_S(x \vee y) = 1$ for all $x, y \in \mathcal{W}$.
Then, $1 = \mu_S(x \vee y) \leq max\{\mu_S(x), \mu_S(y)\} = \mu_S(x) \; or \; \mu_S(y)$.
Thus, $1 \leq \mu_S(x) \; or \; 1 \leq \mu_S(y)$.
We know that $\mu_S(x) = 1$ or $\mu_S(y) = 1$, then from the definition of I_μ, we have $x \in I_\mu \; or \; y \in I_\mu$.
Thus, I_μ is a prime implicative filter of $\mathcal{W}$.
Similarly, we prove for I_γ.
Therefore, I_γ is a prime implicative filter of $\mathcal{W}$.
Conversely, $I_\mu, I_\gamma \neq \emptyset$ and I_μ, I_γ are prime implicative filters of lattice Wajsberg algebra $\mathcal{W}$.
To prove: $S = (\mu_S, \gamma_S)$ be an IFPIF of lattice Wajsberg algebra $\mathcal{W}$.
Let $\mu_S(x), \mu_S(y) \in I_\mu$ then $\mu_S(x) = 1$ and $\mu_S(y) = 1$ so that $\mu_S(x \vee y) = 1$ and $\max\{\mu_S(x), \mu_S(y)\} = 1$ for all $x, y \in \mathcal{W}$. Thus, $\mu_S(x \vee y) \leq max\{\mu_S(x), \mu_S(y)\}$ for all $x, y \in \mathcal{W}$.

Similarly, we prove if $\gamma_S(x), \gamma_S(y) \in I_\gamma$ then, we have $\gamma_S(x \vee y) \geq min\{\gamma_S(x), \gamma_S(y)\}$ for all $x, y \in \mathcal{W}$.
Since I_μ and I_γ are prime implicative filters, they are implicative filters of $\mathcal{W}$. As μ_S and γ_S are in I_μ and I_γ, μ_S and γ_S are intuitionistic fuzzy implicative filters of $\mathcal{W}$.
Therefore, $S = (\mu_S, \gamma_S)$ is an IFPIFof $\mathcal{W}$.□

Theorem 3.6. Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy implicative filter of lattice Wajsberg algebra $\mathcal{W}$. Then $S = (\mu_S, \gamma_S)$ is an IFPIF if and only if the sets $\mathcal{W}_\mu = \{x \in \mathcal{W} \,/\, \mu_S(x) = \mu_S(1)\}$ and $\mathcal{W}_\gamma = \{x \in \mathcal{W} \,/ \gamma_S(x) = \gamma_S(1)\}$ are prime implicative filters of $\mathcal{W}$.

Proof. Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy implicative filter of lattice Wajsberg algebra $\mathcal{W}$.
Let $S = (\mu_S, \gamma_S)$ be an IFPIF of lattice Wajsberg algebra $\mathcal{W}$.
We want to prove that the sets $\mathcal{W}_\mu$ and $\mathcal{W}_\gamma$ are prime implicative filters of $\mathcal{W}$.
Let $x, y \in \mathcal{W}$ and $\mu_S(x \vee y) = \mu_S(1)$.
Then $\mu_S(1) = \mu_S(x \vee y) \leq max\{\mu_S(x), \mu_S(y)\} = \mu_S(x) \; or \; \mu_S(y)$.
Thus, we have $\mu_S(1) \leq \mu_S(x) \; or \; \mu_S(1) \leq \mu_S(y)$.
We know that $\mu_S(x) = \mu_S(1) \; or \; \mu_S(y) = \mu_S(1)$ then $x \in \mathcal{W}_\mu \; or \; y \in \mathcal{W}_\mu$.
Therefore, $\mathcal{W}_\mu$ is a prime implicative filter of $\mathcal{W}$.
Similarly, $x \vee y \in \mathcal{W}_\gamma$ for any $x, y \in \mathcal{W}$. Therefore $\gamma_S(x \vee y) = \gamma_S(1)$.
Then $\gamma_S(1) = \gamma_S(x \vee y) \geq min\{\gamma_S(x), \gamma_S(y)\} = \gamma_S(x) \; or \; \gamma_S(y)$.
Hence, we get $\gamma_S(1) \geq \gamma_S(x) \; or \; \gamma_S(1) \geq \gamma_S(y)$.
We know that $\gamma_S(x) = \gamma_S(1) \; or \; \gamma_S(y) = \gamma_S(1)$ then $x \in \mathcal{W}_\gamma \; or \; y \in \mathcal{W}_\gamma$.
Thus, $\mathcal{W}_\gamma$ is a prime implicative filter of $\mathcal{W}$.
Conversely, if $\mathcal{W}_\mu = \{x \in \mathcal{W}/\mu_S(x) = \mu_S(1)\}$ and $\mathcal{W}_\gamma = \{x \in \mathcal{W}/\gamma_S(x) = \gamma_S(1)\}$ are prime implicative filters of $\mathcal{W}$.
To prove: $S = (\mu_S, \gamma_S)$ be an IFPIF of $\mathcal{W}$.
Now, $(x \to y) \vee (y \to x) = 1$ for any $x, y \in \mathcal{W}_\mu$.
Thus $\mu_S((x \to y) \vee (y \to x)) = \mu_S(1)$.Then $\mu_S(x \to y) = \mu_S(1) \; or \; \mu_S(y \to x) = \mu_S(1)$.
If $\mu_S(x \to y) = \mu_S(1)$ and $\mathcal{W}_\mu$ is a fuzzy implicative filter.
Then $\mu_S(y) \geq min\{\mu_S(x), \mu_S(x \to y)\} = min\{\mu_S(x), \mu_S(1)\} = \mu_S(x)$. That is, $\mu_S(x) \leq \mu_S(y)$.
Thus, $\mu_S(x \vee y) \leq max\{\mu_S(x), \mu_S(y)\} = \mu_S(y)$. Therefore, $\mu_S(x \vee y) \leq \mu_S(y)$.
Hence, $\mu_S(x \vee y) \leq max\{\mu_S(x), \mu_S(y)\}$.
Similarly, $\gamma_S((x \to y) \vee (y \to x)) = \gamma_S(1)$. Then, $\gamma_S(x \to y) = \gamma_S(1) \; or \; \gamma_S(y \to x) = \gamma_S(1)$.
If $\gamma_S(x \to y) = \gamma_S(1)$ and $\mathcal{W}_\gamma$ is a fuzzy implicative filter.
Then,$\gamma_S(y) \leq max\{\gamma_S(x), \gamma_S(x \to y)\} = max\{\gamma_S(x), \gamma_S(1)\} = \gamma_S(x)$.That is $\gamma_S(y) \leq \gamma_S(x)$.
Thus, $\gamma_S(x \vee y) \geq min\{\gamma_S(x), \gamma_S(y)\} = \gamma_S(y)$. Therefore, $\gamma_S(x \vee y) \geq \gamma_S(y)$.
Hence, $\gamma_S(x \vee y) \geq min\{\gamma_S(x), \gamma_S(y)\}$.
Thus, $S = (\mu_S, \gamma_S)$ is an IFPIF of $\mathcal{W}$.□

Proposition 3.7. Let $S = (\mu_S, \gamma_S)$ be an IFPIF of a lattice Wajsberg algebra $\mathcal{W}$ if and only if the fuzzy subsets μ_S, γ_S^c are FPIFs of $\mathcal{W}$, where $\gamma_S^c(x) = 1 - \gamma_S(x)$ for all $x \in \mathcal{W}$.

Proof. Let $S = (\mu_S, \gamma_S)$ be an IFPIF of $\mathcal{W}$.

From Proposition 3.3, we have $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter of $\mathcal{W}$.
From (ii) of Definition 3.1, the fuzzy subset μ_S is a FPIF of $\mathcal{W}$.
Now $\gamma_S^c(1) = 1 - \gamma_S(1) \geq 1 - \gamma_S(x) = \gamma_S^c(x)$
and $\gamma_S^c(x \vee y) = 1 - \gamma_S(x \vee y) \leq 1 - min\{\gamma_S(x), \gamma_S(y)\}$

$$= max\{1 - \gamma_S(x), 1 - \gamma_S(y)\}$$
$$= max\{\gamma_S^c(x), \gamma_S^c(y)\}.$$

Thus, $\gamma_S^c(1) \geq \gamma_S^c(x)$ and $\gamma_S^c(y) \leq max\{\gamma_S^c(x), \gamma_S^c(y)\}$.
Hence, γ_S^c is a FPIF of $\mathcal{W}$.
Conversely, if μ_S and γ_S^c are FPIFs of $\mathcal{W}$.
Then we have $\mu_S(1) \geq \mu_S(x)$ and

$$1 - \gamma_S(1) = \gamma_S^c(1) \geq \gamma_S^c(x) = 1 - \gamma_S(x) \text{ implies that } \gamma_S(x) \geq \gamma_S(1).$$

From the Definition 2.6, we have $\mu_S(x \vee y) \leq max\{\mu_S(x), \mu_S(y)\}$ and

$$\gamma_S^c(x \vee y) \leq max\{\gamma_S^c(x), \gamma_S^c(y)\}$$
$$1 - \gamma_S(x \vee y) \leq max\{1 - \gamma_S(x), 1 - \gamma_S(y)\}$$
$$= 1 - min\{\gamma_S(x), \gamma_S(y)\}$$
$$\gamma_S(x \vee y) \geq min\{\gamma_S(x), \gamma_S(y)\}.$$

Thus, $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter of $\mathcal{W}$.
Hence $S = (\mu_S, \gamma_S)$ is an FPIF of $\mathcal{W}$.□

Proposition 3.8. Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy subset of a lattice Wajsberg algebra $\mathcal{W}$. Then $S = (\mu_S, \gamma_S)$ is an IFPIF of $\mathcal{W}$ if and only if (μ_S, μ_S^c) and (γ_S^c, γ_S) are IFPIFs of $\mathcal{W}$.

Proof. Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy subset of $\mathcal{W}$.
Then by Definition 2.3, $0 \leq \mu_S(x) + \mu_S^c(x) = 1$ and $0 \leq \gamma_S(x) + \gamma_S^c(x) = 1$.
Thus (μ_S, μ_S^c) and (γ_S^c, γ_S) are intuitionistic fuzzy subsets of $\mathcal{W}$.
Assume that $S = (\mu_S, \gamma_S)$ is an IFPIF of $\mathcal{W}$.
Then by Proposition 2.5, (μ_S, μ_S^c) and (γ_S^c, γ_S) are intuitionistic fuzzy implicative filters of $\mathcal{W}$.
To prove: $\mu_S^c(x \vee y) \geq min\{\mu_S^c(x), \mu_S^c(y)\}$ and $\gamma_S^c(x \vee y) \leq max\{\gamma_S^c(x), \gamma_S^c(y)\}$.
From (ii) of Definition 3.1, $\mu_S^c(x \vee y) = 1 - \mu_S(x \vee y) \geq 1 - max\{\mu_S(x), \mu_S(y)\}$

$$= min\{1 - \mu_S(x), 1 - \mu_S(y)\}$$
$$= min\{\mu_S^c(x), \mu_S^c(y)\}.$$

Thus, $\mu_S^c(x \vee y) \geq min\{\mu_S^c(x), \mu_S^c(y)\}$.
Similarly, $\gamma_S^c(x \vee y) = 1 - \gamma_S(x \vee y) \leq 1 - min\{\mu_S(x), \mu_S(y)\}$

$$= max\{1 - \mu_S(x), 1 - \mu_S(y)\}$$
$$= max\{\mu_S^c(x), \mu_S^c(y)\}.$$

Thus, $\gamma_S^c(x \vee y) \leq max\{\gamma_S^c(x), \gamma_S^c(y)\}$.
Hence, (μ_S, μ_S^c) and (γ_S^c, γ_S) are IFPIFs of $\mathcal{W}$.
Conversely, (μ_S, μ_S^c) and (γ_S^c, γ_S) are IFPIFs of $\mathcal{W}$.
From the Definition 3.1, $S = (\mu_S, \gamma_S)$ is an IFPIF of $\mathcal{W}$.□

Proposition 3.9. An intuitionistic fuzzy set $S = (\mu_S, \gamma_S)$ of a lattice Wajsberg algebra $\mathcal{W}$ is an IFPIF of $\mathcal{W}$ if and only if for each $m, n \in [0, 1]$, the sets μ_{S_n} and ${\gamma_S}^m$ is either empty or FPIFs of $\mathcal{W}$.

Proof. Let $S = (\mu_S, \gamma_S)$ be an IFPIF of a lattice Wajsberg algebra $\mathcal{W}$.

From the Proposition 3.3, we have $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter of a lattice Wajsberg algebra $\mathcal{W}$.
From the Proposition 3.4, we have $\mu_{S_n} \neq \phi$, ${\gamma_S}^m \neq \phi$ for all $m, n \in [0,1]$, the sets μ_{S_n} and ${\gamma_S}^m$ are fuzzy implicative filters of $\mathcal{W}$.
Let $x, y \in A$ be such that $x \vee y \in \mu_{S_n}$.
Then, we have $\mu_S(x \vee y) \geq n$.
Thus, $n \leq \mu_S(x \vee y) \leq max\{\mu_S(x), \mu_S(y)\}$ this implies $\mu_S(x) \geq n$ or $\mu_S(y) \geq n$.
Hence $x \in \mu_{S_n}$ or $y \in \mu_{S_n}$.
Thus, μ_{S_n} is a FPIF of $\mathcal{W}$.
Similarly, ${\gamma_S}^m$ is a FPIF of $\mathcal{W}$.
Conversely, for each $m, n \in [0,1]$, the sets μ_{S_n} and ${\gamma_S}^m$ are either empty or FPIFs of $\mathcal{W}$.
For any $x \in \mathcal{W}$, let $\mu_S(x) = n$ and $\gamma_S(x) = m$. Then, $x \in \mu_{S_n} \cap {\gamma_S}^m$ and so that $\mu_{S_n} \neq \phi \neq {\gamma_S}^m$.
From the Definition 2.6, μ_{S_n} and ${\gamma_S}^m$ are fuzzy implicative filters of $\mathcal{W}$.
From the Proposition 3.4, $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy implicative filter of a lattice Wajsberg algebra $\mathcal{W}$.
Suppose that $S = (\mu_S, \gamma_S)$ is not an IFPIF of a lattice Wajsberg algebra $\mathcal{W}$.
Then there exists $x, y \in \mathcal{W}$ such that $\mu_S(x \vee y) > max\{\mu_S(x), \mu_S(y)\}$ and $\gamma_S(x \vee y) < min\{\gamma_S(x), \gamma_S(y)\}$.
Taking $a = \frac{1}{2}[\mu_S(x \vee y) + max\{\mu_S(x), \mu_S(y)\}]$, $b = \frac{1}{2}[\gamma_S(x \vee y) + min\{\gamma_S(x), \gamma_S(y)\}]$,
we have $\mu_S(x \vee y) > a > max\{\mu_S(x), \mu_S(y)\}$ and $\gamma_S(x \vee y) < b < min\{\gamma_S(x), \gamma_S(y)\}$.
These implies i. $x \vee y \in \mu_{S_a}$ but $x \notin \mu_{S_a}$ and $y \notin \mu_{S_a}$ ii. $x \vee y \in {\gamma_S}^b$ but $x \notin {\gamma_S}^b$ and $y \notin {\gamma_S}^b$, which is a contradiction to our assumption.
Hence, $S = (\mu_S, \gamma_S)$ is an IFPIF of a lattice Wajsberg algebra $\mathcal{W}$.

Proposition 3.10. Let $\mathcal{W}$ be a lattice Wajsberg algebra, V a non-empty subset of $[0,1]$ and $J_t = \{x/J(x) \geq t, x \in \mathcal{W}\}$ such that $t \in V$ and J_t is a FPIF of $\mathcal{W}$ satisfies the following:
(i) $\mathcal{W} = \cup_{t \in V} J_t$
(ii) $r > t$ if and only if $J_r \subseteq J_t$ for all $r, t \in V$
(iii) Let $\mu_P(x) = Sup\{t \in V/x \in J_t\}$ and $\gamma_P(x) = Inf\{t \in V/x \in J_t\}$ for all $x \in \mathcal{W}$ then $P = (\mu_P, \gamma_P)$ is an IFPIF of $\mathcal{W}$.

Proof. Let $P = (\mu_P, \gamma_P)$ be an intuitionistic fuzzy subset of $\mathcal{W}$.
From the Proposition 2.7, $P = (\mu_P, \gamma_P)$ is an intuitionistic fuzzy implicative filter of $\mathcal{W}$. (3.1)
To prove: $\mu_P(x \vee y) \leq max\{\mu_P(x), \mu_P(y)\}$ and $\gamma_P(x \vee y) \geq min\{\gamma_P(x), \gamma_P(y)\}$ for all $x, y \in \mathcal{W}$.
(i) If $\mu_P(x \vee y) > max\{\mu_P(x), \mu_P(y)\}$ for all $x, y \in \mathcal{W}$.
Let $max\{\mu_P(x), \mu_P(y)\} = t_1$. So, $\mu_P(x) \leq t_1$ and $\mu_P(y) \leq t_1$.
So $\mu_P(x \vee y) > t_1$.
Then there exist a FPIF J_{t_1} such that $x \vee y \in J_{t_1}$ but $x \notin J_{t_1}, y \notin J_{t_1}$.
Which is a contradiction, therefore $\mu_P(x \vee y) \leq max\{\mu_P(x), \mu_P(y)\}$. (3.2)
(ii) If $\gamma_P(x \vee y) < min\{\gamma_P(x), \gamma_P(y)\}$ for all $x, y \in \mathcal{W}$. Let $min\{\gamma_P(x), \gamma_P(y)\} = t_2$.
So, $\gamma_P(x) \geq t_2$ and $\mu_P(y) \geq t_2$. Hence $\gamma_P(x \vee y) < t_2$.
Then there exist a FPIF J_{t_2} such that $x \vee y \in J_{t_2}$ but $x \notin J_{t_1}, y \notin J_{t_1}$.

Which is a contradiction, therefore $\gamma_P(x \vee y) \geq min\{\gamma_P(x), \gamma_P(y)\}$. (3.3)
From (3.1), (3.2) and (3.3), $P = (\mu_P, \gamma_P)$ is an IFPIF of $\mathcal{W}$. □

IV. CONCLUSION

In this paper, we have introduced the notion of an intuitionistic fuzzy prime implicative filter of lattice Wajsberg algebra. Also, we have discussed some of their properties with illustrations. Further, we have investigated the relation between an intuitionistic fuzzy prime implicative filter and fuzzy prime implicative filter.

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