Effect of thermal diffusion on unsteady laminar free convective flow of dusty viscous fluid through porous medium along a moving porous hot vertical plate with hall current

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Abstract- The purpose of the present problem is to study the effect of thermal diffusion on unsteady laminar free convection flow of dusty viscous fluid through porous medium along a moving porous hot vertical plate in the presence of heat source and Hall current. The governing equations of motion are solved by finite difference technique. The numerical results are presented graphically for different values of the parameters entering into the problem on the velocity profiles of dusty fluid, dust particles, temperature, concentration profile and skin friction profile.

Keywords – Unsteady, Dusty viscous fluid, MHD, Porous medium, Thermal diffusion

I. INTRODUCTION

The problem of free convection flow of an electrically conducting fluid past a vertical plate under the influence of a magnetic field attracted many scientists, in view of its application in Aerodynamics Astrophysics, Geophysics and Engineering. Laminar natural convection and heat transfer in fluids flow with and without heat source in channels with constant wall temperature was discussed by Ostrach [5]. An analysis of laminar free convective flow and heat transfer on a flat plate parallel to the direction of governing body force was studied by Ostrach [6]. Combined natural and forced convection laminar flow and heat transfer in fluid, with and without source channels, with linearly varying wall temperature was discussed by Ostrach [7]. Sastri [10] dealts heat transfer in the flow over a flat plate with suction and constant heat source. Also Sastri [11] studied the problem of heat transfer in the presence of temperature dependent heat source in the flow over a flat plate with suction. Forced and natural flows were discussed by Bansal [1], magnetohydrodynamics of viscous fluid were discussed by Bansal [2], Eckert and Drake [3] and Schlitchting [12]. Free convection effects on stoke's problem for an infinite vertical plate has been studied by Soundalgekar [14]. Pop and Soundalgekar [8] investigated free convection flow past an accelerated vertical infinite plate. Raptis et.al [9] observed the effect of free convection currents on the flow of an electrically conducting fluid of an accelerated vertical infinite plate with variable suction. Sharma [13] discussed free convection effect on the flow past an infinite vertical porous plate with constant suction and heat flux. Kumar and Varshney [4] studied steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source with mass transfer. Varshney and Kumar [15] have discussed effect of thermal diffusion on steady laminar free convective flow along moving porous hot vertical plate in the presence of heat source with mass transfer. Varshney and Singh [16] have studied the unsteady effect on laminar free convection flow along a moving porous hot vertical plate in the presence of heat source and thermal diffusion with mass transfer. Recently, Sexena et. al. [17] have studied on Effect of the dusty viscous fluid on unsteady free convective flow along a porous hot vertical plate with thermal diffusion and mass transfer solved by perturbation techniques.

The aim of the present paper is to investigate the effects of thermal diffusion and porosity parameter on a dusty viscous fluid on unsteady laminar free convection flow along a moving porous hot vertical plate in the presence of heat source and hall current with mass transfer. The governing equations of motion are solved by finite difference technique. The velocity of dusty fluid, temperature, concentration profile and skin friction profiles for different parameters entering into the problem are analyzed graphically.

II. FORMULATION OF THE PROBLEM

An infinitely long non conducting hot vertical, thin porous plate is situated in an electrically conducting dusty viscous fluid. The x^*-axis is taken along the plate in upward direction and y^*-axis is normal to it. A transverse constant magnetic field is applied i.e. in the direction of y^*-axis. Since the motion is tow dimensional and length of the plate is large, therefore, all the physical variables are independent of x^*. The governing equations of continuity, momentum, dust particles, energy and concentration for a free convective flow through porous medium of an electrically conducting fluid along a hot, non conducting porous vertical plate in the presence of heat source with mass transfer are given by:

$$\frac{\partial v^*}{\partial y^*} = 0 \qquad \text{i. e.} \qquad v^* = - v_0 \text{ (Constant)} \qquad \ldots(1)$$

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=g\beta(T^*-T_\infty)+g\beta'(C^*-C_\infty)$$

$$+\upsilon\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{KN_0}{\rho}(V-u^*)-\frac{\sigma B^2{}_0}{\rho(1+m^2)}u^*-\frac{\upsilon}{K_0^*}u^* \quad ...(2)$$

$$m_1\frac{\partial V}{\partial t^*}=K(u^*-V) \quad ...(3)$$

$$\frac{\partial p^*}{\partial y^*}=0 \qquad i.\,e. \qquad p^*=\text{Constant} \quad ...(4)$$

$$\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{S^*}{\rho C_p}(T^*-T_\infty) \quad ...(5)$$

$$\frac{\partial C^*}{\partial t^*}+v^*\frac{\partial C^*}{\partial y^*}=D\frac{\partial^2 C^*}{\partial y^{*2}}+D_1\frac{\partial^2 T^*}{\partial y^{*2}} \quad ...(6)$$

Where ρ is the density, g is the acceleration due to gravity, u^* and v^* are the velocities of dusty fluid, V is the velocity of dust particles, β is the coefficient of volume expansion, β' is the coefficient of concentration expansion, υ is the kinematic viscosity, T_∞ is the temperature of the fluid in the free stream, σ is the electric conductivity, B_0 is the magnetic induction, D is the chemical molecular diffusivity, k is the thermal conductivity, K is the stoke's resistance coefficient, N_0 is the number density of the dust particles which is taken to be constant, m is the hall current parameter, m_1 is the mass of dust particles, S^* is the coefficient of heat source, C^* is the concentration at infinity, D_1 is the thermal diffusivity, K_0 is the porosity of the porous medium and C_p is the specific heat at constant pressure.

The boundary conditions of the problems are

$$\left.\begin{aligned} &u^*=u_0, \quad T^*=T_w, \quad C^*=C_w \quad at \quad y^*=0 \\ &u^*\to 0, \quad T^*\to T_w, \quad C^*\to C_w \quad at \quad y^*\to\infty \end{aligned}\right\} \quad ...(7)$$

On introducing the following non-dimensional quantities

$$\left.\begin{aligned} & y=\frac{v_0 y^*}{v}, \qquad u=\frac{u^*}{v_0}, \qquad t=\frac{v_0^2 t^*}{v} \\ & \theta=\frac{T^*-T_\infty}{T_w-T_\infty}, \qquad \phi=\frac{C^*-C_\infty}{C_w-C_\infty}, \qquad v=\frac{V}{v_0} \end{aligned}\right\} \qquad ...(8)$$

With the help of equation (8), the equation (2), (3), (5) and (6) becomes.

$$\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}+B_1(v-u)-\left(\frac{M}{(1+m^2)}+\frac{1}{K_0}\right)u=-Gr\theta-Gm\phi \qquad ...(9)$$

$$B\frac{\partial v}{\partial t}=(u-v) \qquad ...(10)$$

$$\frac{\partial^2 \theta}{\partial y^2}+\Pr\frac{\partial \theta}{\partial y}-\Pr\frac{\partial \theta}{\partial t}+\Pr S\theta=0 \qquad ...(11)$$

$$\frac{\partial^2 \phi}{\partial y^2}+Sc\frac{\partial \phi}{\partial y}-Sc\frac{\partial \phi}{\partial t}+ASc\frac{\partial^2 \theta}{\partial y^2}=0 \qquad ...(12)$$

And corresponding boundary conditions are

$$\left.\begin{aligned} & u=Q, \quad \theta=1, \quad \phi=1 \quad at \quad y=0 \\ & u\to 0, \quad \theta\to 0, \quad \phi\to 0 \quad as \quad y\to\infty \end{aligned}\right\} \qquad ...(13)$$

Where, $M=\frac{\sigma\upsilon B_0^2}{\rho v_0^2}$, $\Pr=\frac{\mu Cp}{k}$, $Q=\frac{u_w}{v_0}$, $Sc=\frac{\upsilon}{D}$,

$A=\frac{D_1(T_w-T_\infty)}{\upsilon(C_w-C_\infty)}$, $B_1=\frac{\upsilon KN_0}{\rho v_0^2}$ (Dusty fluids parameter)

$Gr=g\beta\upsilon\frac{(T_w-T_\infty)}{v_0^3}$ (Grashoff Number)

$Gm=g\beta'\upsilon\frac{(C_w-C_\infty)}{v_0^3}$ (Modified Grashoff Number)

$B = \frac{m_1 v_0^2}{VK}$ (Dusty Particle parameter)

$K_0 = \frac{K_0^* v_0^2}{\upsilon^2}$ (Porosity parameter)

III. SOLUTION OF THE PROBLEM

The governing Equations (9) to (12) are to be solved under the initial and boundary conditions of equation (13). The finite difference method is applied to solve these equations.

The equivalent finite difference scheme of equations (9) to (12) are given by

$$\left[\frac{u_{i,j+1} - u_{i,j}}{\Delta t}\right] = \left[\frac{u_{i+1,j} - u_{i,j}}{\Delta y}\right] + \left[\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta y)^2}\right] + B_1\left(v_{i,j} - u_{i,j}\right)$$

$$-\left(\frac{M}{(1+m^2)} + \frac{1}{K_0}\right)u_{i,j} + Gr\theta_{i,j} + Gm\phi_{i,j} \quad ...(14)$$

$$\left[\frac{v_{i,j+1} - v_{i,j}}{\Delta t}\right] = \frac{1}{B}\left(u_{i,j} - v_{i,j}\right) \quad ...(15)$$

$$\left[\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t}\right] = \left[\frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta y}\right] + \frac{1}{\Pr}\left[\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2}\right] + S\theta_{i,j} \quad ...(16)$$

$$\left[\frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta t}\right] = \left[\frac{\phi_{i+1,j} - \phi_{i,j}}{\Delta y}\right]$$

$$+\frac{1}{Sc}\left[\frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{(\Delta y)^2}\right] + A\left[\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2}\right] \quad ...(17)$$

Here, index *i* refers to *y* and *j* to time. The mesh system is divided by taking, $\Delta y = 0.1.$

From the boundary conditions in Equation (13), we have the following equivalent.

$$\left.\begin{array}{l} u(0,0)=Q, \quad \theta(0,0)=1, \quad \phi(0,0)=1 \\ u(i,0)=0, \quad \theta(i,0)=0, \quad \phi(i,0)=0, \quad \textit{for all } i \textit{ except } \ i=0 \end{array}\right\} \quad ...(18)$$

The boundary conditions from equation (13) are expressed in finite difference form are as follows:

$$\left.\begin{array}{l} u(0,j)=Q, \quad \theta(0,j)=1, \quad \phi(0,j)=1 \quad \textit{for all } \ j \\ u(1,j)=0, \quad \theta(1,j)=1, \quad \phi(1,j)=1 \quad \textit{for all } \ j \end{array}\right\} \quad ...(19)$$

Here, infinity is taken as y = 4.1. First, the velocity of dusty fluid at the end of time step namely $u(i, j+1),\ i=1$ to 10 is computed from equation (14), the velocity of dust particle at the end of time step namely $v(i, j+1),\ i=1$ to 10 is computed from equation (15) and temperature $\theta(i, j+1), i=1$ to 10 from equation (16) and concentration $C(i, j+1), i=1$ to 10 from equation (17). The procedure is repeated until t = 1 (i.e., j = 800). During computation, Δt was chosen to be 0.00125. These computations are carried out for different values of parameters Gr, Gm, Pr, Sc, M, m, K_0, A, S (heat source parameter), B (dust particle parameter), B_1 (dusty fluid parameter), Q (velocity parameter) and t (time). To judge the accuracy of the convergence of the finite difference scheme, the same program was run with smaller values of Δt, i.e., Δt = 0.0009, 0.001 and no significant change was observed. Hence, we conclude that the finite difference scheme is stable and convergent.

IV. RESULTS AND DISCUSSION

Numerical calculations have been carried out for dimensionless velocity of dusty fluid, temperature and concentration profiles for different values of parameters and are displayed in Figures-(1) to (17).

Figures-(1) to (12) represent the velocity profiles of dusty fluid for different parameters. Figure-(1) shows the variation of velocity u with magnetic parameter M. It is observed that the velocity decreases as M increases. In figure – (2), shows the effects of hall current parameter on the velocity of dusty fluid u. It is clear that the velocity of dusty fluid increases with increasing the value of hall current parameter. Figure-(3) shows that an increase in permeability parameter K_0 causes an increase in velocity profile of dusty fluid. From Figure-(4), it is observed that the velocity of dusty fluid increases as the Grashoff number *Gr* increase. The variation of *u* with modified Grashoff number *Gm* is shown in Figure-(5). It is noticed that increase in *Gm* leads to increase in velocity of dusty fluid. From Figure-(6) shows the variation of velocity *u* with Prandtl number Pr. It is observed that the velocity of dusty fluid decreases as Pr increases. The velocity profile of dusty fluid for Schmidt number *Sc* is shown in Figure-(7). It is clear that velocity of dusty fluid *u* decreases with increasing in *Sc*. In figure-(8), the velocity profile of dusty fluid increases due to increasing thermal diffusion parameter *A*. Figure-(9), shows the variation of velocity profile of dusty fluid *u* with heat source parameter *S*. It is observed that the velocity increases as *S* increases. From Figure-(10) shows the variation of velocity profile of dusty fluid *u* with dust particle parameter *B*. It is observed that the velocity of dusty fluid decreases as *B* increases. The velocity profile of dusty fluid for B_1 (dusty fluid parameter) is shown in Figure-(11). It is clear that velocity of dusty fluid *u* decreases with increasing in B_1. The velocity profile for time variable *t* is shown in Figure-(12). It is clear that an increase in *t* leads to an increase in *u*. From Figure-(13), it is observed that increase in Prandtl number Pr causes decrease in temperature profile of dusty fluid. Figure-(14) shows that an increase in heat source parameter *S* causes an increase in velocity profile of dusty fluid. From Figure-(15), it is noticed that an increase in Schmidt number *Sc* leads to decrease in concentration profile of dusty fluid. Figure-(16) shows that an increase in thermal diffusion parameter *A* causes an increase in concentration profile of dusty fluid. Figure-(17) shows the variation of concentration profile of dusty fluid ϕ with Prandtl number Pr. It is observed that the concentration profile of dusty fluid increases as Pr increases. Figure-(18) shows the variation of concentration profile of dusty fluid ϕ with heat source parameter *S*. It is observed that the concentration decreases as *S* increases.

Figure-(19) shows the skin friction. Knowing the velocity field, the skin friction is evaluated in non-dimensional form using, $\tau = \left[-\frac{\partial u}{\partial y} \right]_{y=0}$. The numerical values of τ are calculated by applying Newton's interpolation formula for 11 points and are presented. From figure-(19), it is observed that an increase in Grashoff number *Gr*, Modified Grashoff number *Gm*, porosity parameter K_0 and thermal diffusion parameter *A* causes decrease in skin friction, and an increase in magnetic parameter *M* leads an increase in skin friction.

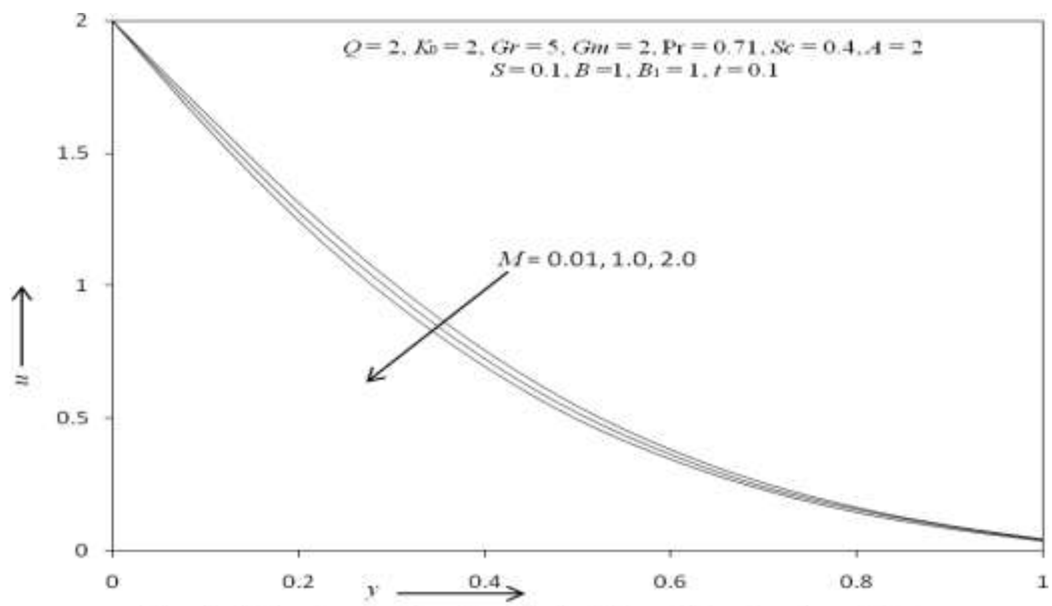

Fig.- 1: Velocity profile of dusty fluid for different value of M.

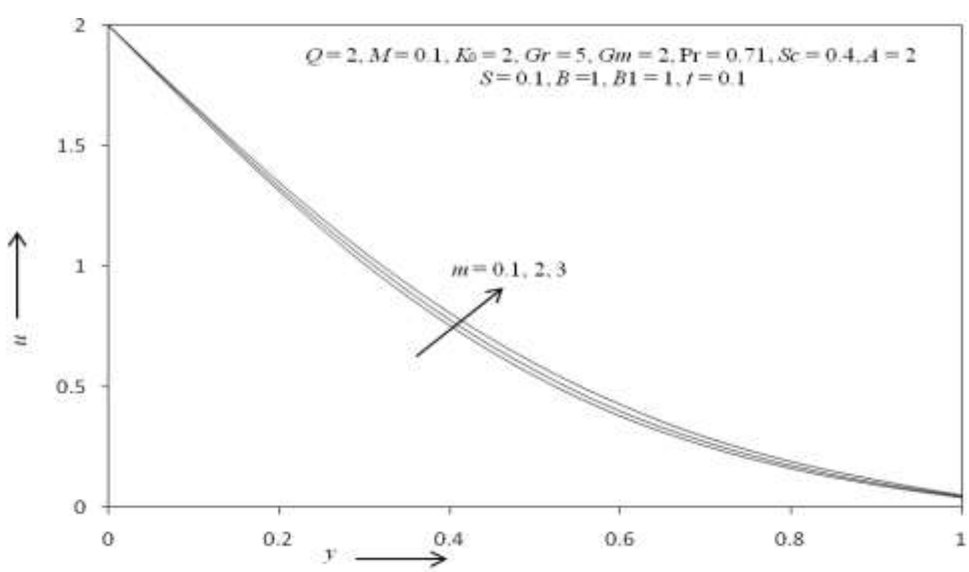

Fig.- 2: Velocity profile of dusty fluid for different value of m.

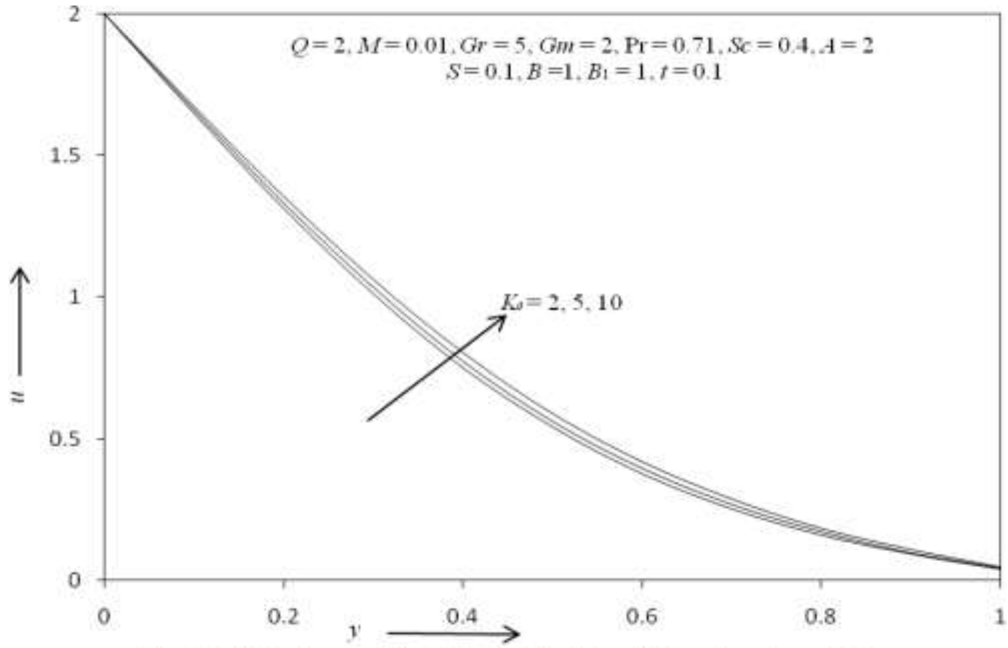

Fig.-3: Velocity profile of dusty fluid for different value of K.

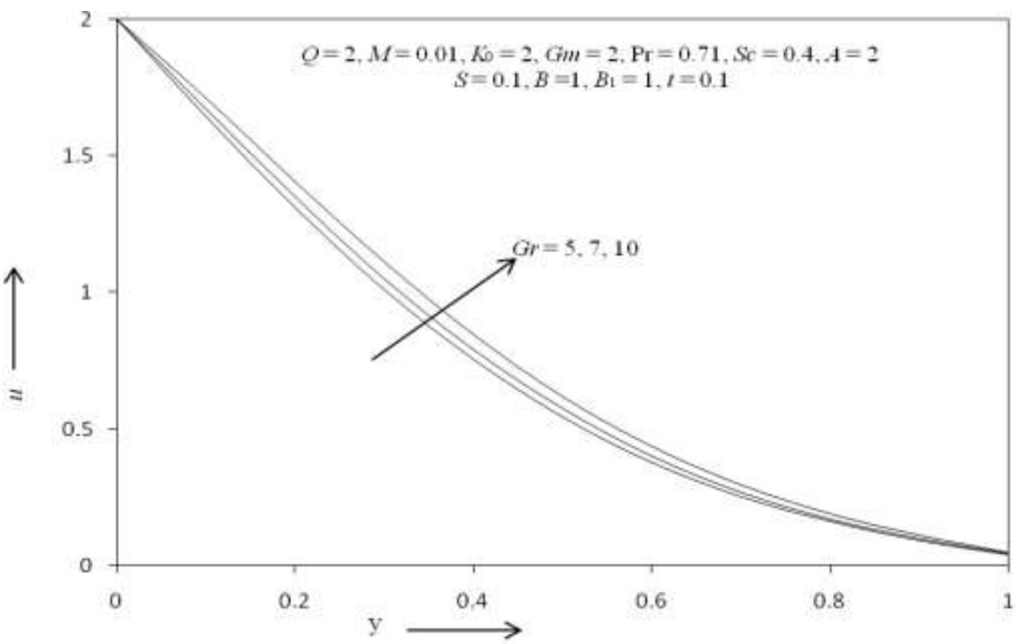

Fig.-4: Velocity profile of dusty fluid for different value Gr.

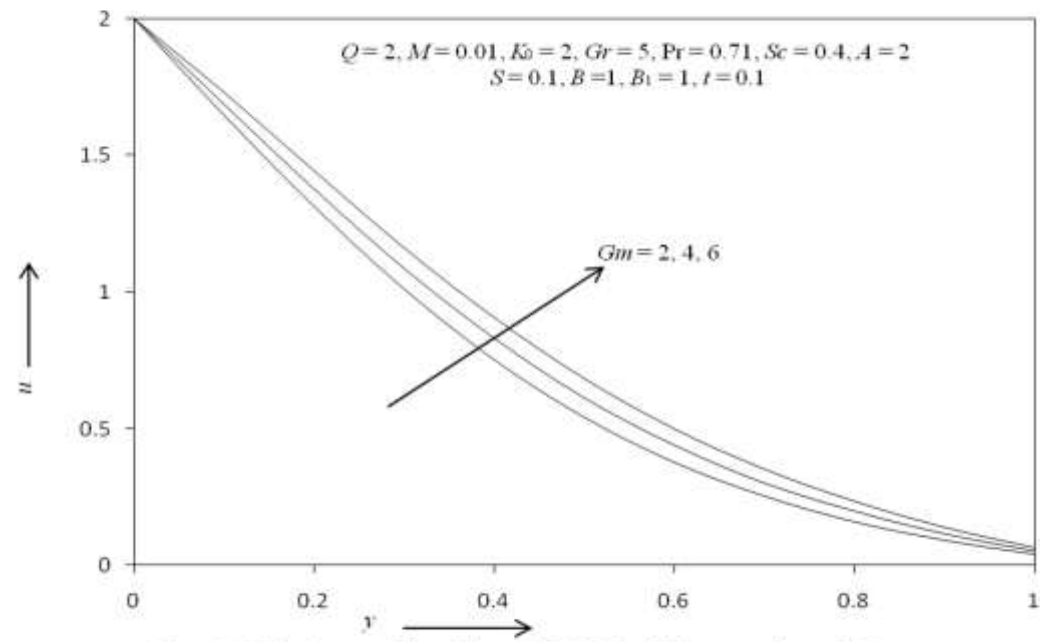

Fig.-5: Velocity profile of dusty fluid for different value of Gm.

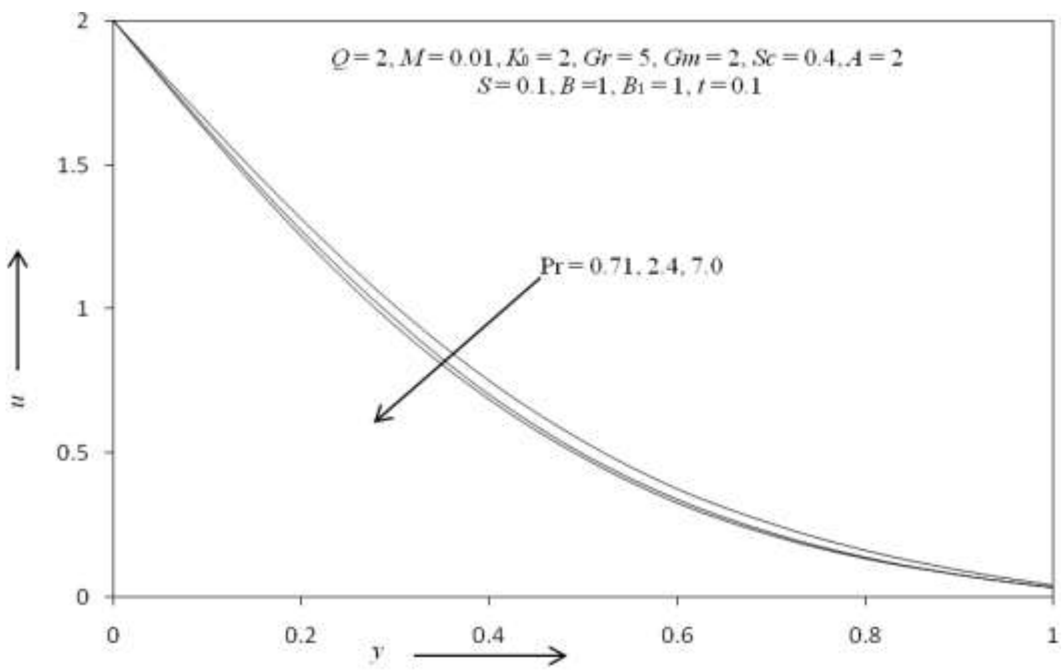

Fig.-6: Velocity profile of dusty fluid for different value of Pr.

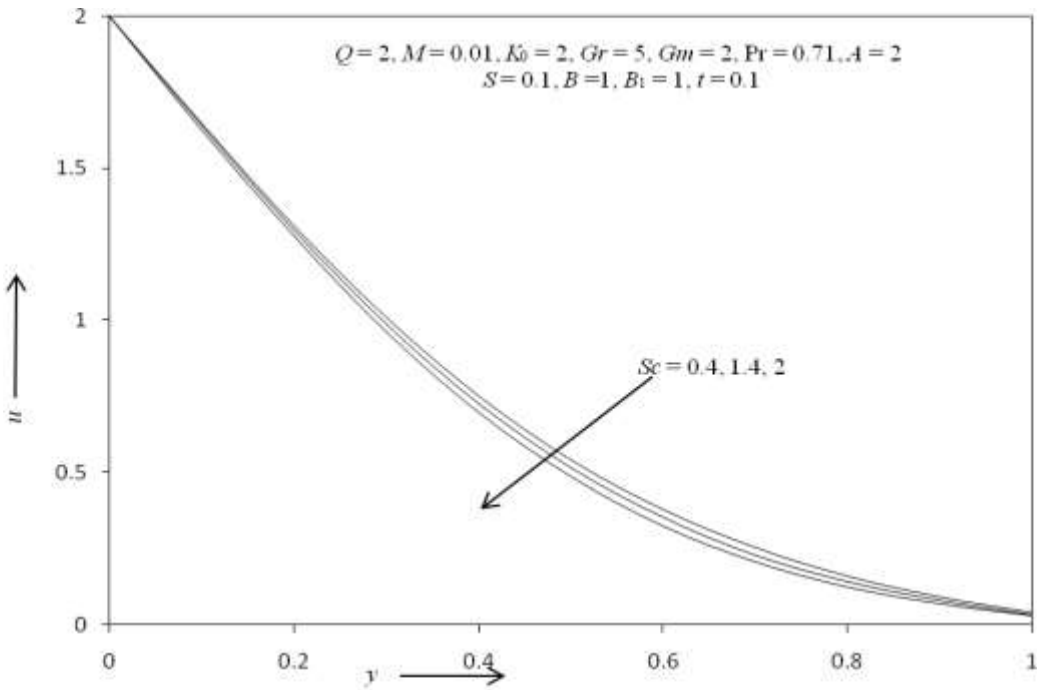

Fig.-7: Velocity profile of dusty fluid for different value Sc.

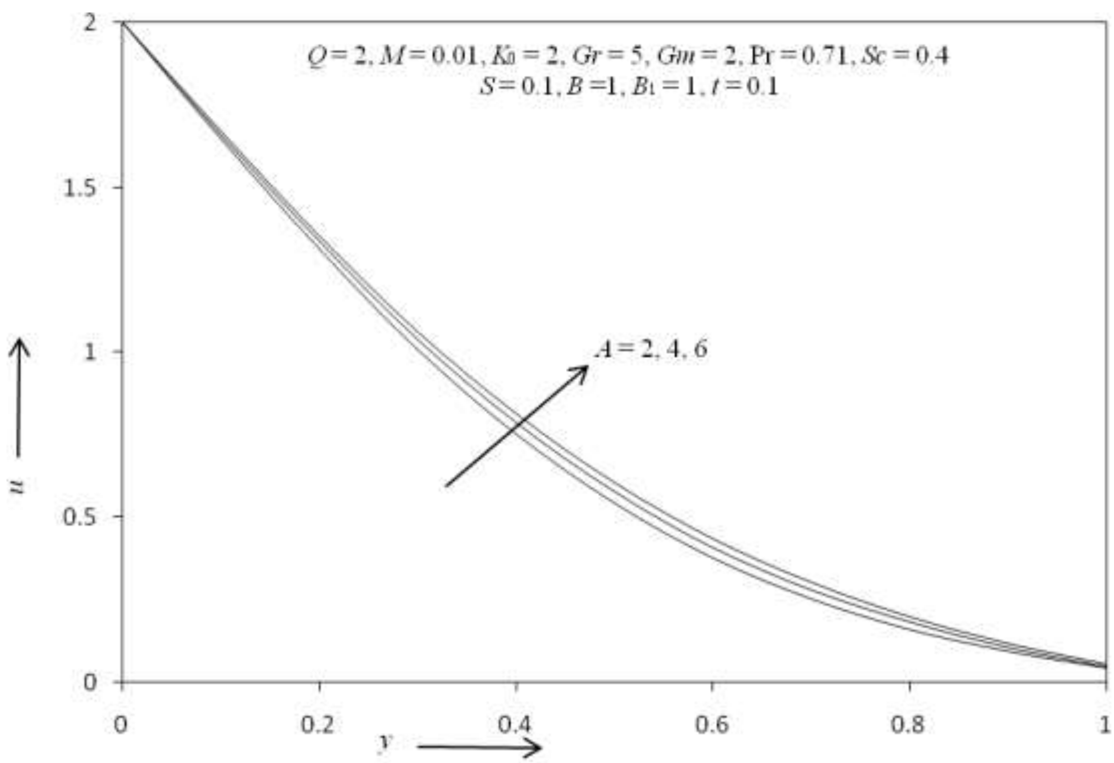

Fig.-8: Velocity profile of dusty fluid for different value of A.

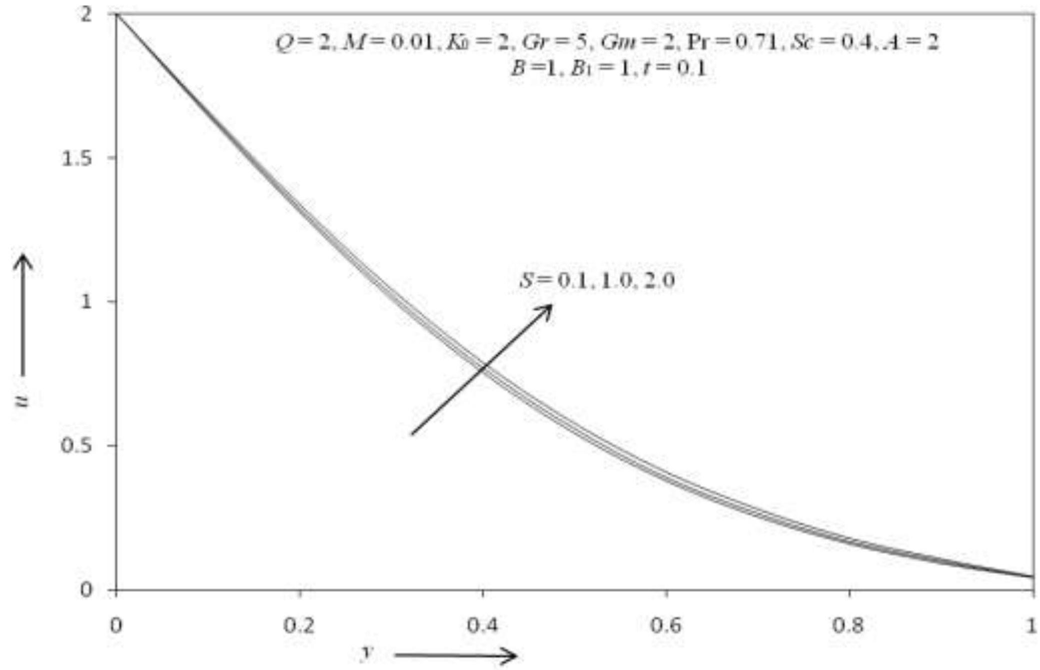

Fig.-9: Velocity profile of dusty fluid for different of S.

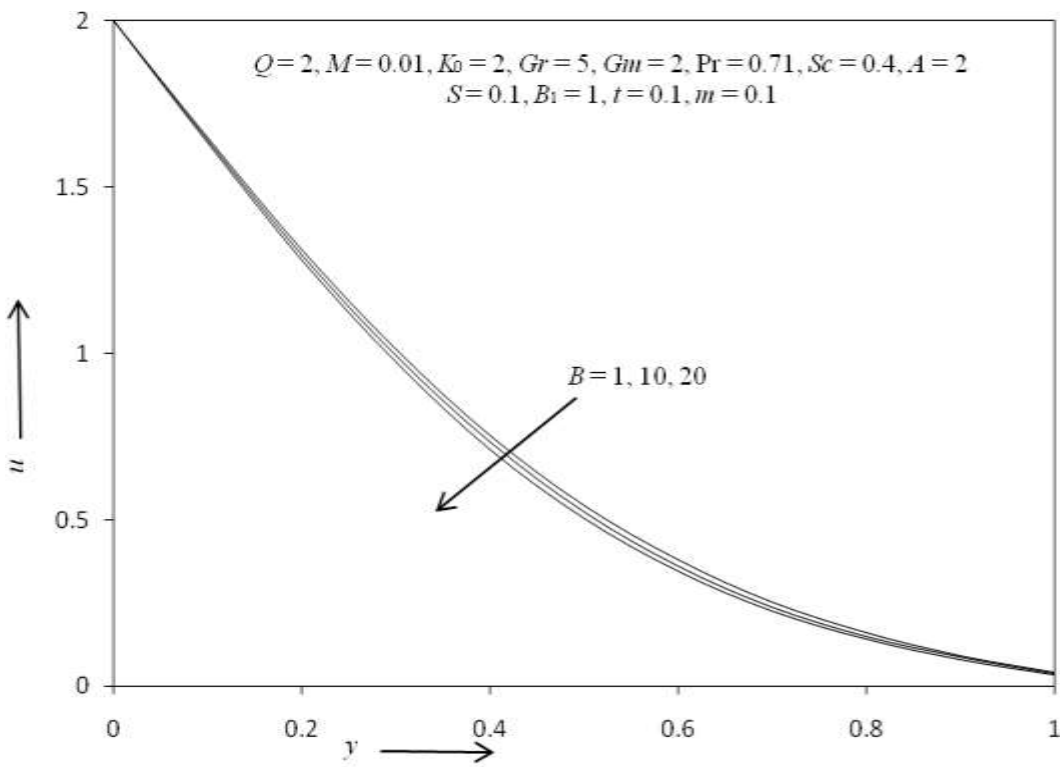

Fig.-10: Velocity profile of dusty fluid for different value of B.

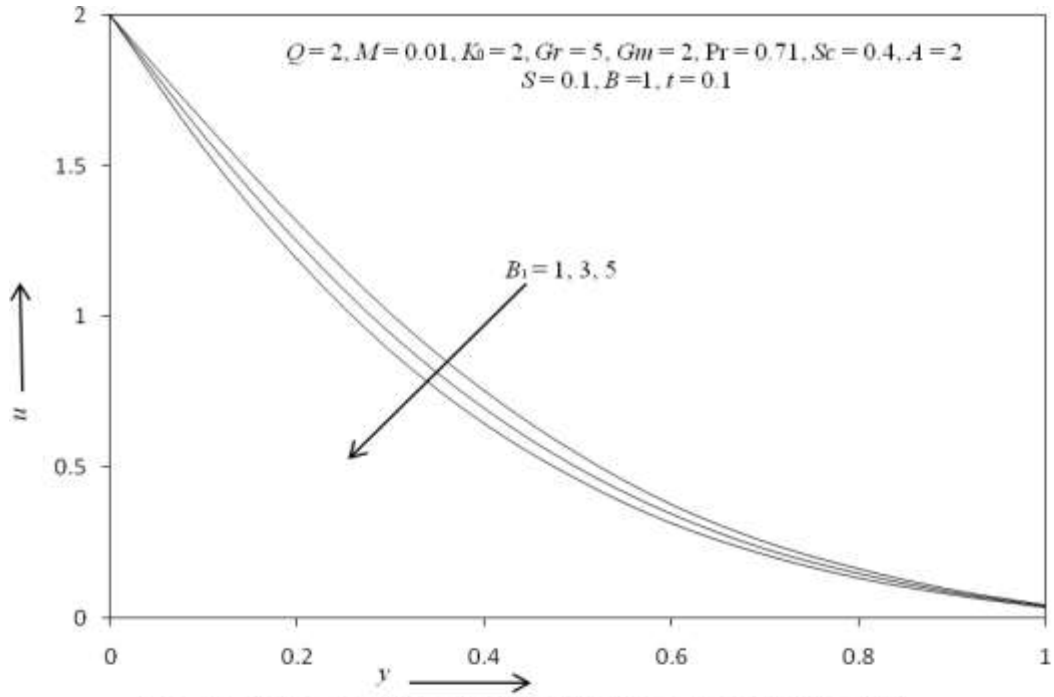

Fig.-11: Velocity profile of dusty fluid for different value of B_1.

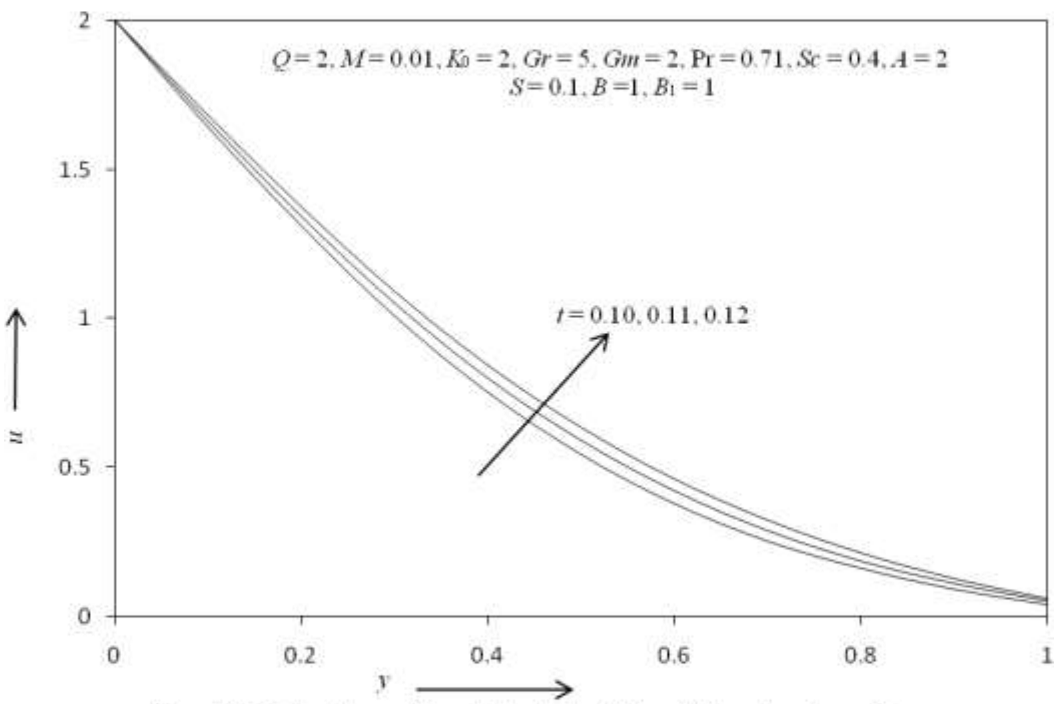

Fig.-12: Velocity profile of dusty fluid for different value of t.

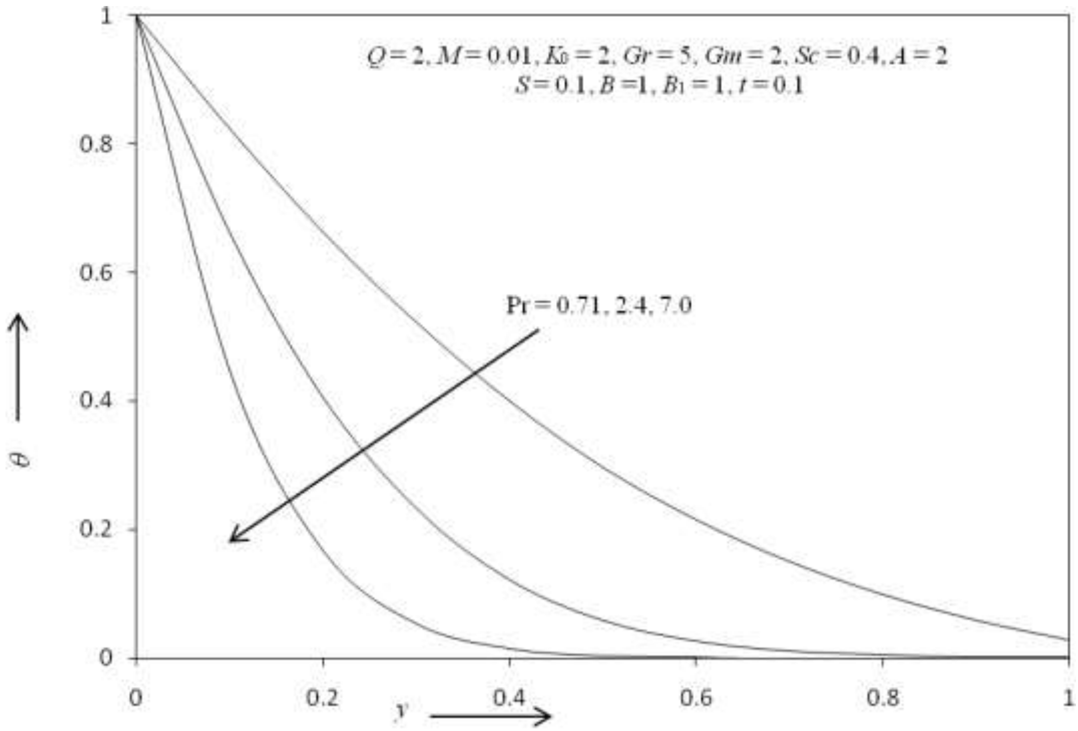

Fig.-13: Temprature profile of dusty fluid for different value of Pr.

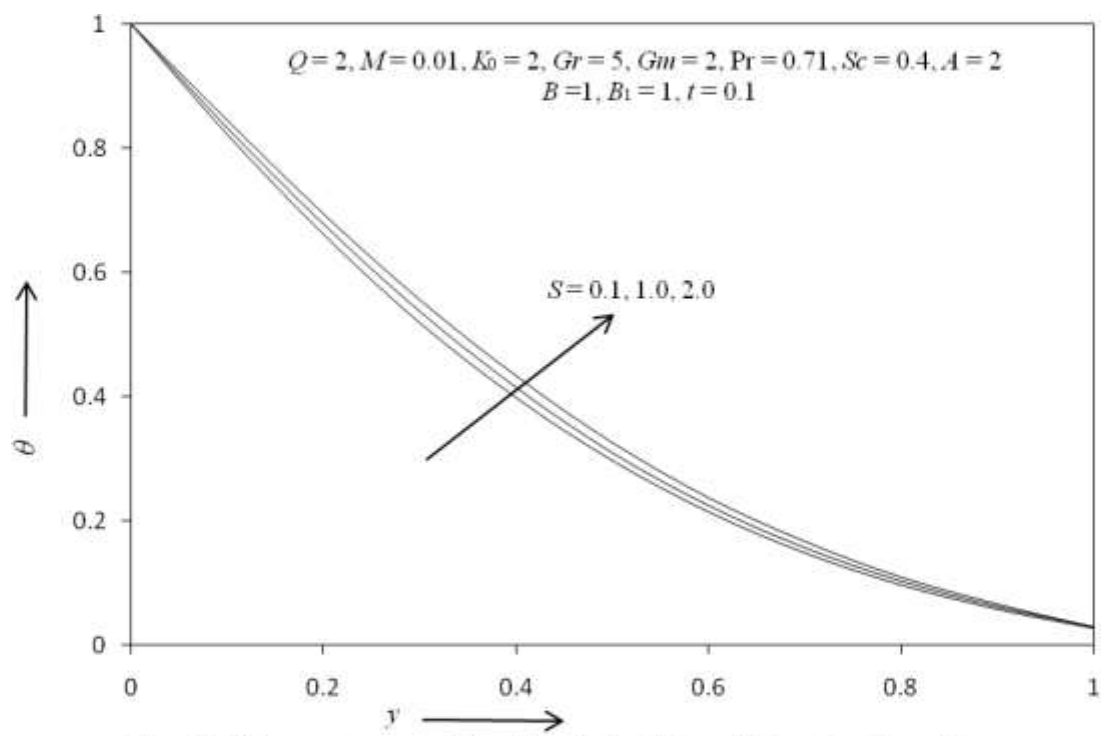

Fig.-14: Temprature profile of dusty fluid for different value of S.

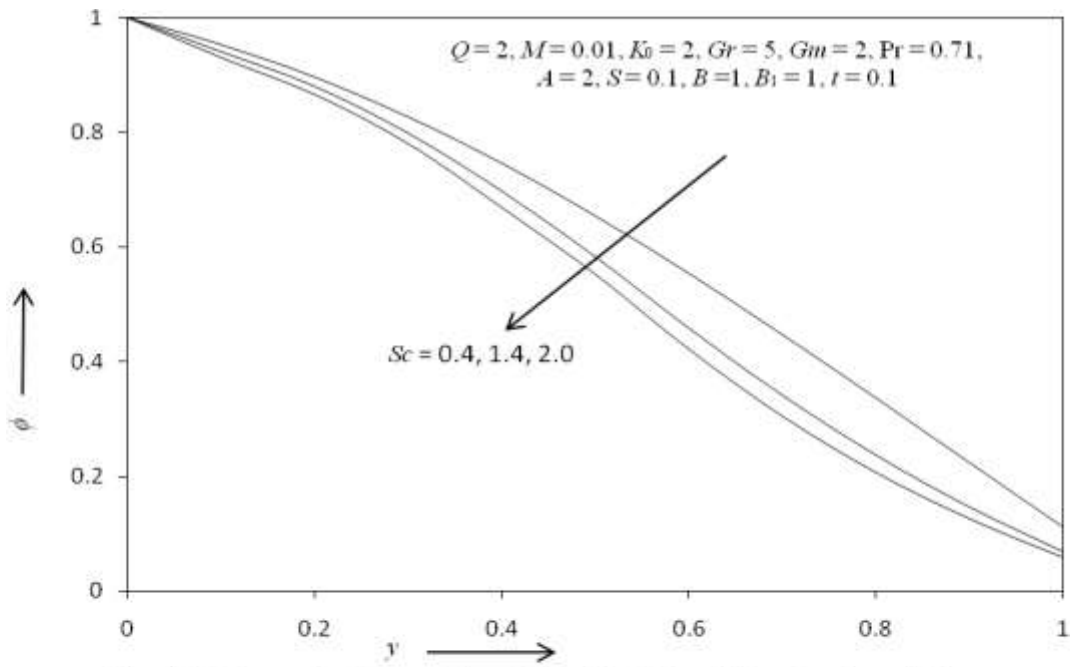

Fig.-15: Concentration profile of disty fluid for different value of Sc.

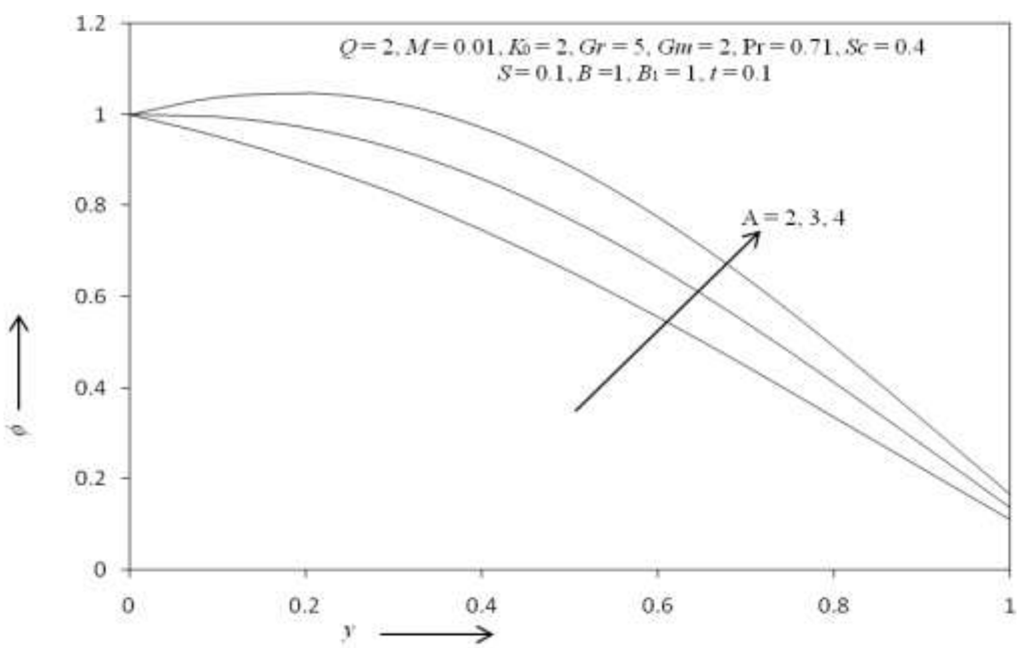

Fig.-16: Concentration profile of dusty fluid for different value of A.

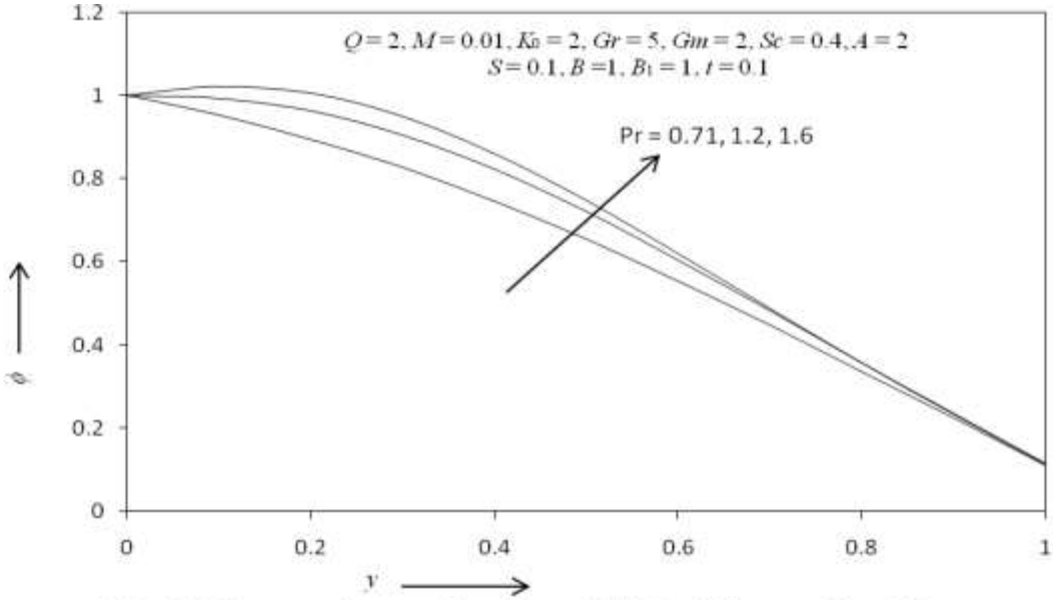

Fig.-17: Concentration profile of dusty fluid for different value of Pr.

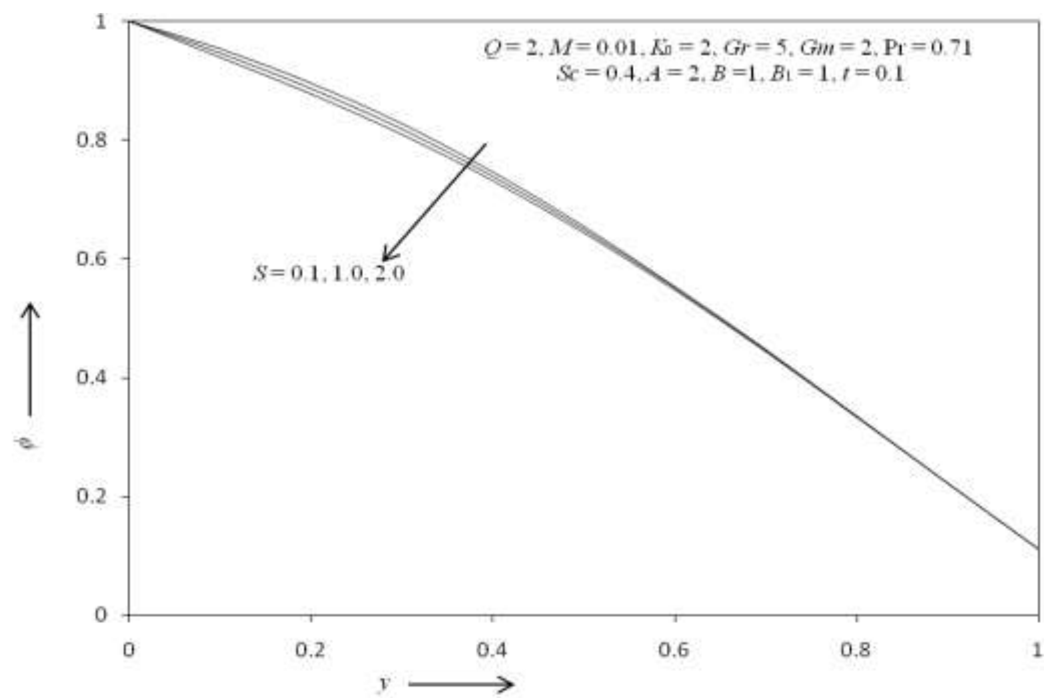

Fig.-18: Concentration profile of dusty fluid for different value of S.

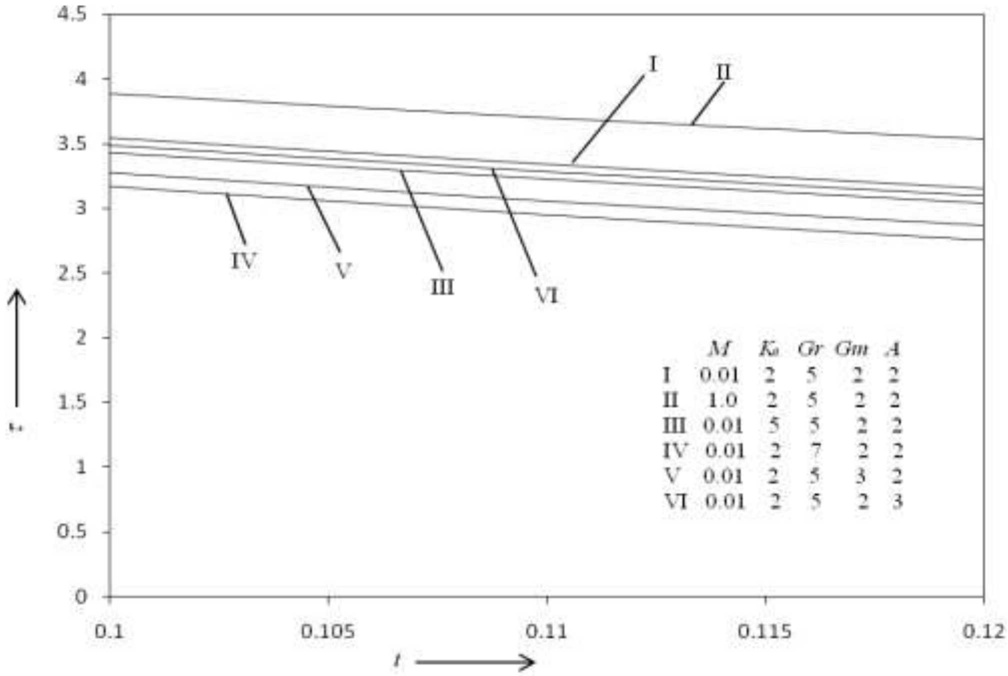

Fig.-19: Skin friction of dusty fluid for different value of M, K_0, Gr, Gm and A.

V.CONCLUSION

REFERENCES

[1] Bansal, J. L. (1977): Viscous Fluid Dynamics, Oxford and IBH Pub. Comp., Calcutta (India).
[2] Bansal, J. L. (1994): MHD of viscous Fluid, Jaipur Pub. House, Jaipur (India).
[3] Eckert, E. R. C. and Drake, R. M. (1974): Heat and Mass Transfer. Tata McGraw Hill Pub. Comp. Ltd, New Delhi (India).
[4] Kumar, Rakesh and Varshney, N. K. (1999): Journal of M.A.C.T., Vol. 32, pp. 49 – 65.
[5] Ostrach, S. (1952): NACTA, TN, pp. 2863.
[6] Ostrach, S. (1953): NACTA, TN, pp. 1111.
[7] Ostrach, S. (1954): NACTA, TN, pp. 3141.
[8] Pop, I. and Soundalgekar, V. M. (1980): ZAMM, Vol. 60, pp. 167.
[9] Raptis, A. (1981): ZAMM, Vol. 61, pp. 341.
[10] Sastri, K. S. (1965): Appl. Sci. Res., vol. 14, pp. 126.
[11] Sastri, K. S. (1965): J. Phy. Soc. Japan, vol. 2, pp. 1711.
[12] Schlichting, H. (1966): Boundary Layer Theory, Mc Graw Hill Book Co. Inc., New York.
[13] Sharma, P. R. (1991): J. Ultra Scientist Phy. Sci. India, Vol. 3, pp. 88.
[14] Soundalgekar, V.M. (1977): Trans. ASME, J. Heat Transfer, Vol. 99, pp. 499.
[15] Varshney, N. K. and Kumar Shilendra (2004): Acta Ciencia Indica, vol. XXXM, No.2, pp. 489 -495.
[16] Varshney, N. K. and Singh, Sooraj Pal (2006): Thesis, Dr. B. R. A. University, Agra, pp. 78-91.
[17] Sexena, S. S., Dubey, G. K. and Varshney, N. K. (2009): Effect of the dusty viscous fluid on unsteady free convective flow along a moving a porous hot vertical plate with thermal diffusion, and mass transfer, J. Purvanchal Academy of Sciences, Vol. 15, pp. 1-12.