NEUTROSOPHIC BIPOLAR VAGUE REGULAR WEAKLY CLOSED SETS IN NEUTROSOPHIC BIPOLAR VAGUE TOPOLOGICAL SPACES

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ABSTRACT: In this research article, a new class of sets known as neutrosophic bipolar vague regular weakly closed sets and open sets are introduced in neutrosophic bipolar vague topological spaces. Also some of its characteristics has been analysed and compared.

KEYWORDS: NBV-regular weakly closed set; NBV-regular weakly open set; NBV-rw connected space; NBV-regular weakly continuous functions.

AMS SUBJECT CLASSIFICATION: 54A05, 54A99

I. INTRODUCTION

Various issues in real-life problems have fluctuations, one has to handle these vulnerabilities, and vague set is introduced. Vague set theory was first proposed by Gau and Buehre[5] which is an extension of fuzzy set theory. Neutrosophic sets are the more generalized sets introduced by Salama .A.A and Alblowi. S.A[13]; one can manage with uncertain information in a more successful way with a progressive manner when appeared differently when related to fuzzy sets. The neutrosophic sets has three completely independent parts, truth-membership degree, indeterminacy-membership degree, falsity-membership degree with the sum of these values lies between0 and 3.

Bipolar fuzzy sets are an extension of fuzzy sets was introduced by Lee[9] whose membership degree range is extended from the interval [0,1] to [-1,1]. Later Deli et. al. [4] announced the concept of bipolar neutrosophic sets; as an extension of neutrosophic sets. In the bipolar neutrosophic sets, the positive membership degree $T^+(x)$, $I^+(x)$, $F^+(x)$ signifies the truth-membership degree, indeterminacy-membership degree, falsity-membership degree of an element $x \in X$ analogous to a bipolar neutrosophic set A and the negative membership degree $T^-(x)$, $I^-(x)$, $F^-(x)$ signifies the truth-membership degree, indeterminacy-membership degree, falsity-membership degree of an element $x \in X$ to some implied counter property analogous to a bipolar neutrosophic set A.

The author Shawkat Alkhazaleh [16] introduced the concept of neutrosophic vague set theory. As a continuation, Satham hussain [14] introduced neutrosophic bipolar vague sets in bipolar vague topological spaces. Authors Savithiri.D, Janaki.C [15] introduced Neutrosophic RW closed sets in Neutrosophic topological spaces.

In this article, a new class of sets known as neutrosophic bipolar vague regular weakly closed sets and open sets are introduced in neutrosophic bipolar vague topological spaces. Also some of its characteristics has been analysed and compared.

II. PRELIMINARIES

Definition 2.1[3]: Let X be the universe. Then a **bipolar valued fuzzy set** A on X is defined by positive membership function μ_A^+: X →[0,1] and a negative membership function μ_A^- :X →[-1,0]. For sake of easiness we shall practice the symbol $A = \{\langle x, \mu_A^+, \mu_A^- \rangle : x \epsilon X$.

Definition 2.2[7]: Let X be a non-empty fixed set. A **Neutrosophic set (NS for short)** A is an object having the form $A = < x, \mu_A(x), \sigma_A(x), \gamma_A(x) >, x \in X$ where $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element x∈X to the set A. For the sake of simplicity A neutrosophic set A = {x, $\mu_A(x),\sigma_A(x),\gamma_A(x)$> ; x∈X} can be identified to be an ordered triple < μ_A,σ_A,γ_A> . For each point x in X, $0 \le \mu_A(x) + \sigma_A(x) + \gamma_A(x) \le 3$.

Definition 2.3[16]: A **vague set** A in the universe of discourse U is a pair (t_A,f_A) where t_A : U → [0,1], f_A : U →[0,1] denote the mapping such that $t_A + f_A \le 1$ for all u∈ U. The function t_A and f_A are called true membership function and false membership function respectively. The interval $[t_A,1\text{-}f_A]$ is called the vague value of A, and denoted by $\upsilon_A(u)$.

Definition 2.4:[11] A **neutrosophic vague set** A_{NV} (NVS in short) on the universe X written as $A_{NV} = \{\langle x, \hat{T}_{A_{NV}}(x); \hat{I}_{ANV}(x); \hat{F}_{ANV}(x)\rangle; x\epsilon X\}$,whose truth membership, indeterminacy membership and false membership functions is defined as:

$$\hat{T}_{A_{NV}}(x) = [T^-, T^+], \hat{I}_{A_{NV}}(x) = [I^-, I^+], \hat{F}_{A_{NV}}(x) = [F^-, F^+]\ where$$

(1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ (3) $0^- \le T^- + I^- + F^- \le 2^+$

Definition 2.5 [11]: Let A_{NV} and B_{NV} be two NVSs of the universe U. If ∀ u_i ∈ U, $\hat{T}_{A_{NV}}(u_i) \le \hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i) \ge \hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i) \ge \hat{F}_{B_{NV}}(u_i)$ then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where 1≤ i ≤n.

Definition 2.6[11]: The complement of NVS A_{NV} is denoted by A_{NV}^C and is defined by $\hat{T}_{A_{NV}}^C(x) = [1 - T^+, 1 - T^-], \hat{I}_{A_{NV}}^C(x) = [1 - I^+, 1 - I^-], \hat{F}_{A_{NV}}^C(x) = [1 - F^+, 1 - F^-]$.

Definition 2.7[11]:i) A function f: (X,τ) → (Y,σ) is NBV continuous if the inverse image of every NBVC set in (Y,σ) is NBVC set in (X,τ). T

ii)A function f: (X,τ) → (Y,σ) is GNBV-continuous if the inverse image of every NBVC set in (Y,σ) is GNBVC set in (X,τ).

Definition 2.8[12]: A neutrosophic bipolar vague set A_{NBV} (NBVS in short) on the universe X written as
$A_{NBV} = \{\langle x, [T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^- \rangle\}$ where
$(T^+)^+ = 1 - (F^-)^+$, $(F^+)^+ = 1 - (T^-)^+$ and $(T^+)^- = {}^-1 - (F^-)^-$, $(F^+)^- = {}^-1 - (T^-)^-$, T^+, I^+, F^+ : X → [0,1] and T^-, I^-, F^- : X →[-1,0].

Definition 2.9 [12]: Let A_{NBV} and B_{NBV} be two NVSs of the universe U where
$A_{NBV} = \{\langle x, [T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^- \rangle\}$ and
$B_{NBV} = \{\langle x, [T_B^-, T_B^+]^+, [I_B^-, I_B^+]^+, [F_B^-, F_B^+]^+, [T_B^-, T_B^+]^-, [I_B^-, I_B^+]^-, [F_B^-, F_B^+]^- \rangle\}$
Then we say that **A ⊆ B** if and only if
$[T_A^- \le T_B^-]^+, [T_A^+ \le T_B^+]^+, [I_A^- \ge I_B^-]^+, [I_A^+ \ge I_B^+]^+, [F_A^- \ge F_B^-]^+, [F_A^+ \ge F_B^+]^+, [T_A^- \ge T_B^-]^-, [T_A^+ \ge T_B^+]^-, [I_A^- \le I_B^-]^-, [I_A^+ \le I_B^+]^-, [F_A^- \le F_B^-]^-, [F_A^+ \le F_B^+]^-$.

Definition 2.10 [12]: Let A_{NBV} and B_{NBV} be two NVSs of the universe U where
$A_{NBV} = \{\langle x, [T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^- \rangle\}$ and
$B_{NBV} = \{\langle x, [T_B^-, T_B^+]^+, [I_B^-, I_B^+]^+, [F_B^-, F_B^+]^+, [T_B^-, T_B^+]^-, [I_B^-, I_B^+]^-, [F_B^-, F_B^+]^- \rangle\}$.
Then their union, intersection and complement are well defined as follows:

$$1. A \cup B = \begin{Bmatrix} max[T_A^-, T_B^-]^+, max[T_A^+, T_B^+]^+, min[I_A^-, I_B^-]^+ min[I_A^+, I_B^+]^+, min[F_A^-, F_B^-]^+, min[F_A^+, F_B^+]^+, \\ min[T_A^-, T_B^-]^-, min[T_A^+, T_B^+]^-, max[I_A^-, I_B^-]^-, max[I_A^+, I_B^+]^- max[F_A^-, F_B^-]^-, max[F_A^+, F_B^+]^- \end{Bmatrix}$$

$$2. A \cap B = \begin{Bmatrix} min[T_A^-, T_B^-]^+, min[T_A^+, T_B^+]^+, max[I_A^-, I_B^-]^+ max[I_A^+, I_B^+]^+, max[F_A^-, F_B^-]^+, max[F_A^+, F_B^+]^+, \\ max[T_A^-, T_B^-]^-, max[T_A^+, T_B^+]^-, min[I_A^-, I_B^-]^-, min[I_A^+, I_B^+]^- min[F_A^-, F_B^-]^-, min[F_A^+, F_B^+]^- \end{Bmatrix}$$

3. $A^C = \left\{\begin{matrix}[1-T_A^+]^+,[1-T_A^-]^+,[1-I_A^+]^+,[1-I_A^-]^+,[1-F_A^+]^+,[1-F_A^-]^+,[-1-T_A^+]^-,[-1-T_A^-]^-,\\ [-1-I_A^+]^-,[-1-I_A^-]^-,[-1-F_A^+]^-,[-1-F_A^-]^-\end{matrix}\right\}$

Definition 2.11[12]: A neutrosophic bipolar vague topology **(In short NBVT)** on a nonempty set X is a family τ of neutrosophic bipolar vague sets in X sustaining the following axioms:

1. 0,1 ∈ τ.
2. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
3. $\cup G_i \in \tau$ for any arbitrary family $\{ G_i : G_i \in \tau, i \in I\}$.

Under this case the pair (X,τ) is known as the neutrosophic bipolar vague topological space **(In short NBVTS)** and any NBVS in τ is known as neutrosophic bipolar vague open set **(In short NBVOS)**. The complement of a NBVOS $\overline{A}$ is said to be a neutrosophic bipolar vague closed set **(In short NBVCS)** in (X,τ).

Example 2.12[12]: Let X = {a,b}, τ = {0, A, B, 1} where the neutrosophic bipolar vague open sets

$$A = \left\{x, \frac{a}{[0.5,0.7],[0.5,0.5],[0.3,0.5],[-0.4,-0.1],[-0.5,-0.6],[-0.9,-0.6]}, \frac{b}{[0.3,0.6],[0.4,0.4],[0.4,0.7],[-0.2,-0.2],[-0.6,-0.8],[-0.8,-0.8]}\right\},$$

$$B = \left\{x, \frac{a}{[0.5,0.9],[0.3,0.3],[0.1,0.5],[-0.4,-0.3],[-0.4,-0.4],[-0.7,-0.6]}, \frac{b}{[0.4,0.6],[0.2,0.2],[0.4,0.6],[-0.5,-0.3],[-0.5,-0.5],[-0.7,-0.5]}\right\}$$

Then (X,τ) is a NBVTS on X.

Definition 2.13[12]: Let A_{NBV} be NBVS of the universe U where $\forall u_i \in U$, $\hat{T}^P_{A_{NBV}}(x) = [1,1]; \hat{I}^P_{A_{NBV}}(x) = [0,0]; \hat{F}^P_{A_{NBV}}(x) = [0,0]; \hat{T}^N_{A_{NBV}}(x) = [-1,-1]; \hat{I}^N_{A_{NBV}}(x) = [0,0]; \hat{F}^N_{A_{NBV}}(x) = [0,0]$

Then A_{NBV} is called unit NBVS (**1_{NBV} in short**), where 1≤ i ≤n.

Definition 2.14[12]: Let A_{NBV} be NBVS of the universe U where $\forall u_i \in U$, $\hat{T}^P_{A_{NBV}}(x) = [0,0]; \hat{I}^P_{A_{NBV}}(x) = [1,1]; \hat{F}^P_{A_{NBV}}(x) = [1,1]; \hat{T}^N_{A_{NBV}}(x) = [0,0]; \hat{I}^N_{A_{NBV}}(x) = [-1,-1]; \hat{F}^N_{A_{NBV}}(x) = [-1,-1]$

Then A_{NBV} is called zero NBVS (**0_{NBV} in short**), where 1≤ i ≤n.

Definition 2.15[12]: Let A be a NBVS in a NBVTS (X,τ). The neutrosophic bipolar vague interior of A **(In short NBVint(A))** and the neutrosophic bipolar vague closure of A **(In short NBVcl(A))** are defined by,

(1) NBVint(A) = ∪ {G/ G is a NBVOS in X and G ⊆ A},

(2) NBVcl(A) = ∩ {K/K is a NBVCS in X and A ⊆ K}.

Note that for any NBVS A in (X,τ), we have NBVcl(A^c) = (NBV int(A))^c and NBV int (A^c) = (NBV cl(A))^c. It can also be shown that NBVcl(A) is NBVCS and NBVint(A) is NBVOS in X.

(1) A is NBVCS in X if and only if NBVcl(A) = A.

(2) A is NBVOS in X if and only if NBVint(A) = A.

Result 2.16:[12]: Let A be any NBVS in X. Then

(1) NBVint A^C = (NBVcl(A))^C and (2) NBVcl(A^C) = (NBVint(A))^C.

Result 2.17 [10]: Let A_{NBV}, B_{NBV} , C_{NBV} and D_{NBV} be NBVSs.

(a) $A_{NBV} \subseteq B_{NBV}$ and $C_{NBV} \subseteq D_{NBV} \Rightarrow A_{NBV} \cup B_{NBV} \subseteq C_{NBV} \cup D_{NBV}$ and $A_{NBV} \cap B_{NBV} \subseteq C_{NBV} \cap D_{NBV}$

(b) $A_{NBV} \subseteq B_{NBV}$ and $A_{NBV} \subseteq C_{NBV} \Rightarrow A_{NBV} \subseteq B_{NBV} \cap C_{NBV}$

(c) $A_{NBV} \subseteq C_{NBV}$ and $B_{NBV} \subseteq C_{NBV} \Rightarrow A_{NBV} \cup B_{NBV} \subseteq C_{NBV}$

(d) $A_{NBV} \subseteq B_{NBV}$ and $B_{NBV} \subseteq C_{NBV} \Rightarrow A_{NBV} \subseteq C_{NBV}$ (e) $1^C_{NBV} = 0_{NBV}$ (f) $0^C_{NBV} = 1_{NBV}$

Result 2.18:[11]: Let (X,τ) be a NBVTS and A, B be NBVSs in X. Then the following properties hold:

(a) NBVint(A) ⊆ A, (f) A ⊆ NBVcl(A)

(b) A⊆B⇒NBVint(A)⊆NBVint(B) (g) A ⊆ B ⇒ NBVcl(A) ⊆ NBVcl(B)

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(c)NBVint(NBVint(A))=NBVint(A) (h) NBVcl(NBVcl(A)) = NBVcl(A)

(d)NBVint(A∩B)=NBVint(A)∩NBVint(B) (i) NBVcl(A)∪NBVcl(B) = NBVcl(A∪B).

(e)NBVint(1_{NBV})=1_{NBV}. (j)NBVcl(0_{NBV}) = 0_{NBV}

Definition 2.19[12]: Let (X,τ) be a Neutrosophic bipolar vague topological space. A NBV set A is said to be a Generalized Neutrosophic Bipolar Vague closed set **(GNBVCS in short)** if NVBcl(A) ⊆ A whenever A ⊆ U and U is NBVOS in X.

Definition 2.20[12]: (i) A function f: (X,τ) → (Y,σ) is **NBV continuous** if the inverse image of every NBVC set in (Y,σ) is NBVC set in (X,τ).

(ii) A function f: (X,τ) → (Y,σ) is **GNBV-continuous** if the inverse image of every NBVC set in (Y,σ) is GNBVC set in (X,τ).

III. SOME NEW TYPES OF OPEN AND CLOSED SETS IN NEUTROSOPHIC BIPOLAR VAGUE SETS

Definition 3.1: Let (X,τ) be a NBVTS and let
$A_{NBV} = \{\langle x, [T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^- \rangle\}$ be a Neutrosophic bipolar vague set is said to be a

(1) Neutrosophic Bipolar Vague Regular open set **(NBVROS in short)** if NBVint(NBVcl(A)) = A.

(2)Neutrosophic Bipolar Vague Regular semi open set **(NBVRSOS in short)** if there exists a NBVROS U such that U ⊆ A ⊆ NBVcl(U).

(3) Neutrosophic Bipolar Vague Semi open set **(NBVSOS in short)** if A ⊆ NBVcl(NBVint (A)).

(4) Neutrosophic Bipolar Vague Pre-open set **(NBVPOS in short)** if A ⊆ NBVint(NBVcl(A)).

(5) Neutrosophic Bipolar Vague π-open set **(NBVπOS in short)** if it is the finite union of NBVRO sets.

The complement of a NBVO sets are called as NBVC closed sets respectively. A NBV set which is both NBVO set and NBVC set is known as NBVclopen set.

Definition 3.2: Let (X, τ) be a NBVTS and let
$A_{NBV} = \{\langle x, [T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^- \rangle\}$be a Neutrosophic bipolar vague set is said to be a

(1) Neutrosophic Bipolar Vague w-closed set **(NBVWCS in short)** if NVBcl(A) ⊆ U whenever A ⊆ U and U is NBVSOS in X.

(2) Neutrosophic Bipolar Vague wg closed set **(NBVRGCS in short)** if NBVcl(A) ⊆U whenever A ⊆ U and U is NBVROS in X.

(3)Neutrosophic Bipolar Vague gpr closed set **(NBVGPRCS in short)** if NBVpcl(A) ⊆ U whenever A ⊆ U and U is NVBROS in X.

(4)Neutrosophic Bipolar Vague rwg closed set **(NBVRWGCS in short)** if NBVcl(NBVint(A)) ⊆ U whenever A ⊆ U and U is NBVROS in X.

Definition 3.3: Let (X,τ) be a Neutrosophic bipolar vague topological space. A Neutrosophic bipolar vague set A is said to be a **Neutrosophic bipolar vague regular weakly closed (In short NBVRWC) set,** if NBVcl(A) ⊆ U whenever A ⊆ U and U is a Neutrosophic bipolar vague regular semi open set.

The set of all neutrosophic bipolar vague RW closed sets of X is denoted by **NBVRWC(X).**

Definition 3.4: Let (X,τ) and (Y,σ) be any two Neutrosophic Bipolar Vague topological spaces. A function f: (X,τ) → (Y,σ) is said to be a

(i) **NBVRWG-continuous** if the inverse image of every NBVC set in (Y,σ) is NBVRWGC set in (X,τ).
(ii) **NBVW-continuous** if the inverse image of every NBVC set in (Y,σ) is NBVWC set in (X,τ).
(iii) **NBVGPR-continuous** if the inverse image of every NBVC set in (Y,σ) is NBVGPRC set in (X,τ).
(iv) **NBVπ-continuous** if the inverse image of every NBVC set in (Y,σ) is NBVπC set in (X,τ).
(v) **NBVR-continuous** if the inverse image of every NBVC set in (Y,σ) is NBVRC set in (X,τ).

Lemma 3.5: Let A and B be NBV subsets of (X,τ). Then

(i) NBVRWcl(0_{NBV}) = 0_{NBV} and NBVRWcl(1_{NBV}) = 1_{NBV}.

(ii) If A ⊂ B then NBVRWcl(A) ⊂ NBVRWcl(B)

(iii) A ⊆ NBVRWcl(A)

(iv) NBVRWcl(A) = NBVRWcl(NBVRWcl(A)) and

(v) NBVcl(A∪B) ⊃ NBVRWcl(A) ∪ NBVRWcl(B)

Lemma 3.6: Let A and B be NBV subsets of (X,τ). Then NBVRWcl(A∩B) ⊂ NBVRWcl(A) ∩ NBVRWcl(B).

Proof: Since A∩B ⊂ A,B, by lemma 3.16(ii), NBVRWcl(A∩B) ⊂ NBVRWcl(A), and NBVRWcl(A∩B) ⊂ NBVRWcl(B). Thus NBVRWcl(A∩B) ⊂ NBVRWcl(A) ∩ NBVRWcl(B).

Theorem 3.7: Every NBVCS is NBVRWC set.

Proof: Let A be a NBV closed set. Let U be a NBVRSO set such that A ⊆ U. Since A is a NBVC set NBVcl(A) = A ⊆ U implies that A is a NBVRWC set.

Remark 3.8: The converse of the above theorem need not be true as shown in the following example.

Example 3.9: Let X = {a,b}, τ = {0_{NBV},1_{NBV}, F }. Clearly (X,τ) be a NBVT space where the NBVOS

$$F = \left\{x, \frac{a}{[0.2,0.3],[0.5,0.6],[0.7,0.8],[-0.3,-0.4],[-0.5,-0.5],[-0.6,-0.7]}, \frac{b}{[0.4,0.5],[0.5,0.7],[0.5,0.6],[-0.4,-0.5],[-0.5,-0.6],[-0.5,-0.6]}\right\}$$

Then the neutrosophic bipolar vague set

$$A = \left\{x, \frac{a}{[0.4,0.5],[0.5,0.6],[0.5,0.6],[-0.3,-0.4],[-0.6,-0.7],[-0.6,-0.7]}, \frac{b}{[0.4,0.6],[0.5,0.5],[0.4,0.6],[-0.5,-0.5],[-0.6,-0.6],[-0.5,-0.5]}\right\}$$ is a NBVRWC set but it is not a NBVC set in (X,τ).

Theorem 3.10: (i) NBVWC set ⇒ NBVRWC set (ii) NBVRC set ⇒ NBVRWC set (iii) NBVπC set ⇒ NBVRWC set.

Proof: (i) Let A be a NBVWC set. Let A ⊆ U and U be a NBVRSO set. Since A is a NBVWC set and every NBVRO set is NBVO set, NBVcl(A) ⊆ U. Thus A is a NBVRWC set.

(ii) Let A be a NBVRC set i.e., A = NBVcl(NBVint(A)). Let U be a NBVRSO set such that A ⊆ U. NBVcl(A) ⊆ NBVcl(NBVcl(NBVint(A)) ⊆ (NBVint(A)) ⊆ NBVint(U) ⊆ U implies A is a NBVRWC set.

(iii) Let A be a NBVπC set. Every NBVπCset is NBVRC set which implies A is a NBVRWC set by using (ii).

Remark 3.11: The converse of the above theorem need not be true as shown in the following example.

Example 3.12: Let X = {a,b}, τ = {0_{NBV},1_{NBV} ,G,}. Clearly (X,τ) be a NBVT space where the NBVOS

$$G = \left\{x, \frac{a}{[0.2,0.3],[0.5,0.6],[0.7,0.8],[-0.3,-0.4],[-0.5,-0.5],[-0.6,-0.7]}, \frac{b}{[0.4,0.5],[0.5,0.7],[0.5,0.6],[-0.4,-0.5],[-0.5,-0.6],[-0.5,-0.6]}\right\}$$ Then the neutrosophic bipolar vague set

$$A = \left\{x, \frac{a}{[0.4,,0.5],[0.5,0.6],[0.5,0.6],[-0.3,-0.4],[-0.6,-0.7],[-0.6,-0.7]}, \frac{b}{[0.4,0.6],[0.5,0.5],[0.4,0.6],[-0.5,-0.5],[-0.6,-0.6],[-0.5,-0.5]}\right\}$$ is a NBVRWC set but it is not a NBVRC set, NBVWC set and NBVπC set in (X,τ).

Theorem 3.13: The finite union of two NBVRWC subsets of (X,τ) is also a NVBRWC subset of X.

Proof: Suppose that A and B are two NBVRWC sets in X such that A∪B ⊆ U where U is a NBVRSO set in X. Then A ⊆ U and B ⊆ U. Since A and B are NBVRWC sets, NVBcl (A) ⊆ U and NVBcl (B) ⊆ U. By lemma 3.5, NBVcl(A∪B) = NBVcl(A) ∪ NBVcl(B) ⊆ U which implies A∪B is also a NBVRWC set in X.

Remark 3.14: The intersection of two NBVRWC sets need not be a NBVRWC set in (X,τ) which is proved by following example.

Example 3.15: Let X = {u,v}, τ = {0_{NBV},1_{NBV} ,G}. Clearly (X,τ) be a NBVT space where the NBVOS

$$G = \left\{x, \frac{a}{[0.4,0.6],[0.6,0.6],[0.4,0.6],[-0.4,-0.6],[-0.6,-0.6],[-0.4,-0.6]}, \frac{b}{[0.4,0.6],[0.6,0.4],[0.4,0.6],[-0.5,-0.5],[-0.7,-0.3],[-0.5,-0.5]}\right\}$$

Then the neutrosophic bipolar vague sets

$$A_1 = \left\{x, \frac{a}{[0.3,0.5],[0.4,0.6],[0.5,0.7],[-0.3,-0.4],[-0.4,-0.4],[-0.6,-0.7]}, \frac{b}{[0.3,0.4],[0.6,0.4],[0.6,0.7],[-0.3,-0.2],[-0.7,-0.5],[-0.8,-0.7]}\right\},$$
$$A_2 = \left\{x, \frac{a}{[0.2,0.3],[0.6,0.4],[0.7,0.8],[-0.3,-0.5],[-0.6,-0.6],[-0.5,-0.7]}, \frac{b}{[0.2,0.5],[0.6,0.5],[0.5,0.8],[-0.3,-0.3],[-0.7,-0.8],[-0.7,-0.7]}\right\}$$

are NBVRWC sets but their intersection $A_1 \cap A_2$ is not a NBVRWC set in (X,τ).

Theorem 3.16: If A is NBVRWC subset of X such that A ⊆ B ⊆ NBVcl(A), then B is a NBVRWC set in X.

Proof: Let A be NBVRWC set of X such that A ⊂ B ⊆ NBVcl(A). Let U be a NBVRSO set of X such that B ⊆ U. Then by hypothesis, A ⊆ U. Since A is NBVRWC, NBVcl(A) ⊆ U. Now NBVcl(B) ⊆ NBVcl(NBVcl(A)) = NBV cl(A) ⊆ U. Hence B is a NVBRWC set in X.

Remark 3.17: The converse of the above theorem need not be true in general as seen in the following example.

Example 3.18: Let X = {a,b}, τ = {0_{NBV},1_{NBV} ,A, B}. Clearly (X,τ) is NBVT on X. The NBVO sets A and B are

$$A = \left\{x, \frac{a}{[0.5,0.7],[0.5,0.5],[0.3,0.5],[-0.4,-0.1],[-0.5,-0.6],[-0.9,-0.6]}, \frac{b}{[0.3,0.6],[0.4,0.4],[0.4,0.7],[-0.2,-0.2],[-0.6,-0.8],[-0.8,-0.8]}\right\},$$

$$B = \left\{x, \frac{a}{[0.5,0.9],[0.3,0.3],[0.1,0.5],[-0.4,-0.3],[-0.4,-0.4],[-0.7,-0.6]}, \frac{b}{[0.4,0.6],[0.2,0.2],[0.4,0.6],[-0.5,-0.3],[-0.5,-0.5],[-0.7,-0.5]}\right\}$$

Then the NBV sets

$$F_1 = \left\{x, \frac{a}{[0.1,0.1],[0.6,0.6],[0.9,0.9],[-0.2,-0.4],[-0.5,-0.6],[-0.6,-0.8]}, \frac{b}{[0.2,0.3],[0.8,0.8],[0.7,0.8],[-0.1,-0.3],[-0.5,-0.6],[-0.7,-0.9]}\right\},$$

$$F_2 = \left\{x, \frac{a}{[0.1,0.2],[0.6,0.6],[0.8,0.9],[-0.3,-0.5],[-0.4,-0.5],[-0.5,-0.7]}, \frac{b}{[0.2,0.4],[0.8,0.8],[0.6,0.8],[-0.1,-0.4],[-0.3,-0.3],[-0.6,-0.9]}\right\}.$$

Clearly F_1 and F_2 are NBVRWC sets and $F_1 \subseteq F_2$ but F_2 is not a subset of NBVcl(F_1).

Theorem 3.19: If A is NBVRO and NBVRWC set then A is NBVRC set and hence NBVclopen.

Proof: Suppose A is NBVRO and NBVRWC set. As every NBVRO set is NBVRSO and A ⊆ A, then NBVcl(A) ⊆ A and also A ⊆ NBVcl(A) implies NBVcl(A) = A. Thus A is NBVC set. Since A is NBVRO then A is NBVO set, NBVcl(NBV int(A)) = NBVcl(A) = A. Therefore A is NBVRC and hence NBV clopen.

Theorem 3.20: If A is NBVRO and NBVRGC set in a NBVT space (X,τ), then A is NBVRWC set in X.

Proof: Let A be a NBVRO and NBVRGC set in X. Let U be a NBVRSO in X such that A ⊆ U. By hypothesis, NBVcl(A) ⊆ A. Then NVBcl(A) ⊆ A ⊆ U which implies A is a NVBRWC set in X.

Theorem 3.21: If A is NBVRSO and NBVRWC set in (X,τ), then A is a NVBC set in X.

Proof: Since A is both NBVRSO and NBVRWC set, A ⊆ A implies NBVcl(A) ⊆ A. Thus A is NBVC set in X.

Corollary 3.22: If A is a NBVRSO and NBVRWC set in NBVT space (X,τ) and F is a NBVC set in X then A ∩ F is a NBVRWC set in X.

Proof: Let A be a NBVRSO and NBVRWC set. F be a NBVC set of X. Then by theorem 4.16, A is also a NVBC set, A ∩ F is also a NBVC set. Hence it is NBVRWC set in (X,τ).

Theorem 3.23: If A is both NBVO and NBVGC set in (X,τ). Then A is NBRWC set in X.

Proof: Let A be both NBVO and NBVGC set in NBVT space (X,τ). Let A ⊆ U where U is NBVRSO in X. Now A ⊆ A by hypothesis, NBVcl(A) ⊆ A, i.e., NBVcl(A) ⊆ U which implies A is NBVRWC set in X.

Theorem 3.24: NBVRWC set ⇒ (1) NBVRWGC set

(2) NBVGPRC set.

Proof: (i) Let A be a NBVRWC set. Let U be a NBVRO set such that A ⊆ U. Since every NBVRO set is NBVRSO set and by hypothesis, NBVcl(NBVint(A)) ⊆ NBVcl (A) ⊆ U. Hence A be a NBVRWGC set.

(ii) Let A be a NBVRWC set. Let A ⊆ U, U is NBVRO set. Since every NBVC set is NBVPC set and every NBVRO set is NBVRSO set, NBVpcl(A) ⊆ NBVcl(A) ⊆ U. Hence A is NBVGPRC set.

Remark 3.24: The converse of the above theorem need not be true as shown in the following example.

Example 3.25: Let X = {a,b}, τ = {0_{NBV},1_{NBV} ,G}. Clearly (X,τ) be a NBVT space where the NBVO set

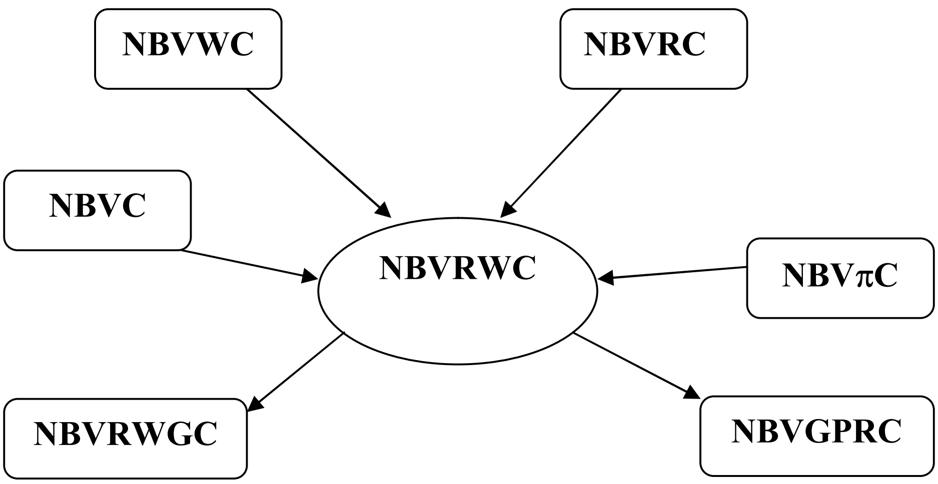
$G = \left\{x, \frac{a}{[0.2,0.3],[0.5,0.6],[0.7,0.8],[-0.3,-0.4],[-0.5,-0.5],[-0.6,-0.7]}, \frac{b}{[0.4,0.5],[0.5,0.7],[0.5,0.6],[-0.4,-0.5],[-0.5,-0.6],[-0.5,-0.6]}\right\}$. In the NBVT space (X,τ), the NBV set

$A = \left\{x, \frac{a}{[0.1,0.2],[0.5,0.7],[0.8,0.9],[-0.1,-0.2],[-0.6,-0.6],[-0.8,-0.9]}, \frac{b}{[0.3,0.4],[0.6,0.6],[0.6,0.7],[-0.2,-0.3],[-0.5,-0.7],[-0.7,-0.8]}\right\}$ is both NBVRWGC set and NBVGPRC set but not NBVRWC set.

Definition 3.26: A neutrosophic bipolar vague set A of a NBVT space (X,τ) is called a **neutrosophic bipolar vague regular weakly open (In short NBVRWO)** set, if its complement A^C is NBVRWC set in (X,τ).

The set of all NBVRWO sets of X is denoted by **NBVRWO(X).**

Theorem 3.27: If a subset A of a topological space is both NBVO and NBVWGC set, then it is NBVRWC set.
Proof: Suppose a subset A of X is both NBVO and NBVRWC set. Let A ⊆ U, U is NBVRSO in X. A = NBVcl (NBVint(A)) ⊆ A, as A is NBVO implies that NBVcl(A) ⊆ A ⊆ U. Thus A is a NBVRWC set.
The above discussions are implemented in the following diagram.



where A ⟶ B means that A implies B but not conversely.

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Theorem 3.27: Let A be a NBVRWO set of a NBVT space (X,τ) and NBV int(A) ⊆ B ⊆ A. Then B ∈ NBVRWO(X).

Proof: Let A be a NBVRWO in X such that NBVint(A) ⊆ B ⊆ A, which implies $A^C \subseteq B^C \subseteq (\text{NBV int } A)^C \Rightarrow A^C \subseteq B^C \subseteq$ NBVcl(A^C). By theorem 3.16, B^C is NBVRWC set implies that B is NBVRWO set.

Definition 3.28: A neutrosophic bipolar vague topological space is called **NBVRW connected space** if there is no proper NBV set of X which is both NBVRWC set and NVBRWO set.

Theorem 3.29: A NBVT space is NBVRW connected if and only if there exists no non-zero NBVRWO sets A and B in X such that A = B^C.

Proof: Necessity: Suppose that A and B are NBVRWO set such that $A \neq 0_{NBV} \neq B$ and A = B^C. Since A = B^C, B is a NBVRWO set, B^C = A is NBVRWC set and $B \neq 0$ implies that $B^C \neq 1_{NBV}$ $i.e., A \neq 1$. Hence there exists a proper NBV set A ($A \neq 0_{NBV}, A \neq 1_{NBV}$) such that A is both NBVRWC and NBVRWO set contradicts the hypothesis that X is NBVRW connected.
Sufficiency: Let (X,τ) be NBVT and A is both NBVRWC and NBVRWO set in X such that $0_{NBV} \neq A \neq 0_{NBV}$. Let B=A^C. Then B is NBVRWO set and $A \neq 1_{NBV}$ implies that$B = A^C \neq 0_{NBV}$, a contradiction. Therefore there is no proper NBV set of X which is both NBVRWC and NBVRWO. Thus the NBVT (X,τ) is NBVRW connected.

IV. APPLICATION OF NEUTROSOPHIC BIPOLAR VAGUE REGULAR WEAKLY CLOSED SETS

Definition 4.1: A mapping f: (X,τ) → (Y,σ) is neutrosophic Bipolar vague RW- continuous **(In short NBVRW- continuous)** if the inverse image of every neutrosophic Bipolar vague closed (NBVC) set of Y is neutrosophic Bipolar vague RW-closed (NBVRWC) set in X.

Theorem 4.2: A mapping f: (X,τ) → (Y,σ) is NBVRW continuous if and only if the inverse image of every NBVO set of Y is NBVRWO set in X.

Proof: The proof is obvious because $f^{-1}(U^C) = (f^{-1}(U))^C$ for every NBV set U.

Remark 4.3: (i) NBV-continuous ⇒ NBVRW-continuous

(ii) NBVW-continuous ⇒ NBVRW-continuous but the converse is not true as seen in the following example.

Example 4.4: Let X = {a,b}, Y = {p,q}, the neutrosophic topologies τ = {0_{NBV},1_{NBV} ,F} and σ = {0_{NBV},1_{NBV},G} where NBVO sets,

$$F = \left\{x, \frac{a}{[0.4,0.5],[0.6,0.7],[0.5,0.6],[-0.4,-0.4],[-0.6,-0.8],[-0.6,-0.6]}, \frac{b}{[0.2,0.4],[0.5,0.6],[0.6,0.8],[-0.3,-0.4],[-0.7,-0.8],[-0.6,-0.7]}\right\},$$

$$G = \left\{x, \frac{p}{[0.5,0.6],[0.5,0.5],[0.4,0.5],[-0.6,-0.4],[-0.6,-0.7],[-0.6,-0.4]}, \frac{q}{[0.6,0.8],[0.6,0.6],[0.2,0.4],[-0.8,-0.9],[-0.2,-0.2],[-0.1,-0.2]}\right\}$$

Define a function f: (X,τ) → (Y,σ) by f(a) = q, f(b) =p. Then f is NBVRW-continuous but it is not both NBV-continuous and NBVW-continuous function.

Theorem 4.5: If a function f: (X, τ) → (Y,σ) is NBVRW-continuous, then f(NBVRWcl(A) ⊆ NBVcl[f(A)] for every NBVsubset A of X.

Proof: Let A be a NBVset of X. Then NBVcl[f(A)] is NBVC set of Y. Since f is NBVRW-continuous, then f^{-1}(NBVRWcl(A)) is a NBVRWC set of X. Clearly A ⊆ f^{-1}(NBVcl[f(A)]) implies NBVRWcl(A) ⊆ NBVRWcl(f^{-1}(NBVcl[f(A)]) = f^{-1}(NBVcl[f(A)]). Thus f(NBVRWcl(A) ⊆ NBVcl[f(A)] for every NBV subset A of X.

Theorem 4.6: If f: (X, τ) → (Y,σ) is NBVRW-continuous, g: (Y, σ) → (Z,η) is NBV-continuous, then $g°f$: (X, τ) → (Z,η) is NBVRW-continuous.

Proof: Let A be a NBVC set in (Z,η), then $g^{-1}(A)$ is NBVC set in Y. Since f is NBVRW-continuous, therefore $(g°f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is NBVRW-closed set in X. Hence $g°f$ is NBVRW-continuous.

Remark 4.7: Every NBVRW-continuous function is (i) NBVGPR-continuous and (ii) NBVRWG-continuous but the converse is not true as seen in the following example.

Example 4.8: Let X = {a,b}, Y = {p,q}, the neutrosophic topologies $\tau = \{0_{NBV}, 1_{NBV}, F\}$ and $\sigma = \{0_{NBV}, 1_{NBV}, G\}$ where NBVO sets

$$F = \left\{x, \frac{a}{[0.4,0.5],[0.6,0.7],[0.5,0.6],[-0.4,-0.4],[-0.6,-0.8],[-0.6,-0.6]}, \frac{b}{[0.2,0.4],[0.5,0.6],[0.6,0.8],[-0.3,-0.4],[-0.7,-0.8],[-0.6,-0.7]}\right\},$$

$$G = \left\{x, \frac{p}{[0.8,0.8],[0.3,0.3],[0.2,0.2],[-0.6,-0.7],[-0.2,-0.4],[-0.3,-0.4]}, \frac{q}{[0.8,0.8],[0.4,0.4],[0.2,0.2],[-0.7,-0.7],[-0.2,-0.2],[-0.3,-0.3]}\right\}$$

Define a function f: $(X,\tau) \to (Y,\sigma)$ by f(a) = p, f(b) = q. Then f is NBVGPR-continuous and NBVRWG-continuous but it is not NBVRW-continuous function.

Remark 4.9 (i) NBVR-continuous ⇒ NBVRW-continuous F

(ii) NBVπ-continuous ⇒ NBVRW-continuous but the converse is not true as seen in the following example.

Example 4.10: Let X = {a,b}, Y = {p,q}, the neutrosophic topologies $\tau = \{0_{NBV}, 1_{NBV}, F\}$ and $\sigma = \{0_{NBV}, 1_{NBV}, G\}$ where NBVO sets,

$$F = \left\{x, \frac{a}{[0.2,0.3],[0.5,0.6],[0.7,0.8],[-0.3,-0.4],[-0.5,-0.5],[-0.6,-0.7]}, \frac{b}{[0.4,0.5],[0.5,0.7],[0.5,0.6],[-0.4,-0.5],[-0.5,-0.6],[-0.5,-0.6]}\right\},$$

$$G = \left\{x, \frac{p}{[0.5,0.6],[0.4,0.5],[0.4,0.5],[-0.6,-0.7],[-0.3,-0.4],[-0.3,-0.4]}, \frac{q}{[0.6,0.4],[0.5,0.5],[0.6,0.4],[-0.5,-0.6],[-0.4,-0.4],[-0.6,-0.5]}\right\}$$

Define a function f: $(X,\tau) \to (Y,\sigma)$ by f(a) = p, f(b) = q. Then f is NBVRW-continuous but it is not both NBVR-continuous and NBVπ-continuous function.

Theorem 4.11: If f: $(X, \tau) \to (Y,\sigma)$ is NBVRW-continuous, g: $(Y, \sigma) \to (Z,\eta)$ is NBVG-continuous and (Y,σ) is NBV $T_{1/2}$ space then $g \circ f$: $(X, \tau) \to (Z,\eta)$ is NBVRW-continuous.

Proof: Let A be a NBVC set in (Z,η), then by hypothesis, $g^{-1}(A)$ is NBVGC set in (Y,σ). Since Y is NBV $T_{1/2}$ space, then $g^{-1}(A)$ is NBVC set, obviously NBVRWC set in (X,τ). Hence $g \circ f$ is NBVRW-continuous function.

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