Comparing Different Fuzzy reliability Estimation of Exponential Rayleigh Probability Distribution

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Abstract:This paper deals with comparing three different fuzzy estimators of hazard function of new mixture distribution from exponential (2γ) and Rayleigh with (β), after the mixed p.d.f found, mixed CDF, mixed reliability, and hazard rate, found, we work on estimating its two parameters (α, β) by three different methods, which are maximum likelihood, Moments, and ordinary least square method, then fuzzy reliability estimator compared and the results of comparison using simulation procedure explained in special tables.

Keywords: Maximum Likelihood Estimator, Moments Method, ordinary least square, Rayleigh Distribution, Exponential Distribution.

I. Introduction

The development in medical and engineering fields and another scientific research depend on related data analysis and extract information, that will help us to analyze data and to develop many statistical methods in processing and extracting data, one of them is to obtain a new mixture of distribution (Gauss 2013), also to expand the given distribution (Gauss M., Corderio etal 2016), as well as Kumaraswamy exponential Weibull distribution (2016), Faton Mervoci and Ibrahim Elbatal (2015) introduce a new distribution called (Weibull Rayleigh distribution), many articles are introduced to analyze and estimate parameters of new mixture distribution. We continue the work in this subject and explain one of mixed distribution called exponential Rayleigh, the parameters (γ, β) are estimated, then fuzzy reliability function is compared.

II. Theoretical Aspect

We introduce the p.d.f and CDF and reliability function of mixed exponential Rayleigh with two parameters (γ, β), then work on estimation these two parameters by three different methods like, MLE,MOM, ordinary least square.

Let;

$$f(x) = (2\gamma + \beta x)e^{\left(2\gamma x + \frac{\beta}{2}x^2\right)} \quad \gamma, \beta, x > 0 \qquad (1)$$

The corresponding CDF is;

$$G(x) = 1 - e^{\left(2\gamma x + \frac{\beta}{2}x^2\right)} \qquad (2)$$

While the reliability and hazard functions are;

$$R_X(x) = e^{-\left(2\gamma x + \frac{\beta}{2}x^2\right)} \qquad (3)$$

$$hX^x = 2\gamma + \beta x$$

The two parameters (γ, β) are estimated by MLE, MOM, and PEC.

$$\hat{R}_X(k_i, x_i) = e^{-\left(2\hat{\gamma}k_i x_i + \frac{\hat{\beta}}{2}(k_i x_i)^2\right)} \quad (4)$$

After estimating the two parameters (γ, β), then $[\hat{R}_X(k_i x_i)]$ compared using given values of $(k_i = 0.3, 0.6)$, $(\gamma = 1.5, 3)$, and $(\beta = 2.5, 4)$.

III. Estimation methods

The two parameters (γ, β), are estimated by MLE, ordinary least square, and MOM.

3.1 Estimation by MLE

Let $(x_1, x_2, \dots, x_n)$ be a random sample from p.d.f in equation (1), then;

$$L = \prod_{i=1}^{n} f(x_i, \gamma, \beta) = \prod_{i=1}^{n} (2\gamma + \beta x_i)\, e^{-\sum_{i=1}^{n}\left(2\hat{\gamma}x_i + \frac{\hat{\beta}}{2}x_i^2\right)} \quad (5)$$

$$\log L = \sum_{i=1}^{n} \log(2\gamma + \beta x_i) - \sum_{i=1}^{n}\left(2\hat{\gamma}x_i + \frac{\hat{\beta}}{2}x_i^2\right)$$

$$\frac{\partial \log L}{\partial \gamma} = \sum_{i=1}^{n} \frac{2}{(2\gamma + \beta x_i)} - 2\sum_{i=1}^{n} x_i = 0$$

$$n\bar{x} = \sum_{i=1}^{n} \frac{1}{(2\hat{\gamma} + \hat{\beta} x_i)} \quad (6)$$

$$\frac{\partial \log L}{\partial \beta} = \sum_{i=1}^{n} \frac{x_i}{(2\gamma + \beta x_i)} - \sum_{i=1}^{n} \frac{x_i^2}{2} = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{(2\gamma + \beta x_i)} = \sum_{i=1}^{n} \frac{x_i^2}{2} \quad (7)$$

3.2 Estimation by MOM

In this method $(\hat{\gamma}_{MOM}, \hat{\beta}_{MOM})$ are obtained by solving $[\mu'_r = E(x^r)]$, since;

$$\mu'_r = E(x^r) = \frac{\gamma\Gamma\left(\frac{1}{\beta}+1\right)\Gamma(\gamma)}{\Gamma\left(\frac{1}{\beta}+\gamma+1\right)} \quad (8)$$

$$\frac{\sum_{i=1}^{n} x_i}{n} = r \;\; \frac{\left[\Gamma\left(\frac{1}{\beta}+1\right)\Gamma(\gamma+1)\right]}{\Gamma\left(\frac{1}{\beta}+\gamma+1\right)} \quad (9)$$

$$\frac{\sum_{i=1}^{n} x_i^2}{n} = r \;\; \frac{\left[\Gamma\left(\frac{2}{\beta}+1\right)\Gamma(\gamma+1)\right]}{\Gamma\left(\frac{1}{\beta}+\gamma+1\right)} \quad (10)$$

Then taking $(\gamma, \beta \geq 1)$ integer and solving equations (9) and (10) numerically we get, $(\hat{\gamma}_{MOM}, \hat{\beta}_{MOM})$.

3.3 Estimation by Ordinary Least Square (OLS)

The estimators $(\hat{\gamma}, \hat{\beta})$ are obtained by minimizing total sum square of the difference between sample values (i) and the value $[E(p_i)]$.

$$\sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n} [p_i - E(p_i)]^2$$

$$= \sum_{i=1}^{n} \left(1 - e^{-\left(2\gamma x_i + \frac{\beta}{2} x_i^2\right)} - \frac{i}{n+1}\right)^2 \quad (11)$$

$$\frac{\partial \sum_{i=1}^{n} \epsilon_i^2}{\partial \gamma} = 2 \sum_{i=1}^{n} \left(1 - e^{-\left(2\gamma x_i + \frac{\beta}{2} x_i^2\right)} - \frac{i}{n+1}\right) (2x_i e^{-2\gamma x_i})$$

$$\sum_{i=1}^{n} x_i\, e^{-2\hat{\gamma} x_i} - \sum_{i=1}^{n} x_i\, e^{-4\,\hat{\gamma} x_i} - \sum_{i=1}^{n} \frac{i}{n+1}\;\; x_i\, e^{-2\hat{\gamma} x_i} = 0 \quad (12)$$

$$\frac{\partial \sum_{i=1}^{n} \epsilon_i^2}{\partial \beta} = 2 \sum_{i=1}^{n} \left(1 - e^{-\left(2\gamma x_i + \frac{\beta}{2} x_i^2\right)} - \frac{i}{n+1}\right) \left(-\frac{x_i^2}{2} e^{-\left(\frac{\beta}{2} x_i^2\right)}\right) = 0$$

$$\sum_{i=1}^{n} x_i^2\, e^{-\left(\frac{\beta}{2} x_i^2\right)} - \sum_{i=1}^{n} x_i^2\, e^{-\left(2\gamma x_i + \frac{\beta}{2} x_i^2\right)} + \sum_{i=1}^{n} \frac{i}{n+1}\;\; x_i^2\, e^{-\left(\frac{\beta}{2} x_i^2\right)} = 0 \quad (13)$$

From equations (12) and (13) we can find $\hat{\beta}_{OLS}$ and $\hat{\gamma}_{OLS}$

IV. Simulation Aspect

The fuzzy reliability function of mixed Exp – Rayliegh is compared by three different methods which are; MLE, MOM, and OLS, were;

$$\hat{R}_X(k_i, x_i) = e^{-\left(2\hat{\gamma} k_i x_i + \frac{\hat{\beta}}{2} (k_i x_i)^2\right)}$$

$$f(x_i, \gamma, \beta) = 1 - e^{-\left(2\gamma x_i + \frac{\beta}{2} x_i^2\right)}$$

$$u_i = 1 - e^{-\left(2\gamma x_i + \frac{\beta}{2} x_i^2\right)}$$

Let $(z_i = u_i)$

$$z_i = 1 - e^{-\left(2\gamma x_i + \frac{\beta}{2} x_i^2\right)}$$

$$\ln\, z_i = -\left(2\gamma x_i + \frac{\beta}{2} x_i^2\right)$$

$$x_i^2 = -\frac{1}{\beta} \ln\, z_i$$

To find the estimator's $(MLE, MOM\ \&\ OLS)$ and $(n = 25,50,75)$, $(R = 300)$, we perform simulation experiments using Monte Carlo assuming that;

k_i	**0.3**	**0.6**
β	**1.5**	**4**
γ	**1.5**	**3**

Table (1): Estimator of Fuzzy Reliability when ($\beta = 1.5, \gamma = 1.5, k_i = 0.3$)

n	t_i	Real R_i	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{OLS}$	Best
25	1.5	0.4889	0.4886	0.3396	0.3384	MLE
	2.5	0.5032	0.4577	0.3974	0.3976	MOM
	3.5	0.5082	0.5029	0.4345	0.4361	MOM
	4.5	0.5324	0.5219	0.4806	0.4802	MOM
	5.5	0.5614	0.5536	0.4961	0.4952	MLE
	6.5	0.5662	0.5702	0.5008	0.5068	MLE
	7.5	0.5782	0.5778	0.4242	0.5136	MLE
	8.5	0.5899	0.5853	0.5308	0.5242	MLE
	9.5	0.6032	0.5864	0.5306	0.5963	OLS
	10.5	0.6208	0.6019	0.5301	0.5308	OLS
50	1.5	0.4889	0.4006	0.3262	0.3116	MLE
	2.5	0.5032	0.4134	0.3844	0.3749	MLE
	3.5	0.5082	0.4743	0.4266	0.4166	MLE
	4.5	0.5324	0.5202	0.4538	0.5452	OLS
	5.5	0.5614	0.5485	0.4726	0.4657	OLS
	6.5	0.5662	0.5664	0.4891	0.4812	OLS
	7.5	0.5782	0.5709	0.5012	0.4936	OLS
	8.5	0.5899	0.5873	0.5109	0.5123	OLS
	9.5	0.6032	0.5993	0.5123	0.5224	OLS
	10.5	0.6208	0.6103	0.5266	0.5148	OLS
75	1.5	0.4889	0.4889	0.3263	0.3102	OLS
	2.5	0.5032	0.5032	0.3815	0.3802	OLS
	3.5	0.5082	0.5082	0.4214	0.4212	OLS
	4.5	0.5324	0.5324	0.4493	0.4483	OLS
	5.5	0.5614	0.5614	0.4687	0.4699	OLS
	6.5	0.5662	0.5662	0.5856	0.4852	MOM
	7.5	0.5782	0.5782	0.5980	0.4873	MOM
	8.5	0.5899	0.5899	0.5981	0.5928	MOM
	9.5	0.6032	0.6032	0.5166	0.5080	MOM
	10.5	0.6208	0.6208	0.5221	0.5162	MOM

Table (2): Estimator of Fuzzy Reliability when ($\beta = 4, \gamma = 1.5, k_i = 0.6$)

n	t_i	Real R_i	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{OLS}$	Best
25	1.5	0.2668	0.2667	0.3126	0.2955	MOM
	2.5	0.3173	0.3504	0.3507	0.3342	MLE
	3.5	0.3453	0.3768	0.3769	0.3642	MOM
	4.5	0.3667	0.3861	0.4108	0.3802	MOM
	5.5	0.3826	0.4033	0.4226	0.4002	MOM
	6.5	0.3974	0.4266	0.4319	0.4115	MOM
	7.5	0.4045	0.4338	0.4296	0.4303	MLE
	8.5	0.4132	0.4409	0.4463	0.4375	MOM
	9.5	0.4262	0.4467	0.4402	0.4426	MLE
	10.5	0.4468	0.4458	0.4522	0.4006	MLE
50	1.5	0.2668	0.2867	0.2982	0.4107	OLS
	2.5	0.3173	0.3280	0.3368	0.4188	OLS
	3.5	0.3453	0.3543	0.3634	0.4247	OLS
	4.5	0.3667	0.3768	0.3829	0.4288	OLS
	5.5	0.3826	0.3826	0.3978	0.4269	OLS
	6.5	0.3974	0.4052	0.4062	0.5021	OLS
	7.5	0.4045	0.4152	0.4192	0.5223	OLS
	8.5	0.4132	0.4235	0.4272	0.5312	OLS
	9.5	0.4262	0.4304	0.4337	0.5422	OLS
	10.5	0.4468	0.4354	0.4396	0.5472	OLS
75	1.5	0.2668	0.2967	0.3225	0.2863	MOM
	2.5	0.3173	0.3367	0.3506	0.3367	MOM
	3.5	0.3453	0.3563	0.2983	0.2956	MLE
	4.5	0.3667	0.3724	0.3347	0.3363	MLE
	5.5	0.3826	0.3776	0.3523	0.3542	MLE
	6.5	0.3974	0.4052	0.3649	0.3846	MLE
	7.5	0.4045	0.4125	0.3977	0.4006	MLE
	8.5	0.4132	0.4235	0.4097	0.4126	MLE
	9.5	0.4262	0.4506	0.4183	0.4305	MLE
	10.5	0.4468	0.4364	0.4272	0.4434	OLS

Table (3): Estimator of Fuzzy Reliability when ($\beta = 1.5, \gamma = 3, k_i = 0.3$)

n	t_i	Real R_i	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{OLS}$	Best
25	1.5	0.4856	0.4886	0.4102	0.2998	MLE
	2.5	0.5033	0.4558	0.5333	0.3546	MOM
	3.5	0.3566	0.5029	0.5641	0.3552	MOM
	4.5	0.5324	0.5319	0.5521	0.3762	MOM
	5.5	0.5614	0.5536	0.5712	0.5532	OLS
	6.5	0.5662	0.5701	0.5872	0.4505	MOM
	7.5	0.5782	0.6776	0.6011	0.4612	MLE
	8.5	0.5899	0.5021	0.5502	0.4613	MLE
	9.5	0.6032	0.4943	0.4073	0.5625	OLS
	10.5	0.6208	0.6019	0.4762	0.4673	MLE
50	1.5	0.4856	0.4066	0.4892	0.4778	MLE
	2.5	0.5033	0.4743	0.5201	0.4892	MOM
	3.5	0.3566	0.5201	0.5496	0.2912	MOM
	4.5	0.5324	0.5485	0.5311	0.3190	MLE
	5.5	0.5614	0.5864	0.5862	0.3464	MLE
	6.5	0.5662	0.5709	0.5440	0.3765	MLE
	7.5	0.5782	0.5873	0.6102	0.3802	MOM
	8.5	0.5899	0.6103	0.6246	0.4025	MOM
	9.5	0.6032	0.6224	0.6372	0.4066	MOM
	10.5	0.6208	0.6324	0.6524	0.4267	MLE
75	1.5	0.4856	0.5868	0.6552	0.4305	MOM
	2.5	0.5033	0.5664	0.6561	0.6302	MOM
	3.5	0.3566	0.5824	0.7021	0.6227	MOM
	4.5	0.5324	0.4048	0.5133	0.4015	MOM
	5.5	0.5614	0.4130	0.5244	0.4026	MOM
	6.5	0.5662	0.4206	0.5336	0.3995	MOM
	7.5	0.5782	0.4256	0.5456	0.3998	MOM
	8.5	0.5899	0.4346	0.3546	0.3999	MOM
	9.5	0.6032	0.4451	0.4548	0.4260	MOM
	10.5	0.6208	0.4566	0.4600	0.4444	MOM

Table (4): Estimator of Fuzzy Reliability when ($\beta = 1.5, \gamma = 3, k_i = 0.6$)

n	t_i	Real R_i	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{OLS}$	Best
25	1.5	0.2668	0.2998	0.2809	0.2806	MLE
	2.5	0.3162	0.3546	0.3201	0.3202	MLE
	3.5	0.3664	0.3552	0.4466	0.4461	MOM
	4.5	0.3825	0.3761	0.4462	0.4465	OLS
	5.5	0.3904	0.4233	0.5013	0.5016	OLS
	6.5	0.4048	0.4505	0.5026	0.5066	OLS
	7.5	0.4130	0.4606	0.5031	0.5036	OLS
	8.5	0.4200	0.4612	0.5036	0.5033	MOM
	9.5	0.4256	0.4613	0.5036	0.5036	MOM
	10.5	0.4346	0.4625	0.5047	0.5067	MOM
50	1.5	0.2668	0.4362	0.2902	0.2868	MLE
	2.5	0.3162	0.3282	0.3192	0.3286	OLS
	3.5	0.3664	0.3445	0.3464	0.3566	OLS
	4.5	0.3825	0.3606	0.3765	0.3776	OLS
	5.5	0.3904	0.3621	0.3802	0.3927	OLS
	6.5	0.4048	0.4225	0.4025	0.4251	OLS
	7.5	0.4130	0.3162	0.4252	0.4152	MOM
	8.5	0.4200	0.3664	0.4326	0.4235	MOM
	9.5	0.4256	0.3904	0.4331	0.4325	MOM
	10.5	0.4346	0.4048	0.4362	0.4364	OLS
75	1.5	0.2668	0.4132	0.2868	0.4364	OLS
	2.5	0.3162	0.4201	0.3284	0.4308	OLS
	3.5	0.3664	0.4256	0.3563	0.3606	MLE
	4.5	0.3825	0.4346	0.3766	0.3728	MLE
	5.5	0.3904	0.4421	0.3927	0.3607	MLE
	6.5	0.4048	0.5001	0.4051	0.4606	MLE
	7.5	0.4130	0.5161	0.4162	0.4502	MLE
	8.5	0.4200	0.5172	0.4235	0.4241	MLE
	9.5	0.4256	0.6026	0.4364	0.4336	MLE
	10.5	0.4346	0.6043	0.4372	0.4452	MLE

Table (5): Estimator of Fuzzy Reliability when ($\beta = 4, \gamma = 1.5, k_i = 0.3$)

n	t_i	Real R_i	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{OLS}$	Best
25	1.5	0.2666	0.2948	0.2098	0.2809	MLE
	2.5	0.3162	0.3736	0.3564	0.3223	MLE
	3.5	0.3825	0.3871	0.3762	0.4462	OLS
	4.5	0.3904	0.4032	0.4242	0.5023	OLS
	5.5	0.4048	0.4155	0.4505	0.5026	OLS
	6.5	0.4130	0.4255	0.4606	0.5032	OLS
	7.5	0.4200	0.4336	0.5036	0.5466	OLS
	8.5	0.4222	0.4339	0.4612	0.5022	OLS
	9.5	0.4256	0.4409	0.4613	0.5036	OLS
	10.5	0.4468	0.4625	0.4661	0.5047	OLS
50	1.5	0.2666	0.2868	0.2974	0.2921	MLE
	2.5	0.3162	0.3482	0.3199	0.3364	MLE
	3.5	0.3825	0.3563	0.3277	0.3464	MLE
	4.5	0.3904	0.3766	0.3464	0.3566	MLE
	5.5	0.4048	0.3977	0.3765	0.3765	OLS
	6.5	0.4130	0.4052	0.4062	0.4147	OLS
	7.5	0.4200	0.4235	0.4267	0.4326	OLS
	8.5	0.4222	0.4327	0.4315	0.4366	OLS
	9.5	0.4256	0.4365	0.4472	0.4406	OLS
	10.5	0.4468	0.5485	0.4762	0.4566	MLE
75	1.5	0.2666	0.4336	0.5202	0.4073	MOM
	2.5	0.3162	0.4802	0.5486	0.4762	MOM
	3.5	0.3825	0.4816	0.5711	0.4892	MOM
	4.5	0.3904	0.4926	0.5862	0.5201	MOM
	5.5	0.4048	0.5212	0.6002	0.5496	MOM
	6.5	0.4130	0.5412	0.6102	0.5711	MOM
	7.5	0.4200	0.5617	0.6223	0.5862	MOM
	8.5	0.4222	0.6166	0.6446	0.6002	MOM
	9.5	0.4256	0.6277	0.6276	0.6102	MOM
	10.5	0.4468	0.6306	0.6326	0.6246	MOM

Table (6): Estimator of Fuzzy Reliability when ($\beta = 4, \gamma = 1.5, k_i = 0.6$)

n	t_i	Real R_i	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{OLS}$	Best
25	1.5	0.4886	0.4886	0.4336	0.2998	MLE
	2.5	0.5032	0.4577	0.4832	0.3546	MOM
	3.5	0.5082	0.5029	0.5326	0.3552	MOM
	4.5	0.5324	0.5319	0.5624	0.4063	MOM
	5.5	0.5514	0.5936	0.5822	0.4371	MLE
	6.5	0.5662	0.5702	0.5672	0.4828	MLE
	7.5	0.5782	0.5776	0.5882	0.5202	MOM
	8.5	0.5899	0.5821	0.6094	0.5398	MOM
	9.5	0.6032	0.5843	0.6196	0.5422	MOM
	10.5	0.6208	0.6019	0.6203	0.5398	MOM
50	1.5	0.4886	0.4066	0.4792	0.5422	OLS
	2.5	0.5032	0.4743	0.4896	0.5712	OLS
	3.5	0.5082	0.5282	0.5132	0.5872	OLS
	4.5	0.5324	0.5306	0.5103	0.6011	OLS
	5.5	0.5514	0.5526	0.5322	0.6102	OLS
	6.5	0.5662	0.5607	0.5641	0.6203	OLS
	7.5	0.5782	0.5682	0.5521	0.6208	OLS
	8.5	0.5899	0.5694	0.5654	0.6122	OLS
	9.5	0.6032	0.5701	0.5902	0.6145	OLS
	10.5	0.6208	0.5723	0.6203	0.6306	OLS
75	1.5	0.4886	0.4889	0.4063	0.6442	OLS
	2.5	0.5032	0.5032	0.4372	0.5872	MOM
	3.5	0.5082	0.5086	0.4829	0.6011	MOM
	4.5	0.5324	0.5324	0.5202	0.6102	OLS
	5.5	0.5514	0.5614	0.5398	0.6342	OLS
	6.5	0.5662	0.5662	0.5712	0.5806	OLS
	7.5	0.5782	0.5904	0.5872	0.5960	OLS
	8.5	0.5899	0.6208	0.6011	0.5682	MLE
	9.5	0.6032	0.6311	0.6102	0.6022	MLE
	10.5	0.6208	0.6824	0.6204	0.6103	MLE

Table (7): Estimator of Fuzzy Reliability when ($\beta = 4, \gamma = 3, k_i = 0.3$)

n	t_i	Real R_i	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{OLS}$	Best
25	1.5	0.3206	0.3116	0.3226	0.3117	MOM
	2.5	0.3743	0.3262	0,3749	0.3748	MOM
	3.5	0.4165	0.3762	0.4161	0.4166	MOM
	4.5	0.4462	0.3852	0.4452	0.4452	MOM
	5.5	0.4667	0.3864	0.4657	0.4647	MOM
	6.5	0.4942	0.4268	0.4812	0.4722	MOM
	7.5	0.5032	0.4538	0.4836	0.4732	MOM
	8.5	0.5043	0.4726	0.5023	0.4836	MOM
	9.5	0.5126	0.4862	0.5224	0.5123	MOM
	10.5	0.5194	0.5012	0.5324	0.5224	MOM
50	1.5	0.3206	0.5108	0.5364	0.5322	MOM
	2.5	0.3743	0.5242	0.5582	0.5148	MOM
	3.5	0.4165	0.3263	0.5592	0.5235	MOM
	4.5	0.4462	0.3815	0.3102	0.5442	MOM
	5.5	0.4667	0.4214	0.3622	0.5462	OLS
	6.5	0.4942	0.4493	0.3804	0.5472	OLS
	7.5	0.5032	0.4665	0.4212	0.5462	OLS
	8.5	0.5043	0.5166	0.4483	0.5177	OLS
	9.5	0.5126	0.5232	0.4699	0.5362	OLS
	10.5	0.5194	0.3116	0.4852	0.5405	OLS
75	1.5	0.3206	0.3758	0.4852	0.5416	OLS
	2.5	0.3743	0.4172	0.4982	0.5432	OLS
	3.5	0.4165	0.4454	0.5082	0.5442	OLS
	4.5	0.4462	0.4662	0.5226	0.5462	OLS
	5.5	0.4667	0.4703	0.5237	0.5471	OLS
	6.5	0.4942	0.5845	0.5306	0.5472	MLE
	7.5	0.5032	0.5666	0.5416	0.5473	MLE
	8.5	0.5043	0.5632	0.5422	0.5477	MLE
	9.5	0.5126	0.6222	0.6016	0.5532	MLE
	10.5	0.5194	0.6326	0.6166	0.6216	MLE

Table (8): Estimator of Fuzzy Reliability when ($\beta = 4, \gamma = 3, k_i = 0.6$)

n	t_i	Real R_i	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{OLS}$	Best
25	1.5	0.3668	0.3068	0.3261	0.4316	OLS
	2.5	0.4062	0.3676	0.3686	0.3116	MOM
	3.5	0.4153	0.4165	0.4224	0.3758	OLS
	4.5	0.4226	0.4562	0.4463	0.4172	MLE
	5.5	0.5311	0.4665	0.4678	0.4454	MLE
	6.5	0.6062	0.4842	0.4857	0.4662	MOM
	7.5	0.6172	0.5032	0.4685	0.4803	OLS
	8.5	0.6326	0.5036	0.4980	0.4826	MLE
	9.5	0.6736	0.5126	0.5082	0.4837	MLE
	10.5	0.6566	0.5194	0.5166	0.5056	MLE
50	1.5	0.3668	0.3206	0.4231	0.5132	OLS
	2.5	0.4062	0.3747	0.4454	0.5162	OLS
	3.5	0.4153	0.4156	0.4662	0.5224	OLS
	4.5	0.4226	0.4462	0.4803	0.5304	OLS
	5.5	0.5311	0.4667	0.4945	0.5406	OLS
	6.5	0.6062	0.5043	0.5056	0.5106	OLS
	7.5	0.6172	0.5129	0.5130	0.6226	OLS
	8.5	0.6326	0.5194	0.5221	0.6236	OLS
	9.5	0.6736	0.5244	0.5306	0.6306	OLS
	10.5	0.6566	0.5366	0.5321	0.6224	OLS
75	1.5	0.3668	0.7263	0.5104	0.6086	MLE
	2.5	0.4062	0.7815	0.5702	0.6246	MLE
	3.5	0.4153	0.6214	0.5212	0.6308	MLE
	4.5	0.4226	0.6493	0.4463	0.6321	MLE
	5.5	0.5311	0.6687	0.5677	0.6322	MLE
	6.5	0.6062	0.6856	0.4853	0.6344	MLE
	7.5	0.6172	0.6883	0.4667	0.6404	MLE
	8.5	0.6326	0.5082	0.5062	0.6412	OLS
	9.5	0.6736	0.5166	0.5162	0.6422	OLS
	10.5	0.6566	0.5232	0.5227	0.6455	OLS

V. CONCLUSION

From the results of simulation for comparing the fuzzy reliability function of mixed Ex – Rayleigh we find that the first best one is the OLS, second one is MOM, and the third is MLE. The results of comparison indicates that the best fuzzy reliability that the first best one is the OLS, second one is MOM, and the third is MLE, table below summarize the above results.

Table (9): Reliability Results Summery

Tables	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$\hat{R}_{OLS}$
1	**0.266**	**0.266**	**0.466**
2	**0.366**	**0.266**	**0.333**
3	**0.300**	**0.633**	**0.066**
4	**0.300**	**0.233**	**0.400**
5	**0.266**	**0.333**	**0.400**
6	**0.233**	**0.300**	**0.500**
7	**0.166**	**0.466**	**0.366**
8	**0.400**	**0.066**	**0.533**

From table (9) we conclude that OLS is the best with percentage (38%), MOM with percentage (32%), and MLE with percentage (30%).

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