Effect of Damping on the Vibrations of Annular Plate (Ring Shaped Plate) Having Variable Thickness Considering Elastic Foundation

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Abstract - Quintic spline interpolation technique (QSIT) is used to solve governing mathematical model, i.e. the plate equation for annular plates considering the variation of parameters: damping, variable thickness, rigidity ratio, ratio of radii and elastic foundation on it. Three different combinations at boundaries are applied to obtain the results. The limiting conditions are applied for inner edge Clamped-outer edge Clamped (C-C), inner edge Clamped-outer edge simply supported (C-S) and inner edge Clamped-outer edge Free (C-F) combinations. Relationship between various parameters ratio of radii, taper constant, rigidity ratio, damping parameter and elastic foundation parameters have been presented through graphs. The deflection function for uniform and tapered condition for C-C, C-S and C-F plates have also been presented through graphs. The validation of the method QSIT is obtained by equating the results achieved in our discussion with the results available in previous literature. The results are obtained and represented using MATLAB (2015).

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Keywords - Annular plate, taper constant, damping parameter, foundation parameter, rigidity ratio, radii ratio.

1. INTRODUCTION

Every engineering application requires structural components of various shapes. Circular & annular (ring shaped) plates are the predominant constructional elements in engineering structures so Ring shape or annular plates are widely seen in various engineering applications such as car manufacturing industry, aerospace industry, civil engineering industry, packaging industry, agriculture equipment industry etc. Annular plates having thickness variations are widely preferred in structural applications in order to reduced weight and size of the engineering system and improve its structural characteristics. In recent time elastic foundation interaction with plates has achieved great relevance in today's technological world due to its different applications in various structures like in steam turbines, cylinder heads, and piston heads etc. Since free vibrations are practically not possible, so every engineering structure cannot be imagine without considering the damping phenomenon. Damping effect in study of vibration is so important that it can be large enough to stop vibration or so small to produce any appreciable effect on plate frequency, considering damping effect we come closer to real world problem of vibration analysis.

Leissa has presented excellent survey of the information about vibration of plates considering various effects by plate parameters [14]. A large number of researchers has studied isotropic annular and circular plates of uniform thickness [10, 24]. Considering vibration analysis of polar orthotropic plates, we find that their mathematical models in general have no exact solutions i.e. closed form of solutions does not exist. So different approximation methods have been used by various researchers. O'Boy and Krylov analysed property of damping in annular and circular plates and used Rayleigh-Ritz method to study their frequencies [15]. In addition to these methods Duan et al. used DSC element method [4] and Khare &Mittal used finite element method (FEM) for exploring the nature of thin circular and annular plates with general BC's [11]. In literature related to vibration analysis studying interaction between structure and foundation is approximated by different approximation for foundation by Vlasov's, Pasternak and Winkler etc. Bhattacharya studied the Vlasov's foundation on triangular plates [2]. Sharma et al. used Pasternak foundation for circular plates, and Salawu et al. Considered Winkler foundation and Pasternak foundation in their study on circular plates [16, 17]. Since effect of change in thickness have great impact on the frequency of plates of different shapes. So various researcher had studied plates with variable thickness. Zhou et al. [23], Bhardwaj et al. [1] Studied circular and annular plate with varying thickness in two and three directions and with different variation in thickness. Crandall [3] studied the effect of damping in vibration theory. Jaiman and Singh [9] in their study on circular and annular plates used different boundary conditions to calculate frequency and modes. Singh and Jain [19] presented a particular case study on asymmetric vibrations of non-uniform polar orthotropic annular plates. In addition Wang and Wang [22] used differential quadrature method to re-analyse free vibration of annular plates. In recent times, Shi et al. [18] presented their work on free in-plane vibration of an annular sector plate with elastic boundary support and a unified solution for free in-plane vibration having journal boundary conditions for orthotropic circular plate and subcases was given by Wang et al. [21].The effect of damping for isotropic elastic circular plates considering non-homogeneity, parabollically varying thickness and elastic foundation was studied by Gupta [7] using Frobenius method.

The present work deals with the QSIT solution for damped axisymmetric vibration of annular (ring shaped) plates having polar orthotrophy with linear thickness variation and considering Winkler's type elastic foundation. Mathematical model for the plate equation have been presented in polar form and the governing differential equation solution is determined by numerical approximation method QSIT. Generally, classical plate theory (CPT) provide us nearly accurate results for lower modes, so first 3 modes of vibration have been calculated and studied for three boundary conditions (BC's) namely C-C, C-S and C-F, for the annular plate. Here "C" stands for clamped, "S" stands for simply supported and "F" stands for free. Thus these three conditions can be explained as C-C conditions implies inner edge is clamped-outer edge is clamped, C-S conditions implies inner edge is clamped-outer edge is simply supported and C-F conditions implies inner edge is clamped-outer edge is free.

2. STATEMENT OF THE MATHEMATICAL MODEL FOR THE PROBLEM

For the present model, we have considered a thin annular palate which has linearly varying thickness $h(r)$ and here r is radial co-ordinate. The annular plate has inner radii "c" and outer radii "a" respectively and this plate rest on Winkler's foundation with modulus k_f as presented in the mathematical model. Only a few studies have been carryout considering damping variation in annular plates so in our problem we shall be dealing with effect of damping and we have presented the effect of damping with suitable changes in the mathematical model [8].

The governing partial differential equation (PDE) [8] for motion of annular (ring shaped) plates as given in previous literature after addition of damping factor and considering various parameters becomes:

$$D_r \frac{\partial^4 u}{\partial r^4} + \left[2(D_r + rD_{r,r})/r\right]\frac{\partial^3 u}{\partial r^3} + \left[\{-D_\theta / r^2 + \frac{(2+\nu_\theta)D_{r,r}}{r} + D_{r,rr}\}\right]\frac{\partial^2 u}{\partial r^2}$$
$$+\left[\{D_0 - rD_{\theta,r} + r^2\nu_\theta D_{r,rr}\}/r^3\right]\frac{\partial u}{\partial r} + \rho h\frac{\partial^2 u}{\partial t^2} + k_d\frac{\partial u}{\partial r} + k_f u = 0. \quad (1)$$

Here u denotes deflection function and additional symbols used represents their normal meaning. For harmonic solution the transverse deflection function is of the form $u = U(r)e^{-\mu t}\cos\omega t,$ here ω is circular frequency of the plate. Substituting the above function and introducing dimensional less variables $r/a = x$, $H = h/a$ and assuming linear change in thickness given by $H{=}H_0(1{-}\alpha x)$, the dimensionless form of the PDE given in (1) after mathematical calculation is obtain as,

$$V_0\frac{\partial^4 U}{\partial x^4} + V_1\frac{\partial^3 U}{\partial x^3} + V_2\frac{\partial^2 U}{\partial x^2} + V_3\frac{\partial U}{\partial x} + V_4 U = 0,$$

$$
\begin{aligned}
&where\\
&V_0=(1-\alpha x)^3\\
&V_1=\frac{2}{x}(1-\alpha x)^3-6\alpha(1-\alpha x)^2\\
&V_2=-\frac{p}{x^2}(1-\alpha x)^3+\frac{(2+\nu_\theta)(-3\alpha)(1-\alpha x)^2}{x}+6\alpha^2(1-\alpha x)\\
&V_3=\frac{p}{x^3}(1-\alpha x)^3+\frac{3p\alpha}{x^2}(1-\alpha x)^2+\frac{\nu_\theta}{x}6\alpha^2(1-\alpha x) \qquad (2)\\
&V_4=-\Omega^2(1-\alpha x)-\frac{{D_k}^2 mst}{(1-\alpha x)}+K\\
&here: \quad \Omega^2=\frac{12(1-\nu_r\nu_\theta)a^2\omega^2\rho}{E_r{H_o}^2}, \qquad D_k=\frac{3(1-\nu_r\nu_\theta){k_d}^2}{E_r\rho},\\
&\qquad K=\frac{12(1-\nu_r\nu_\theta)ak_f}{E_r{H_o}^2},\ Mst=\frac{1}{{H_o}^2} \quad and \quad p=\frac{E_\theta}{E_r}
\end{aligned}
$$

3. APPROXIMATE SOLUTION TO THE DERIVED MATHEMATICAL MODEL

Equation (2) represents the mathematical model for our problem and we observed that no exact solution with assumed boundary conditions is possible due to occurrence of variable coefficient in the mathematical model. So we apply numerical approximation method viz. QSIT to solve the differential equation (2) with boundary conditions at $x=c/a$, $x=1.$ A transformation is considered as X= (x-c/a)/ (1-c/a) and suitable changes are made to apply QSIT .The method was used by Gupta, Lal and Verma [19] to find results for their problem and accurate results were obtained.

Let $U(X)$ is continuous differentiable function on interval $(0,1)$, and also let this range be divided into m sub intervals by taking $\Delta X=1/m$ and $X_r=r\Delta X$, where $r=0,1,2,3,...,m.$

Again let $U(X)$ be a quintic spline which is approximating function for deflection function, and is given by:

$$U(X)=a_0+\sum_{i=1}^{4}a_i\left(X-X_0\right)^i+\sum_{t=0}^{m-1}b_t\left(X-X_t\right)_+^{\,5}, \qquad (3)$$

$$\left(X-X_t\right)_+=\begin{cases}0, & if\ X\le X_t\\ \left(X-X_t\right), & if\ X>X_t\end{cases},$$

and $a_i's, b_t's$ are coefficients; considered constants.

Thus for j^{th} knot, eq. (2) reduce to

$$
\begin{aligned}
&V_4a_0+\left[V_4\left(X_j-X_0\right)+V_3\right]a_1+\left[V_4\left(X_j-X_0\right)^2+2V_3\left(X_j-X_0\right)+2V_2\right]a_2\\
&+\left[V_4\left(X_j-X_0\right)^3+3V_3\left(X_j-X_0\right)^2+6V_2\left(X_j-X_0\right)+6V_1\right]a_3\\
&+\left[V_4\left(X_j-X_0\right)^4+4V_3\left(X_j-X_0\right)^3+12V_2\left(X_j-X_0\right)^2+24V_1\left(X_j-X_0\right)+24V_0\right]a_4 \qquad (4)\\
&+\sum_{t=0}^{m-1}b_t\begin{bmatrix}V_4\left(X_j-X_t\right)_+^{\,5}+5V_3\left(X_j-X_t\right)_+^{\,4}+20V_2\left(X_j-X_t\right)_+^{\,3}\\ +60V_1\left(X_j-X_t\right)_+^{\,2}+120V_0\left(X_j-X_t\right)_+\end{bmatrix}=0.
\end{aligned}
$$

For $j=0(1)m$, $(m+5)$ equations in $(m+5)$ unknowns are obtained from above system, where a_i , i $=0(1)4$ and the above system of linear equations can be represented in homogeneous matrix equation form as $[P][Q]=0,$ where $[P]$ matrix is of type $(m+1)\times(m+5)$ while $[Q]$ is a matrix having order $(m+5)\times 1$, and O is zero matrix having order $(m+5)\times 1.$

4. FREQUENCY EQUATION USING BC'S (BOUNDARY CONDITIONS)

The following BC's are applied in our problem to find frequency equation:

(i) Clamped edge condition: $U = \frac{dU}{dX} = 0.$

(ii) Simply supported edge condition:

$$U = \frac{d^2U}{dX^2} + (\nu_\theta / X)\left(\frac{dU}{dX}\right) = 0.$$

(iii) Free edge condition:

$$\frac{d^2U}{dX^2} + \left(\frac{\nu_\theta}{X}\right)\left(\frac{dU}{dX}\right) = \frac{d^3U}{dX^3} + \left(\frac{1}{X}\right)\left(\frac{d^2U}{dX^2}\right) - \left(\frac{p}{X^2}\right)\left(\frac{dU}{dX}\right) = 0.$$

The following cases for BC'S are considered: (i) inner edge fixed –outer edge fixed(C-C) (ii)inner edge fixed –outer edge simply supported(C-S) (iii) inner edge fixed – outer edge loose/free(C-F). A collection of four equations which are homogenous in nature are obtained for each BC's. These when combined with above equation (4) provided us m+5 homogenous equations in m+5 undetermined variables. For first combination i.e. C-C combination equation for Ω (frequency parameter) using C-C BC's can be expressed as

$$\left[\frac{P}{A^{CC}}\right][Q]=0 \text{ (Zero Matrix),} \tag{5}$$

where A^{CC} = matrix of type $4\times(m+5)$ derived by applying C-C boundary conditions.

For getting non-zero solutions of (6) the determinant must be zero i. e. it must vanish. So we must have:

$$\left|\frac{P}{A^{CC}}\right| = 0, \tag{6}$$

similarly for C-S plate combination and C-F plate combination, the determinant are expressed as,

$$\left|\frac{P}{A^{CS}}\right| = 0 \ \ and \ \ \left|\frac{P}{A^{CF}}\right| = 0 \tag{7, 8}$$

Where A^{CS} and A^{CF} are the matrix of type $4\times(m+5)$ derived by applying C-S BC's and C-F BC's respectively.

5. RESULTS AND DISCUSSION

Frequency equations (6), (7) and (8) provide us the transcendental equations to obtain infinite numerical values for Ω (frequency parameter) for three different boundary combinations/conditions i.e. C-C combinations, C-S combination & C-F combinations. In this mathematical model, we have calculated first two modes of vibrations of annular plates (ring shaped) for above said boundary conditions. MATLAB program is used to develop programming code and drawing various graphs for the present problem. MS-Excel is also used to draw the various graphs representing the variation in Ω with respect to different combinations of parameter used.

To obtained appropriate interval we have considered $\Delta X = \frac{1}{m}$ where $m = 10(10)110$ but for $m \geq 90$, the results shows no improvement. The results for convergence for number of increasing nodes for C-C, C-S, C-F combinations tacking- $\alpha = 0.3, K = 2, d_k = 0.01, c/a = 0.3, p = 5, \upsilon = 0.3$ are presented in Table 1. The variations in different parameters used for the present problem are considered as Taper constant (α) $[\alpha = 0.0(0.1)0.3]$, Foundation parameter (K) $[K = 0(1)3]$, Damping Parameter (d_k) [$d_k = 0.0(0.005)0.015$], Rigidity ratio (p) $[p=1, 2, 5, 10]$, Ratio of radii (c/a) $[c/a=0.1, 0.3, 0.5, 0.7]$. Also, values fixed for calculation the above variations in plate parameters are considered as $\upsilon=0.3$, $h=0.1$ and $m=90$.

Table 1. Convergence of Ω for number of increasing nodes for C-C combination, C-S combination & C-F combination for- $\upsilon=0.3,\ \alpha=0.3,\ K=2,\ d_k=0.01,\ c/a=0.3,\ p=5.$

Value of n	Combination(C-C)		Combination(C-S)		Combination (C-F)	
	% Error-1 mode	% Error-2 mode	% Error-1 mode	% Error-2 mode	% Error-1 mode	% Error-2 mode
10	0.155	1.165	0.141	1.089	0.111	0.217
20	0.029	0.291	0.026	0.269	0.024	0.045
30	0.012	0.121	0.009	0.112	0.006	0.016
40	0.007	0.061	0.003	0.057	0.00	0.009
50	0.004	0.033	0.003	0.031	0.00	0.004
60	0.002	0.019	0.00	0.017	0.00	0.002
70	0.002	0.009	0.00	0.009	0.00	0.00
80	0.00	0.003	0.00	0.003	0.00	0.00
90	0.00	0.00	0.00	0.00	0.00	0.00
100	0.00	0.00	0.00	0.00	0.00	0.00
110	0.00	0.00	0.00	0.00	0.00	0.00

The calculated results of Ω for the variation on the value on taper parameter (α) for a non-damped annular plate i.e. when damping parameter (d_k) is considered 0.0 and for a damped annular plate when damping parameter (d_k) is considered 0.01 for three limiting conditions i.e. for boundary combinations: C-C plate position, C-S plate position & C-F plate position respectively are presented in fig. 1(a) & 1(b) respectively assuming the fixed value of rigidity ratio $(p=5)$, Radii ratio $(c/a=0.3)$ and foundation parameter $(K=2)$ for vibrational analysis of two modes .Fig. 1(a) presents variation in Ω with respect to taper constant (α) for first mode of frequency parameter. From fig. 1(a) we observed that as there is increase in parameter taper constant the value of Ω decreases and this decreases is more prominent for second mode in comparison with I mode for non- damped annular plate in all the three assumed boundary combinations i.e. C-C, C-S & C-F. We observe from fig. 1(b) that introduction of damping parameter on annular plate increases the value of frequency parameter but the nature of the graph presented remains the same in this case as for the non-damped annular plate for C-C plate combination and C-S plate combination but for the I mode in C-F plate combination we find that there is linear increase in value of Ω but for second mode there is decrease in the value of Ω. The changes in both the modes are regular & linear for both non-damped and damped annular plate.

The numerical values of Ω (frequency parameter) for variation in K for non-damped annular plate and damped annular plate, i.e. considering damping 0.0 & 0.01 respectively for the assumed three boundary positions i.e. C-C plate position, C-S position & C-F plate position are presented by figures 2(a) and 2(b) respectively for non-damped ($d_k=0.0$) and damped ($d_k=0.01$) assuming fixed value of rigidity ratio ($p=5$) , Radii ratio ($c/a=0.3$) and taper parameter (Ω=0.3) for the 1st two modes for From fig. 2(a) it was observed that for non-damped annular plate when value of K (foundation parameter) increases the value of frequency parameter(Ω) also increase and increase is significant for first mode in comparison to the II mode for all the assumed boundary condition. Also the observed variation in value of Ω is linear in nature in this case. Now from fig. 2(b) we observed that on applying damping parameter the numerical value of frequency parameter (Ω) increase but the nature of the graph remains the same for this damped annular plate as was for non-damped annular plate i.e. the Ω increases as the value of K increases (K=0, 1, 2, 3) and nature of variation is again linear.

The calculated results for variation in frequency parameter(Ω) considering the change in damping parameter for assumed three limiting conditions i.e. C-C position, C-S position & C-F position respectively considering rigidity ratio (p=5), foundation parameter (K=2) & taper parameter (α=0.3) respectively are presented in the form of graph given by figure 3. From fig. (3) it's observed that there is steady increase in both the modes for the three limiting conditions and our observation is that the numerical value of I mode increases more rapidly in comparison to the II mode for all the assumed limiting conditions i.e. BC's.

Numerical value of Ω when the numerical value of parameter rigidity ratio (p) changes i.e. when p takes value as p= 1, 2, 5, 10 for the assumed three BC's considering $b/a=0.3,\ K=2,\ \&\ \alpha=0.3$ are presented by fig. (4). Fig. (4) Provides inference that the value of frequency parameter increases parabollically and change in second mode is more prominent than for the first mode. The graph also represent there is steep growth in the value of frequency parameter if we take greater value of rigidity ratio. Next we have considered the variation in ratio for inner radius to outer radius for obtaining values of Ω for all the assumed three limiting conditions for the first 2 modes of vibration and it is depicted by a graph given in fig. (5). By Fig. (5) Our observation is that there is parabollically increment in the value of frequency parameter for both the calculated modes as the value of radii ratio increases and the parabolic increment is more dominant for the II mode as compared with the I mode. In this case we have assumed, $d_k=0.01,\ p=5,\ K=2\&\alpha=0.3.$ Deflection function for uniform and tapered condition for C-C, C-S and C-F plates have also been presented through graphs [fig. 6]. Comparative study results for our present paper shows that results are well compared with those obtained by [8] and different researcher through their findings which are given in references [5,6,12,13,20] and different comparison are presented in the form of tables given by tables 2, 3 and 4 respectively.

Table 2. Comparison of frequency parameter Ω, for orthotropic annular plate of variable thickness in the fundamental mode, $K=0.0,\ \alpha=0.3,\ c/a=0.5,\ \upsilon_\theta=0.3.$

Boundary conditions	Rigidity Ratio			
	$p=0.5$	$p=0.8$	$p=1.0$	$p=10.0$
C-C	109.147^	109.338^	109.466^	115.015^
	109.18⊥	109.37⊥	109.497⊥	115.023⊥
	109.15*	**109.341***	**109.468***	**115.017***
C-S	71.622	71.858	72.016	78.75
	71.638⊥	71.873⊥	72.03⊥	78.749⊥
	71.624*	**71.86***	**72.017***	**78.751***
C-F	14.287^	14.645^	14.87^	22.731^
	14.282⊥	14.64⊥	14.873⊥	22.719⊥
	14.287*	**14.646***	**14.879***	**22.732***

^ Values from references [22], ⊥ values from references [8], *present calculation

Table 3. Comparison of frequency parameter Ω, for C-F Isotropic annular plates of variable thickness in the fundamental mode, $K=0.0,\ \upsilon_\theta=0.3.$

Taper Parameter	Radii Ratio (c/a)			
	$c/a=0.1$	$c/a=0.3$	$c/a=0.5$	$c/a=0.7$
0.5	3.992Q	5.789Q	10.102Q	24.683Q
	3.975⊥	5.78⊥	10.093⊥	24.705⊥
	3.991*	**5.791***	**10.103***	**24.704***
0.3	4.051Q	6.097Q	11.229Q	29.559Q
	4.037⊥	6.091⊥	11.224⊥	29.551⊥
	4.052*	**6.097***	**11.229***	**29.56***
0.1	4.167Q	6.464Q	12.415Q	34.48Q
	4.151⊥	6.458⊥	12.414⊥	34.476⊥
	4.166*	**6.464***	**12.417***	**34.48***

Q Values from references [5], ⊥ values from references [8], *present calculation

Table 4. Comparison of frequency parameter Ω, for Isotropic annular plates of variable thickness in the fundamental mode, $K = 0.0, \upsilon_\theta = \upsilon_r = 0.3, c/a = 0.3.$

	Taper Constant (α)						
Boundary conditions	–0.5	–0.3	–0.1	0	0.1	0.3	0.5
C-C	60.139†	54.269†	48.331†	45.348#	42.297†	36.122†	29.714†
	60.202⊥	54.306⊥	48.347⊥	45.346!	42.3⊥	36.12⊥	29.72⊥
	60.263*	**54.343***	**48.365***	**45.346***	**42.303***	**36.117***	**29.727***
C-S	38.075†	34.868†	31.613†	29.979#	28.288†	24.859†	21.259†
	38.123⊥	34.898⊥	31.627⊥	29.978!	28.291⊥	24.858⊥	21.264⊥
	38.17*	**34.928***	**31.642***	**29.978***	**28.295***	**24.857***	**21.269***
C-F	7.695†	7.266†	6.85†	6.662#	6.453†	6.086†	5.771†
	7.708⊥	7.276⊥	6.856⊥	6.6604!	6.458⊥	6.091⊥	5.78⊥
	7.722*	**7.286***	**6.864***	**6.66***	**6.464***	**6.097***	**5.791***

Values from reference [6], † values from references [20], ! Values from references [23],

⊥ values from references [8], * Present calculations

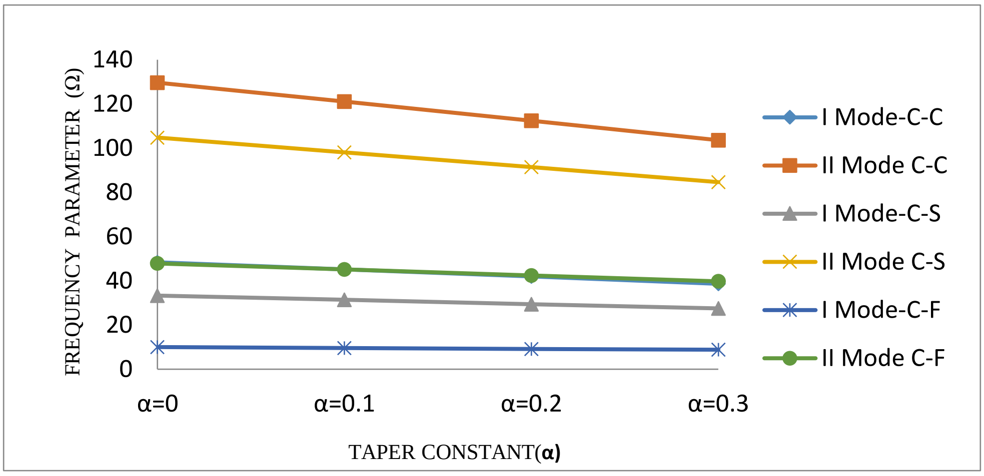


Fig (1a). Graph showing variation in Frequency Parameter w.r.t. Taper Constant when, υ=0.3, h=0.1, $d_k = 0.0$, c/a=0.3, n=90, K=2, p=5

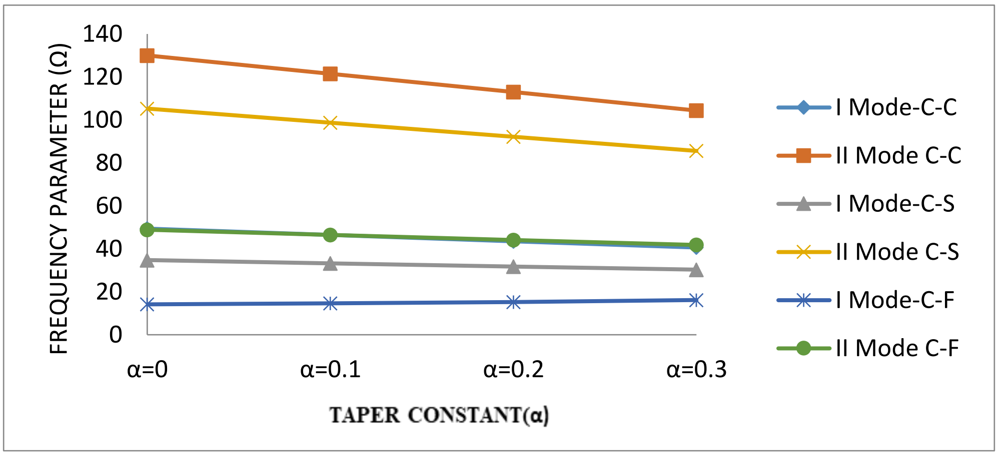


Fig (1b). Graph showing variation in Frequency Parameter w.r.t. Taper Constant when, υ=0.3, h=0.1, $d_k = 0.01$, c/a=0.3, n=90, K=2, p=5

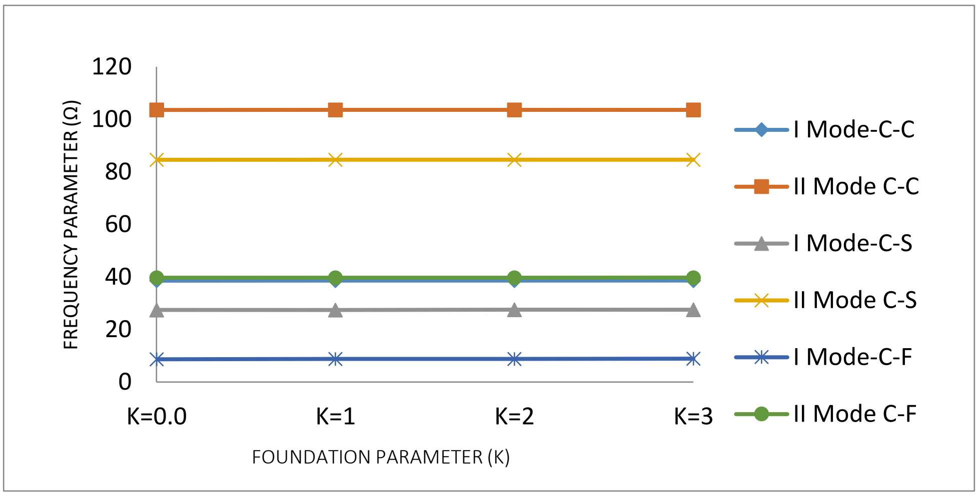


Fig (2a). Graph showing variation in Frequency Parameter w.r.t. Foundation Parameter when, $h = 0.1,\ \upsilon = 0.3,\ d_k = 0.0,\ c/a = 0.3,\ n = 90,\ p = 5,\ \alpha = 0.3.$

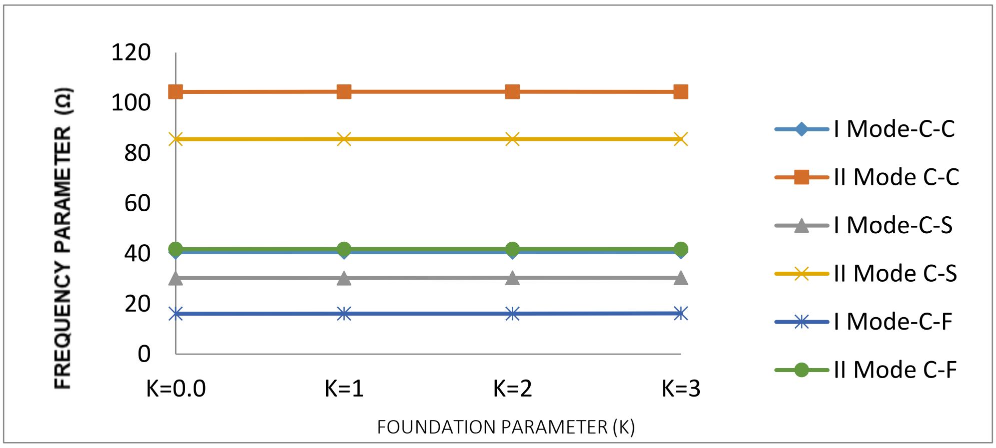


Fig (2b). Graph showing variation in Frequency Parameter w.r.t. Foundation parameter when, $d_k = 0.01,\ h = 0.1,\ \upsilon = 0.3,\ c/a = 0.3,\ n = 90,\ p = 5,\ \alpha = 0.3.$

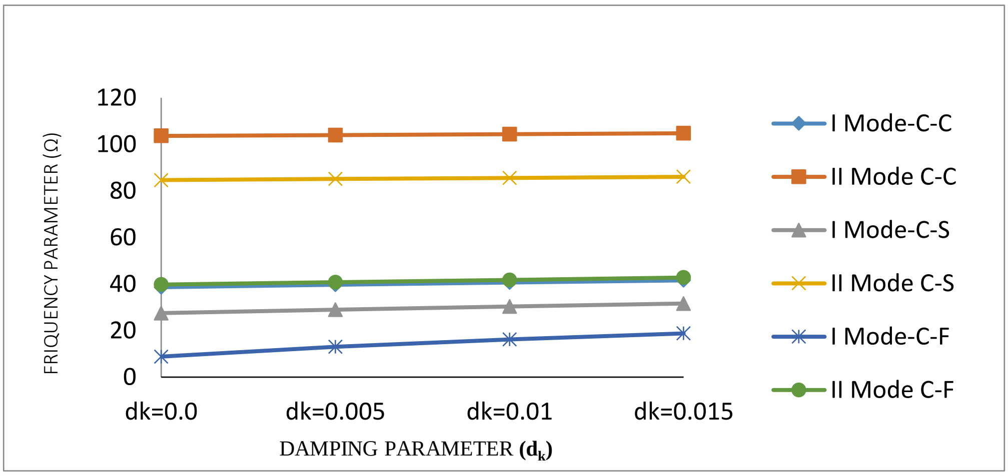


Fig (3). Graph showing variation in Frequency Parameter w.r.t Damping Parameter when, $K = 2,\ h = 0.1,\ \upsilon = 0.3,\ c/a = 0.3,\ n = 90,\ p = 5,\ \alpha = 0.3.$

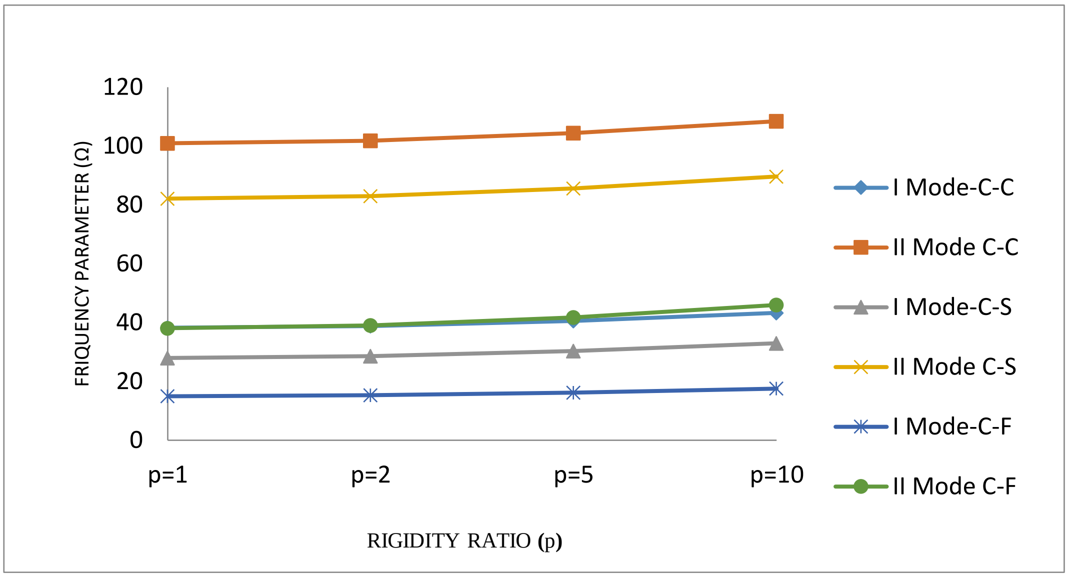


Fig (4). Graph showing variation in Frequency Parameter w.r.t. rigidity ratio when,
$d_k = 0.01,\ h = 0.1,\ \upsilon = 0.3,\ c/a = 0.3,\ n = 90,\ K = 2,\ \alpha = 0.3.$

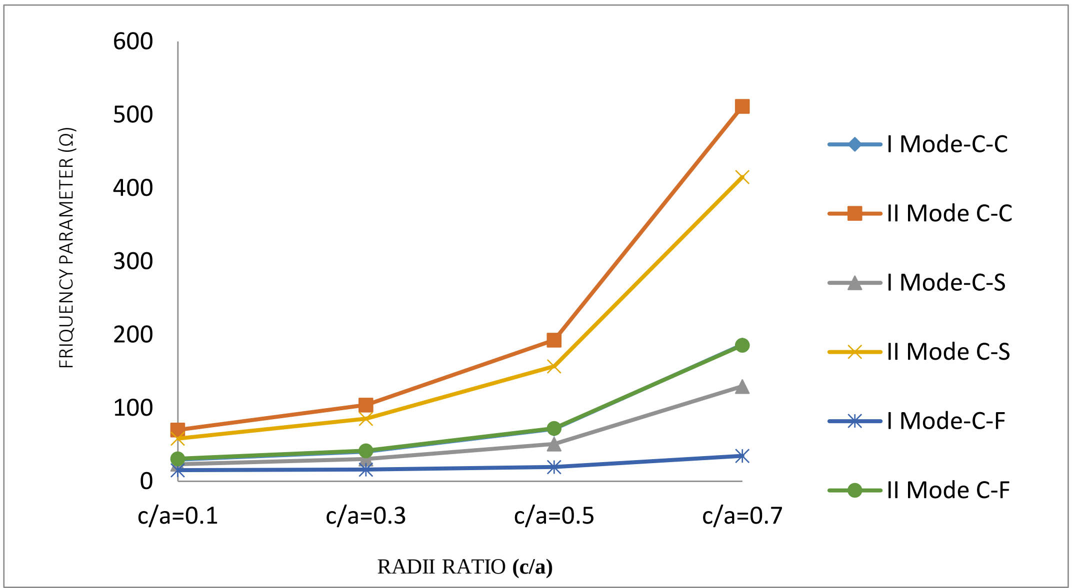
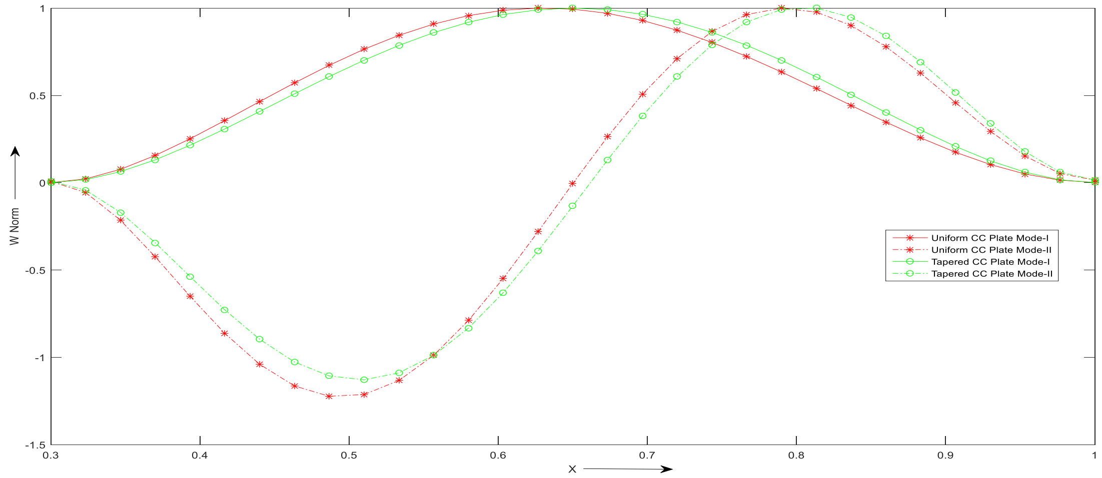
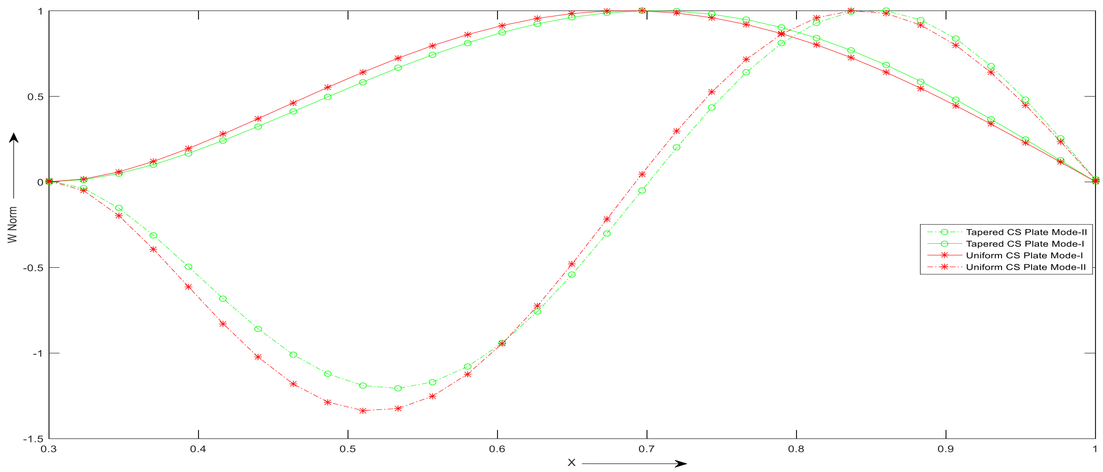


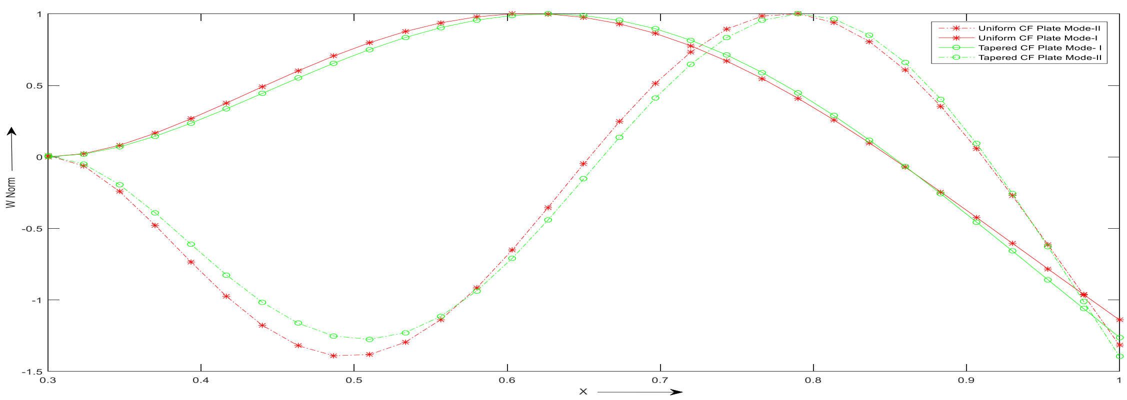
Fig (5). Graph showing variation in Frequency Parameter w.r.t. Radii ratio when,
$d_k = 0.01,\ h = 0.1,\ \upsilon = 0.3,\ K = 2,\ n = 90,\ p = 5,\ \alpha = 0.3.$



(a)



(b)



(c)

Figure 6. Normalized displacements for uniform and tapered - (a) C-C plate, (b) C-S plate and (c) C-F plate for first two modes of vibration for $K=2$, $d_k=0.01$, $p=5$, $c/a=0.3$. O, $\alpha=0.0$; $*$, $\alpha=0.3$, —, First mode; - - -, Second mode.

6. CONCLUSION

New designs for various structures are always required by engineers and stability of structure depends upon its frequency variation and free vibrations are practically impossible, so vibration analysis for plate structure can't be imagine without damping. So damping phenomenon for vibrational analysis study is considered so important that its effect can be so large enough to stop vibration or so small to produce any appreciable effect on plate frequency. Thus considering damping effect we come closer to real world problem of vibration analysis so the present model deals with the study of effect of damping on Vibrations of Annular plates (ring shaped plates) having variable thickness that rest on elastic foundation by QSIT which will be useful for assessment and modification of design structure generally by civil and mechanical engineers. This assessment and modifications helps in preventing fatigue in structure and also complete failure of the design and also helps in reduction of the production cost. MATLAB programming is used to compute the results for the assumed model within permissible range and as per desired accuracy which helps to validate our results. The results so obtained for present problem helps us to obtain desired frequency by changing various parameters and help to make design structure strong and shock proof.

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