STUDY ON VELOCITY OF TEMPERATURE
CONSIDERING INCOMPRESSIBLE DUSTY FLUID.

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ABSTRACT

The velocity of temperature considering incompressible dusty fluid has been studied. The problem is investigated considering the temperature and velocity are differing from surrounding stream and the flow characteristics and properties in fluid mechanics. Taking non dimensionless variable and using perturbation method the non linear differential equation are transferred to linear form for numerical solution. We adopted the laplace transformation technique for solving the differential equation. Result shows that the magnitude of temperature velocity reduce significantly.

Key word and phrase : Dusty Fluid, particulate suspension , boundary layer characteristics , incompressible flow.

INTRODUCTION

We have studied the effect of temperature flow in the incompressible fluid of the dusty fluid in cylindrical polar coordinates. The temperature and the velocity of the dusty fluid are assumed to be different from the surrounding stream, a perturbation method is used to solve the linearize differential equation. The Laplace transformation technique is used to solve the differential equation. The profiles of perturbation particle temperature has been discussed numerically. The finite volume fraction shows the magnitude of temperature fluid particle reduced significantly.

NOMENCLATURE

\( (u, v, w) \) = Velocity components of fluid phase.
\( (u_p, v_p, w_p) \) = Velocity components of particle phase.
\( (\bar{u}, \bar{v}, \bar{w}) \) = Dimensionless velocity components of fluid phase.
\( (\bar{u}_p, \bar{v}_p, \bar{w}_p) \) = Dimensionless velocity components of particle phase.

\( T \) = Temperature of fluid phase.
\( T_p \) = Temperature of particle phase.
\( C_p, C_s \) = Specific heats of fluid and SPM respectively.
\( K \) = Thermal conductivity.
\( R_e \) = Fluid phase Reynolds number.

MATHEMATICAL FORMULATION

The flow of two-phase boundary layer in cylindrical polar co-ordinate can be written as

In Fluid phase the Heat Equation is

\[
(1 - \phi)\rho_p C_p \left( u \frac{\partial T}{\partial z} + v \frac{\partial T}{\partial r} \right) = K \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \rho_p C_s \frac{(T_p - T)}{\tau_t} \quad (1)
\]

In Particle phase the Heat Equation is

\[
\rho_p C_p \left( u_p \frac{\partial T_p}{\partial z} + v_p \frac{\partial T_p}{\partial r} \right) = -\rho_p C_s \frac{(T_p - T)}{\tau_T} \quad (2)
\]

We have taken the dimensionless variables

\[
\bar{r} = \frac{r}{(\tau_m \nu)^\frac{1}{2}}, \bar{u} = \frac{u}{U}, \bar{v} = \nu \left( \frac{\tau_m}{\nu} \right)^\frac{1}{2}, \bar{u}_p = \frac{u_p}{U}, \bar{v}_p = \nu_p \left( \frac{\tau_m}{\nu} \right)^\frac{1}{2}, \alpha = \frac{\rho_p}{\rho} = C
\]
\[ \bar{\rho}_p = \frac{\rho_p}{\rho_{p_0}}, \quad \bar{T} = \frac{T}{T_0}, \quad \bar{T}_p = \frac{T_p}{T_0}, \quad \lambda = \tau_m U, \quad \tau_m = \frac{2}{3} \frac{C_p}{C_s} \frac{1}{p_r}, \quad \mu \frac{\tau_r}{p_r} = \frac{\mu C_p}{K}. \]

Here the thermal conductivity \( K \) and the viscosity \( \mu \) are keeping constant. We also assume that the mixing region to be approximately constant. Hence, the pressure at the exit is equal to that of the surrounding stream. Therefore, both the velocity and the temperature in the jet is different from that of the surrounding stream. Then it is possible to write \( T = T_0 + T_1, \quad T_p = T_{p_0} + T_{p_1}, \quad \rho_p = \rho_{p_1} \) where the basic values with subscripts ‘0’ and the subscripts 1 denotes the perturbed values which is much smaller than the basic value, i.e. \( T_{p_0} >> T_{p_1} \). Using the above dimensional variable and the perturbation method the equations (1) and (2) can be written as

\[ (1 - \phi) u_0 \frac{\partial T_1}{\partial z} = \frac{1}{p_r} \left( \frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} \right) + \frac{2\alpha}{3p_r} \rho_{p_1} \left( T_{p_0} - T_0 \right) \quad (3) \]

\[ u_{p_0} \frac{\partial T_{p_1}}{\partial z} = \frac{2}{3} \frac{1}{p_r} \left[ (T_0 - T_{p_0}) + (T_1 - T_{p_1}) \right] \quad (4) \]

The boundary conditions for \( T_1, T_{p_1} \) are

\[ T_1(0, r) = \begin{cases} T_{10}, & r \leq 1 \\ 0, & r > 1 \end{cases} \quad (5) \]

\[ \frac{\partial T_1}{\partial r}(z, 0) = 0, \quad T_1(z, \infty) = 0 \quad (6) \]

\[ T_{p_1}(0, r) = \begin{cases} T_{p_{10}}, & r \leq 1 \\ 0, & r > 1 \end{cases} \quad (7) \]

The boundary conditions for particle density \( \rho_{p_1} \) are

\[ \rho_{p_1}(0, r) = \begin{cases} \rho_{p_{10}}, & r \leq 1 \\ 0, & r > 1 \end{cases} \quad (8) \]

SOLUTION

The governing equation (4) have been solved by using Laplace transform technique and using the relevant conditions from (5) to (8) we get
\[ u_{p_0} \frac{\partial T_{p_0}}{\partial z} = \frac{2}{3 \beta_r} \left[ (T_0 - T_{p_0}) + (T_1 - T_{p_1}) \right] \]

Taking Laplace Transform on both sides

\[ \Rightarrow u_{p_0} \left( \frac{\partial T_{p_0}}{\partial z} \right) = \frac{2}{3 \beta_r} \left[ L(T_0 - T_{p_0}) + L(T_1 - T_{p_1}) \right] \]

\[ \Rightarrow \frac{\partial T_{p_0}}{\partial z} = \frac{2}{3 \beta_r, u_{p_0}} K + \frac{2}{3 \beta_r, u_{p_0}} T_1 - \frac{2}{3 \beta_r, u_{p_0}} T_{p_1}^*, \text{where } K = \frac{T_0 - T_{p_0}}{p} \]

\[ \Rightarrow \frac{\partial T_{p_1}}{\partial z} + \frac{2}{3 \beta_r, u_{p_0}} T_{p_1}^* = \frac{2}{3 \beta_r, u_{p_0}} \left( K + T_1^* \right) \]

\[ \Rightarrow \frac{\partial T_{p_1}}{\partial z} + CT_{p_1}^* = C \left( K + T_1^* \right) \]

Where \( C = \frac{2}{3 \beta_r, u_{p_0}} \)

Which is linear first order differential equation. To obtain the solution

\[ I.F = e^{C dz} \]

The required Solution is

\[ T_{p_1} e^{C dz} = \int C \left( K + T_1^* \right) e^{C dz} \]

\[ = e^{-C dz} \int C \left( K + T_1^* \right) e^{C dz} \]

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\[ T_{p_1}^* = e^{-C dz} \left[ C \left( K + T_1^* \right) \frac{e^{C dz}}{C} - \int \frac{\partial T_{p_1}^*}{\partial z} e^{C dz} \right] \]

We have

\[ T_1^* = \left( T_{p_0} - \frac{2 \alpha E \rho_0}{3 \beta_r} \right) j_1(p) \frac{e^{Ap^2}}{3 \beta_r} + \frac{2 \alpha E \rho_0}{3 \beta_r} j_1(p) \frac{e^{Ap^2}}{3 \beta_r} \]

Now
\[\int e^{cz} \frac{\partial T^*_p}{\partial z} dz = \int -\left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F p_{10}}{3p_r}\right) J_1(p) e^{-\frac{Ap^2 z}{p r}} e^{cz} dz\]

\[= -\int \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F p_{10}}{3p_r}\right) J_1(p) e^{\left(-\frac{Ap^2 z}{p r}\right)} dz\]

\[= \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F p_{10}}{3p_r}\right) J_1(p) e^{\left(-\frac{Ap^2 z}{p r}\right)} + D\]

\[T^*_p(z, p) = T^*_1(z, p) + \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F p_{10}}{3p_r}\right) J_1(p) e^{\left(-\frac{Ap^2 z}{p r}\right)} + D e^{-cz}\]

When \(z \to 0\) we have

\[T^*_p(0, p) = T^*_1(0, p) + \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F p_{10}}{3p_r}\right) J_1(p) \frac{1}{p} e^{\left(-\frac{Ap^2}{p r}\right)} + D\]

\[D = T_{p10}J_1(p) - T_{10}J_1(p) - \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F p_{10}}{3p_r}\right) J_1(p) \frac{1}{p} e^{\left(-\frac{Ap^2}{p r}\right)}\]

Using the above equation we have

\[T^*_p(z, p) = \left(T_{10} - \frac{2\alpha F p_{10}}{3Ap^2}\right) J_1(p) e^{\frac{Ap^2 z}{p r}} + \frac{2\alpha F p_{10}}{3Ap^2} J_1(p) e^{\frac{Ap^2}{p r}} + \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F p_{10}}{3p_r}\right) J_1(p) e^{\left(-\frac{Ap^2 z}{p r}\right)}\]

\[- \left(T_{10} - T_{p10}\right) J_1(p) e^{-cz} - \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F p_{10}}{3p_r}\right) J_1(p) e^{\left(-\frac{Ap^2 z}{p r}\right)} e^{-cz}\]
\[
\begin{align*}
\rho_{p1}^* &= \rho_{p0} \frac{J_1(p)}{p} \\
T_{p1}^* &= \left[ T_{p10} + \frac{2\alpha F \rho_{p0}}{3C_p} - T_{10} \right] e^{-\alpha z} + \frac{2\alpha F \rho_{p0}}{3C_p} + \frac{T_{10} - 2\alpha F \rho_{p0}}{1 - \frac{Ap^2}{C_p}} e^{-\frac{\Delta z}{r}} \right] \frac{J_1(p)}{p} \\
\text{Where the Bessel function of zero order is } J_0 \text{ and first order is } J_1.
\end{align*}
\]
RESULT AND CONCLUSION
Here we are consider $u_{10} = u p_{10} = T_{10} = T_{p10} = \rho_{p10} = 0.1, \varphi = 0.01$.

Figures 1, 2 and 3 shows the temperature flow velocity particle $T_{p1}$ for $\alpha = 0.1, 0.2$ and 0.3 and for $Z=0.25,0.5$ and 1.0. It is found that the temperature flow particle velocity is high at initially. It is observed that the magnitude of $T_{p1}$ decreases with the increase of concentric parameter and shows that the magnitude of fluid particle temperature reduces significantly.

Reference :


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