Separation Axioms in Vague Bitopological spaces

1.A.Selvi

Assistant Professor,

Providence College for Women,

Coonoor-643104

2. C.Jenita Nancy

Research Scholar,Department of Mathematics,

Nirmala College for Women,

Coimbatore,Tamil Nadu,India.

Email-nancyjenita@gmail.com

3. A.Arokia Lancy

Assistant Professor,Department of Mathematics,

Nirmala College for Women,

Coimbatore,Tamil Nadu,India.

Email-aarokia.lancy@gmail.com

Abstract:

In this paper we are introducing separation axioms in vague bitopological space.The separation axioms based on semi set difference set ,properties of T_0, T_1 and T_2,pairwise regular and pairwise normal in vague bitopological space are discussed.Also the pairwise semi-H_i-spaces and pairwise semi-U_i-spaces in vague bitopological space are studied.

Keywords:

Semi difference set, T_0, T_1, T_2 space,Pairwise regular and pairwise normal bitopological space, pairwise semi-H_i-spaces and pairwise semi-U_i-spaces.

I.Introduction:

A note on five separation axioms in bitopological spaces was introduced by T.M.Nour[8] in 1995.Harjot Singh [3] studied the pairwise separated sets in bitopological spaces in 2018 and he discussed the weak separation properties as hereditary properties during the same year.In addition,he also introduced the topological invariance of separation properties in bitopological space in 2018.Trishla Garg and Krishna[12] Singhal originated study of some mappings in bitopological space in the year 2013.

II.Preliminaries:

Definition:2.1[5]

A subset T of a vague topological space is called as semi difference set if T = $O_1 \cap O_2^c$ with $O_1 \neq$ X and O_1 and O_2 are $\tau_1 \tau_2$ -semi open sets.

Definition 2.2[10]

Assume A to be a non-empty subset of a topological space (X, τ).The class τ_A all intersections of A with τ open subsets of X is a topology on A,it is called the relative topology on A or the relativization of τ to A and the topological space (A, τ_A) is known as a subspace of (X, τ).

Definition:2.3[1]

Let (X, τ_1, τ_2) be a bitopological space.

(i) A set G is called as pairwise open if G are both τ_1-open and τ_2-open in X.

(ii) A set F is called pairwise closed if F are both τ_1-closed and τ_2-closed in X.

(iii) A cover of a bitopological space (X, τ_1, τ_2) is known as pairwise open if their elements are members of τ_1 and τ_2 and if it contains atleast one empty member of τ_1 and τ_2.

Definition: 2.4[11]

A Vague bitopological space (X, τ_1, τ_2) is said to be **pairwise semi-H_0** if for every pair of points r and s such that r ∉ vague-τ_i -scl {s} there exists a vague-τ_i semi-open set R and a vague-τ_j semi-open set V such that r ϵ R, s ϵ S, R ∩ S = ∅.; i ≠j, i, j = 1, 2.

Definition: 2.5[11]

A vague bitopological space (X, τ_1, τ_2) is said to be **pairwise semi-H_1** if for every pair of points r and s such that vague-τ_i -scl {r} ∩ vague-τ_j -scl {s} = ∅, there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that r ϵ R, s ϵ S, R ∩ S = ∅; i ≠ j, i, j = 1, 2.

Definition: 2.6[11]

A vague bitopological space (X, τ_1, τ_2) is said to be **pairwise semi-H_2** if for every vague-τ_i -semi-closed set B and a point r such that vague-τ_i -scl {r} ∩ B = ∅, there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that r ϵ R, B ⊆ S, R ∩ S= ∅; i ≠ j, i, j = 1, 2.

III.Vague - τ_1 τ_2 -semi difference set:

Definition:3.1

A vague bitopological space (X, τ_1, τ_2) is called as pairwise semi T_0 if for every x,z ϵ X, $x \neq$ z there exist a vague- τ_1-semi open set containing one of the points but not the other or there exist a vague- τ_2-semi open set containing one of the points but not the other.

Definition:3.2

A vague bitopological space (X, τ_1, τ_2) is called as pairwise semi T_1 if for every x,z ϵ X, $x \neq$ z there exist a vague- τ_1-semi open set V and a vague- τ_2-semi open set W such that x ϵ V, z ∉ V and z ϵ W, x ∉ W.

Definition:3.3

A vague bitopological space (X, τ_1, τ_2) is called as pairwise T_2 if for every x,z ϵ X, $x \neq$ z there exist a vague- τ_1-semi open set V and a vague- τ_2-semi open set W such that x ϵ V,z ϵ W and V∩ W = ϕ.

Definition:3.4

A subset T of a vague topological space is called as semi difference set if T = $M_1 \cap M_2^c$ with $M_1 \neq$ X and M_1 and M_2 are v- τ_1 τ_2 -semi open sets.

Definition:3.5

A subset of a vague bitopological space (X, τ_1, τ_2) is called as v- τ_1 τ_2 -semi difference set if S = $M_1 \cap M_2^c$ where $M_1 \neq$ X and M_1 is v- τ_1-semi open, M_2 is v- τ_2-semi open.

Example:3.6

Let X = {a},τ_1={0,M_1,1}, τ_2={0,M_2,1},where M_1={<[0.2,0.5]>}, M_2={<[0,0.5]>}.Here S is a v- τ_1 τ_2 -semi difference set if S = $M_1 \cap M_2^c$ where $M_1 \neq$ X and M_1 is v- τ_1-semi open, M_2 is v- τ_2-semi open.

Definition:3.7

A vague bitopological space (X, τ_1, τ_2) is called as pairwise semi D_0 if for every x,z ϵ X, $x \neq$ z,there exist a v- τ_1 τ_2 -semi difference set containing one of the points but not the other or there exist a v- τ_2 τ_1 -semi difference set containing one of the points but not the other.

Definition:3.8

A vague bitopological space (X, τ_1, τ_2) is called as pairwise semi D_1 if for every $x \neq$ z in X,there exist a v- τ_1 τ_2 -semi difference set V such that x ϵ V but z $\notin$ V and a v- τ_2 τ_1-semi difference set W such that z ϵ W, $x \notin$ W.

Definition:3.9

A vague bitopological space (X, τ_1, τ_2) is called as pairwise semi D_2 if for every $x \neq$ z ,there exist a v- τ_1 τ_2 - semi difference set V and a v- τ_2 τ_1-semi difference set W such that V $\cap$ W = ϕ with x ϵ V and z ϵ W.

Theorem:3.10

pairwise semi T_0 $\Longrightarrow$ pairwise semi D_0 in vague bitopological space.

Proof:

Suppose (X, τ_1, τ_2) is pairwise semi T_0 .Then for all x,z ϵ X, $x \neq$ z,there exist a v- τ_1 -semi open set V or there exist a v- τ_2 -semi open set W such that V contains one of the points x and z.

Now V is v- τ_1 -semi open $\Longrightarrow$ V is a v- τ_1 τ_2 -semi difference set .

W is v- τ_2 -semi open $\Longrightarrow$ W is a v- τ_2 τ_1 -semi difference set .

$\Longrightarrow$ there exist a v- τ_1 τ_2 -semi difference set V containing x or y or

there exist a v- τ_2 τ_1 -semi difference set W containing x or y.

$\Longrightarrow$ X is pairwise D_0.

Note:3.11

(a) pairwise semi $T_1$$\Longrightarrow$Vague pairwise D_0 in vague bitopological space.

(b) pairwise semi $T_2$$\Longrightarrow$pairwise D_2 in vague bitopological space.

(c) pairwise semi D_2 $\Longrightarrow$ pairwise D_1 in vague bitopological space.

(d) pairwise semi D_1 $\Longrightarrow$ pairwise D_0.

Theorem:3.12
pairwise semi D_0 $\Longrightarrow$ pairwise T_0 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) to be pairwise semi D_0 in vague bitopological space.Then for every $x \neq$ z there exist a v- τ_1 τ_2 -semi difference set S' with x ϵ S' but $z \notin$ S' or there exist a v- τ_2 τ_1 -semi difference set R' with z ϵ R' but $x \notin$ R'.If there exist a v- τ_1 τ_2 -semi difference set S' with x ϵ S' but $z \notin$ S'.Assume S' = $M_1 \cap M_2^c$ with $M_1 \neq$ X,where M_1 is a v- τ_1-semi open set and M_2 is a v- τ_2-semi open set.

Now, x ϵ S'$\Longrightarrow$ $x \in M_1$ and $x \notin M_2$

$z \notin$ S'$\Longrightarrow$z $\notin M_1$ or z $\in M_1$and $z \notin M_2$.

Case (i):

If $x \in M_1$ but $z \notin M_1$.

$\Longrightarrow$ There exist a v- τ_1-semi open set M_1 containing x but not z.

$\Longrightarrow$ (X, τ_1, τ_2) is pairwise T_0.

Case (ii):

If $z \in M_2$ and $x \notin M_2$.

$\Longrightarrow$ There exist a v- τ_2-semi open set M_1 containing z but not x.Hence the proof.

Theorem:3.13
pairwise -semi $D_1 \Longrightarrow$pairwise -semi D_2 in vague bitopological space.

Proof:

If (X, τ_1, τ_2) is pairwise semi D_1.

$\Longrightarrow \forall x \neq z$,there exist a v- $\tau_1 \tau_2$ -semi difference set S_1' and a v- $\tau_2 \tau_1$ -semi difference set S_2' such that $x\epsilon\, S_1'$, $y \notin S_1'$ and $y\epsilon S_2'$, $x \notin S_2'$.

Assume $S_1' = M_1 \cap M_2^c$ where $M_1 \neq$ X and M_1 is a v- τ_1-semi open set, M_2 is a v- τ_2-semi open set.

$S_2' = M_3 \cap M_4^c$ where $M_3 \neq$ X and M_3 is a v- τ_2-semi open set, M_4 is a v- τ_1-semi open set.

$x \notin S_2' \Longrightarrow x \notin M_3$ or $x\epsilon M_3$ and $x\epsilon M_4$.

$z \notin S_1' \Longrightarrow z \notin M_1$ or $z\epsilon M_1$ and $z\epsilon M_2$.

Case (i):

$x \notin M_3, z \notin M_1$.

Then $x\epsilon\, M_1 \cap M_2^c$

$\Longrightarrow x\epsilon\, M_1 \cap (M_2 \cup M_3)^c$

$z\epsilon\, M_3 \cap M_4^c$

$\Longrightarrow x\epsilon\, M_3 \cap (M_1 \cup M_4)^c$

$\Longrightarrow \{M_1 \cap (M_2 \cup M_3)^c\} \cap \{M_3 \cap (M_1 \cup M_4)^c\} = \phi$.

That is,there exist a v- $\tau_1 \tau_2$ -semi difference set $M_1 \cap (M_2 \cup M_3)^c$ containing x and there exist a v- $\tau_2 \tau_1$ -semi difference set $M_3 \cap (M_1 \cup M_4)^c$ containing z such that

$\{M_1 \cap (M_2 \cup M_3)^c\} \cap \{M_3 \cap (M_1 \cup M_4)^c\} = \phi$.

$\Longrightarrow$ X is vague pairwise semi D_2.

Case (ii):

If $x \notin M_3$ and $z \in M_1$ and $z \in M_2$

$\Longrightarrow x\epsilon\, M_1 \cap M_2^c, z\epsilon\, M_3 \cap M_4^c$

such that $(M_1 \cap M_2^c) \cap (M_3 \cap M_4^c) = \phi$.

$\Longrightarrow X$ is vague pairwise semi D_2.

Case (iii):

If $x\epsilon\, M_3$ and $x\epsilon\, M_4, z \notin M_1$

$\Longrightarrow x\epsilon\, M_1 \cap M_2^c, z\epsilon\, M_3 \cap M_4^c$

such that ($M_1 \cap M_2^c) \cap (M_3 \cap M_4^c$) = ϕ.

$\Longrightarrow X$ is vague pairwise semi D_2.

Case (iv):

If $x\epsilon\, M_3$ and $M_4, z\epsilon M_1$ and M_2.

$x\epsilon\, M_1 \cap M_2^c, z\epsilon\, M_3 \cap M_4^c$

such that ($M_1 \cap M_2^c) \cap (M_3 \cap M_4^c$) = ϕ.

$\Longrightarrow X$ is vague pairwise semi D_2.

Definition:3.14

A mapping f:X⇢Y is called as irresolute if the inverse image of every vague semi open set in Y is vague semi open in X.

Definition:3.15

A mapping f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called pairwise irresolute iff the induced mappings f: (X, τ_1) → (Y, σ_1) and f: (X, τ_2) → (Y, σ_2) are irresolute in vague bitopological space.

Theorem:3.16

If f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is pairwise irresolute surjective mapping and S is a v- σ_1 σ_2 -semi difference set in Y,then f^{-1} (S) is a v- τ_1 τ_2-semi difference set in X.

Proof:

Assume S to be a v- σ_1 σ_2 -semi difference set in Y.

$\Longrightarrow S = M_1 \cap M_2^c$ where $M_1 \neq$ Y and M_1 is v- σ_1-semi open.

M_2 is v- σ_2-semi open.

As f is a pairwise irresolute map f^{-1} (M_1) is M_1 is v- τ_1-semi open and f^{-1} (M_2) is v- τ_2-semi open.

Also f^{-1} $(M_1) \neq$ X ($M_1 \neq$ Y and f is onto).

$\Longrightarrow f^{-1}(S) = f^{-1}(M_1) \cap (f^{-1}(M_2))^c$

$\Longrightarrow f^{-1}(S)$ is a v- $\tau_1\tau_2$-semi difference set in X.

Thus the proof.

Theorem:3.17

Assume (Y, σ_1, σ_2) to be pairwise semi D_1 and g: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is pairwise irresolute and bijective then (X, τ_1, τ_2) is pairwise semi D_1 in vague bitopological space.

Proof:

Assume $x \neq$ z in X $\Longrightarrow g(x) \neq g(z)$ (Since g is injective)

But Y is pairwise semi D_1.

⟹there exist a v- σ_1 σ_2 -semi difference set U' and v- σ_2 σ_1 -semi difference set V' such that $g(x)\epsilon U', g(z) \notin U'$ and $g(z)\epsilon V', g(x) \notin V'$.

$\Longrightarrow x\epsilon\, g^{-1}(U'), z \notin g^{-1}(U')$ and $z \epsilon\, g^{-1}(V'), x \notin g^{-1}(V')$

Since g is pairwise irresolute by theorem (8.4) $g^{-1}(U')$,$g^{-1}(V')$ are v- τ_1 τ_2-semi difference set and v- τ_2 τ_1-semi difference set respectively such that $x\epsilon\ g^{-1}(U')$, $z \notin g^{-1}(U')$ and $z\ \epsilon\ g^{-1}(V')$, $x \notin g^{-1}(V')$.

$\Longrightarrow X$ is pairwise semi D_1 in vague bitopological space.

Thus the proof.

IV. T_0, T_1 and T_2 in vague bitopological space:

Throughout this section X,Y will be a non-empty set. τ_1, τ_2, σ_1, σ_2 will be the topology on X. (X, τ_1, τ_2) and (Y, σ_1, σ_2) are the vague bitopological space.,U', V' are the vague open sets and its elements are $x', y', x_1', y_1', x_2', y_2'$.

Definition:4.1

A mapping g: (X, τ_1, τ_2) → (Y, σ_1, σ_2) is known as P-continuous(resp.P-open ,P-closed) if the induced mappings g: (X, τ_1) → (Y, σ_1) and g: (X, τ_2) → (Y, σ_2) are continuous. (resp.open ,closed) in vague bitopological space.

Definition:4.2

A vague bitopological space (X, τ_1, τ_2) is known as T_0-space if for all $x', y' \epsilon$ X with $x' \neq y'$ then there exist $U' \epsilon\ \tau_1 \cup\ \tau_2$ such that $x' \epsilon\ U'$, $y' \notin\ U'$ or $x' \notin U'$, $y' \epsilon U'$.

Definition:4.3

A vague bitopological space (X, τ_1, τ_2) is known as T_1-space if for all $x', y' \epsilon$ X with $x' \neq y'$,then there exist $U' \epsilon$ v- τ_1 and $V' \epsilon$ v-τ_2 such that $x' \epsilon\ U'$, $y' \notin\ U'$ and $x' \notin V'$, $y' \epsilon V'$.

Definition:4.4

A vague bitopological space (X, τ_1, τ_2) is known as T_2-space if for all $x', y' \epsilon$ X with $x' \neq y'$,then there exist $U'\ \epsilon$ v-τ_1, $V' \epsilon$ v-τ_2 such that $x' \epsilon\ U', y' \epsilon V'$, $U' \cap V' = \phi$.

Properties of T_0, T_1 and T_2 in vague bitopological space:

Theorem :4.5

T_0 is a hereditary property.

Proof:

If (X, τ_1, τ_2) is T_0-space and B⊆ X has to be proved that (B, τ_{1B}, τ_{2B}) is also T_0-space.

Assume $x', y' \epsilon$ B with $x' \neq y'$,then $x', y' \epsilon$ X with $x' \neq y'$.Since (X, τ_1, τ_2) is vague-T_0-space,then there exist $U' \epsilon$ $\tau_1 \cup\ \tau_2$ such that $x' \epsilon\ U'$, $y' \notin\ U'$ or $x' \notin U'$, $y' \epsilon U'$.

Then, $U' \epsilon\ \tau_1 \cup\ \tau_2$

$\Longrightarrow U'\ \epsilon\, \tau_1$ or $U'\ \epsilon\, \tau_2$

$\Longrightarrow U' \cap B\ \epsilon\, \tau_{1B}$ or $U' \cap B\ \epsilon\, \tau_{2B}$

$\Longrightarrow U' \cap B\ \epsilon\, \tau_{1B} \cup \tau_{2B}$.

Again since $x', y' \epsilon$ B, then $x' \epsilon U' \cap B$, $y' \notin U' \cap B$ or $x' \notin U' \cap B$, $y' \epsilon U' \cap B$ .Then (B, τ_{1B}, τ_{2B}) is also T_0-space.

Theorem :4.6

T_1 is a hereditary property.

Proof:

If (X, τ_1, τ_2) is a T_1 -space and B⊆ X has to be proved that (B, τ_{1B}, τ_{2B}) is also T_1-space.

Assume $x', y' \epsilon$ B with $x' \neq y'$,then $x', y' \epsilon$ X with $x' \neq y'$. Since (X, τ_1, τ_2) is T_1-space,then there exist $U' \ \epsilon \ \tau_1$ and $V' \ \epsilon \ \tau_2$ such that $x' \epsilon \ U'$, $y' \notin \ U'$ and $x' \notin V'$, $y' \epsilon V'$.Then $U' \ \epsilon \ \tau_1$ and $V' \ \epsilon \ \tau_2$.

$\Longrightarrow U' \ \cap \ B \ \epsilon \ \tau_{1B}$ or $V' \ \cap \ B \ \epsilon \ \tau_{2B}$.

Again since $x', y' \epsilon$ B then $x' \epsilon \ U' \ \cap \ B$, $y' \notin \ U' \ \cap \ B$ and $x' \notin V' \ \cap \ B$, $y' \epsilon \ V' \ \cap \ B$.

Theorem :4.7

T_2 is a hereditary property.

Proof:

If (X, τ_1, τ_2) is a T_2-space and B$\subseteq$ X has to be proved that (B, τ_{1B},τ_{2B}) is also T_2-space.

Assume $x', y' \epsilon$ B with $x' \neq y'$,then $x', y' \epsilon$ X with $x' \neq y'$. Since (X, τ_1, τ_2) is T_2-space,then there exist $U' \ \epsilon \ \tau_1$ and $V' \ \epsilon \ \tau_2$ such that $x' \epsilon \ U'$, $y' \epsilon V'$ and $U' \cap \ V' = \ \phi$.

Then, $U' \ \epsilon \ \tau_1$ and $V' \ \epsilon \ \tau_2$

$\Longrightarrow U' \ \cap \ B \ \epsilon \ \tau_{1B}$ or $V' \ \cap \ B \ \epsilon \ \tau_{2B}$.

Again since $x', y' \epsilon$ B then $x' \epsilon \ U' \ \cap \ B$, $y' \epsilon \ V' \ \cap \ B$ and $(U' \ \cap \ B) \cap \ (V' \ \cap \ B) = (U' \ \cap \ V') \cap \ B$

$= \emptyset \cap \ B = \emptyset$.Hence (B, τ_{1B},τ_{2B}) is also T_2-space.

Theorem :4.8

T_0 is a vague topological property.

Proof:

Assume g: (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) to be a homeomorphism and (X, τ_1, τ_2) is T_0 space .We are about to prove (Y, σ_1, σ_2) is also a T_0 space .

Assume y_1' , $y_2' \epsilon Y$ with $y_1' \neq y_2'$.Since g is onto then there exist $x_1', x_2' \epsilon X$ with g(x_1') = y_1' and g(x_2') = y_2'.Again,since g is one – one with $y_1' \neq y_2'$.

$\Longrightarrow g(x_1') \ \neq g(x_2') \ \Longrightarrow x_1' \ \neq x_2'$.

Further since (X, τ_1, τ_2) is T_0 space and $x_1', x_2' \epsilon X$ with $x_1' \ \neq x_2'$,then there exist $U' \epsilon \ \tau_1 \cup \ \tau_2$ such that $x_1' \epsilon \ U', x_2' \notin \ U'$ or $x_1' \notin \ U', x_2' \epsilon \ U'$.

Assume $x_1' \epsilon \ U', x_2' \notin \ U'$.Then $U' \epsilon \ \tau_1 \cup \ \tau_2$.

$\Longrightarrow g(U') \epsilon \ g(\tau_1 \cup \ \tau_2 \ )$ as g is vague open and vague continuous,then $g(U') \epsilon \ g(\tau_1) \ \cup g(\ \tau_2) \epsilon \ \sigma_1 \cup \ \sigma_2$.

Also, $x_1' \epsilon \ U', g(x_1') \epsilon \ g(U') \Longrightarrow y_1' \ \epsilon \ g(U')$ and

$x_2' \notin \ U', g(x_2') \notin g(U') \Longrightarrow y_2' \notin g(U')$.

(i.e.) for any y_1' , $y_2' \epsilon Y$ with $y_1' \neq y_2'$, $g(U') \epsilon \ \sigma_1 \cup \ \sigma_2$ is obtained such that $y_1' \ \epsilon \ g(U')$, $y_2' \notin g(U')$.

Therefore (Y, σ_1, σ_2) is a T_0-space (i.e.)every homeomorphic image of T_0-space is a T_0-space.Thus, T_0 is a vague topological property.

Theorem :4.9

T_1 is a vague topological property.

Proof:

Assume g: (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) to be a homeomorphism and (X, τ_1, τ_2) is T_1 space .That (Y, σ_1, σ_2) is also a T_1 space has to be proved.Assume y_1' , $y_2' \epsilon Y$ with $y_1' \neq y_2'$.Since g is onto then there exist $x_1', x_2' \epsilon X$ with g(x_1') = y_1' and g(x_2') = y_2'.Again,since g is one – one with $y_1' \neq y_2'$.

$\Rightarrow g(x_1') \neq g(x_2') \Rightarrow x_1' \neq x_2'$.

Then $x_1', x_2' \epsilon X$ with $x_1' \neq x_2'$.

Again since (X, τ_1, τ_2) is vague-T_1 space then there exist $U' \epsilon \tau_1$ and $V' \epsilon \tau_2$ such that $x_1' \epsilon U', x_2' \notin U'$ and $x_1' \notin V', x_2' \epsilon V'$.Further since g is vague-open, then $g(U') \epsilon \sigma_1$ and $g(V') \epsilon \sigma_2$.Also,

$x_1' \epsilon U' \Rightarrow y_1' = g(x_1') \epsilon g(U'), x_2' \notin U' \Rightarrow y_2' = g(x_2') \notin g(U')$ and

$x_1' \notin V' \Rightarrow y_1' = g(x_1') \notin g(V'), x_2' \notin V' \Rightarrow y_2' = g(x_2') \notin g(V')$

(i.e.)for any y_1' , $y_2' \epsilon Y$ with $y_1' \neq y_2'$, $g(U') \epsilon \sigma_1$ and $g(V') \epsilon \sigma_2$ are obtained such that $y_1' \epsilon g(U'), y_2' \notin g(U')$ and $y_1' \notin g(V'), y_2' \epsilon g(V')$.

Therefore (Y, σ_1, σ_2) is a T_1 space.(i.e.) every homeomorphic image of a T_1 space is also a T_1space.Thus T_1 is a vague topological property.

Theorem :4.10

T_2 is a vague topological property.

Proof:

Assume g: (X, τ_1, τ_2) → (Y, σ_1, σ_2) to be a homeomorphism and (X, τ_1, τ_2) is T_2 space .That (Y, σ_1, σ_2) is also a T_2 space has to be proved.

Assume y_1' , $y_2' \epsilon Y$ with $y_1' \neq y_2'$.Since g is onto then there exist $x_1', x_2' \epsilon X$ with g(x_1') = y_1' and g(x_2') = y_2'.Again,since g is one – one with $y_1' \neq y_2'$.

$\Rightarrow g(x_1') \neq g(x_2') \Rightarrow x_1' \neq x_2'$.

Now $x_1', x_2' \epsilon X$ with $x_1' \neq x_2'$.

Again since (X, τ_1, τ_2) is T_2 space then there exist $U' \epsilon \tau_1$ and $V' \epsilon \tau_2$ such that $x_1' \epsilon U'$, $x_2' \epsilon V'$ and $U' \cap V' = \phi$.Further since g is vague open,then $g(U') \epsilon \sigma_1$ and $g(V') \epsilon \sigma_2$.

Let $g(U') \cap g(V') \neq \phi$,then there exist $z' \epsilon g(U') \cap g(V')$.

$\Rightarrow z' \epsilon g(U')$ and $z' \epsilon g(V')$

$\Rightarrow$there exist $q_1 \epsilon U'$ and $q_2 \epsilon V'$

such that $z' = g(q_1)$ and $z' = g(q_2)$ with $g(q_1) = g(q_2)$

$\Rightarrow q_1 = q_2$ as g is one -one.

$\Rightarrow q_1 \epsilon U'$, $q_1 \epsilon V'$

$\Rightarrow q_1 \epsilon U' \cap V'$

$\Rightarrow U' \cap V' \neq \phi$ which is a contradiction.

The fact is that $U' \cap V' = \phi \Rightarrow g(U') \cap g(V') = \phi$.

Hence,for any y_1' , $y_2' \epsilon Y$ with $y_1' \neq y_2'$, $g(U') \epsilon \sigma_1$ and $g(V') \epsilon \sigma_2$ is obtained such that $y_1' \epsilon g(U'), y_2' \epsilon g(V')$ and $g(U') \cap g(V') = \phi$.(i.e.) (Y, σ_1, σ_2) is a T_2-space.

Thus every homeomorphic image of a T_2-space is also a T_2-space.Hence T_2 is a vague topological property.

Theorem :4.11

Assume (X, τ) to be a T_0-space and (Y, σ) to be any vague topological space then (X, τ_1, τ_2) is a T_0-space.

Proof:

Assume (X, τ) to be a T_0-space then for any $x', y' \epsilon$ X with $x' \neq y'$ then there exist $U' \epsilon\ \tau$ such that
$x' \epsilon\ U'$, $y' \notin U'$ or $x' \notin U'$, $y' \epsilon\ U'$.

Since $U' \epsilon\ \tau \Longrightarrow U' \epsilon\ \tau_1 \cup \tau_2$.

From the above it is clear that (X, τ_1, τ_2) is a T_0-space.

Note:

(i) Assume (X, τ_1, τ_2) to be T_0-space,then (X, τ_1) and (X, τ_2) need not be T_0-space.

(ii) If (X, τ_1) isT_1-space,and (X, τ_2) is any vague topological space then (X, τ_1, τ_2) need not be T_1-space.

V. Pairwise regular and Pairwise normal spaces in vague bitopological space:

Definition:5.1

Assume (X, τ_1, τ_2) to be vague bitopological space.

(i)A set F is called as pairwise open if F are both vague -τ_1-open and vague -τ_2-open in X.

(ii)A set G is called pairwise closed if G are both vague -τ_1-closed and vague -τ_2-closed in X.

(iii)A cover of a vague bitopological space (X, τ_1, τ_2) is known as pairwise open if their elements are members of vague -τ_1 and vague -τ_2 and if it contains atleast one empty member of vague -τ_1 and vague -τ_2.

Definition:5.2

vague -τ_1 is called to be regular with respect to vague -τ_2 in a vague bitopological space,if for every point y ϵX,there will be a vague -τ_1-neighbourhood base of vague -τ_2-closed sets,or,we can say,if for each point y ϵX and each vague -τ_1 -closed set G such that y∉G,there are a vague -τ_1 -open set V' and a vague -τ_2 -open set W'such that y ϵ V', G⊆ $W'$and $V' \cap W' = \phi$.

(X, τ_1, τ_2) is pairwise regular if vague -τ_1 is regular with respect to vague -τ_2 and vice-versa.

Example:5.3

(X, τ_1, τ_2) is a vague bitopological space.Assume X = {y}. τ_1={0,M_1,1}, τ_2={0,M_2,1},where M_1={<[0.1,0.4]>}, M_2={<[0.2,0.5]>}.G ={<[0.6,0.9]>}is a vague-τ_1-closed set.Let V' = {<[0.1,0.8]>}and W' = {<[0.2,0.5]>}.Here vague -τ_1 is regular with respect to vague -τ_2.

Theorem: 5.4

In a vague bitopological space (X, τ_1, τ_2), vague -τ_1 is regular with respect to vague -τ_2 iff for each point y ϵX and vague -τ_1-open set J containing y,there exist a vague -τ_1-open set V such that y ϵ V ⊆ vague -τ_2-cl(V) ⊆ J.

Proof:

Necessity:

If vague -τ_1 is regular with respect to vague -τ_2.Assume y ϵX and J is a vague -τ_1-open set containing y.Then F = J^c is a vague -τ_1-closed set which y∉F.As vague -τ_1 is regular with respect to vague -τ_2,then there exist vague -τ_1-open set V and vague -τ_2-open set W such that y ϵ V,F⊆ W and V∩ W = ϕ.As V ⊆ W^c,then vague -τ_2-cl(V) ⊆ vague -τ_2-cl(W^c) = W^c⊆ F^c = J.Hence, y ϵ V⊆ vague -τ_2-cl(V) ⊆ J.

Sufficiency:

If the condition holds.Assume y ϵX and G is a vague -τ_1-closed set such that y∉G.Then y ϵ G^c, and by hypothesis there will exist a vague -τ_1-open set V such that y ϵ V ⊆ vague -τ_2-cl(V) ⊆ G^c ,which follows that y ϵ V, G ⊆ (vague -τ_2-cl(V)) ^{c} and V∩ (vague -τ_2-cl(V)) ^{c}= ϕ which concludes the proof.

Remark :5.5

Also in other words,by theorem(2.3) stated that vague -τ_1 is regular with respect to vague -τ_2 if, for each point y ϵX,there is a vague -τ_1-neighbourhood base of vague -τ_2-closed sets containing y.This is the equivalent definition of definition (2.2).

Corollary:5.6

In a vague bitopological space (X, τ_1, τ_2), vague -τ_2 is regular with respect to vague -τ_1 iff for each point y ϵX and a vague -τ_2-open set J containing y,there exist a vague -τ_2-open set V such that y ϵ V⊆vague -τ_2-cl(V) ⊆ J.

Suppose Z ⊆ X,then the collection $(\tau_1)_z$ ={ B ∩ Z: B $\epsilon\, \tau_1$ } and

$$(\tau_2)_z = \{ C \cap Z: B\ \epsilon\ \tau_2 \}$$

are the relative topology on Z.

A vague bitopological space (Z, $(\tau_1)_z$, $(\tau_2)_z$) is then known as a subspace of (X, τ_1, τ_2).Moreover,Z is called as vague pairwise closed subspace of X if Z is both $(\tau_1)_z$ closed and $(\tau_2)_z$ closed in X.The vague pairwise open subspace is defined in the same way.

Theorem :5.7

Every subspace of a pairwise regular vague bitopological space (X, τ_1, τ_2) is pairwise regular.

Proof:

Assume (X, τ_1, τ_2) to be pairwise regular and assume (Z, $(\tau_1)_z$, $(\tau_2)_z$) to be a subspace of (X, τ_1, τ_2).Also,assume G to be vague - $(\tau_1)_z$ - closed set in Z,then G = B ∩ Z where B is a vague -τ_1-closed set in X.Now suppose z ϵ Z and z∉G,then z∉ B,so there are vague -τ_1-open set V and vague -τ_2-open set W such that z ϵ V,B⊆ W and V∩W= ϕ.But V∩ Z and W∩ Z are $(\tau_1)_z$ - open set and $(\tau_2)_z$- open set in Z,respectively.Furthermore z ϵ V∩ Z, G⊆ W∩ Z and (V∩ Z) ∩ (W∩ Z) = (V∩ W) ∩ Z = ϕ.

Similarly,Assume F to be a $(\tau_2)_z$ – closed set in Z,then F = C∩ Z where C is a vague -τ_2-closed set in X.Now if z ϵ Z and Z∉F,then z∉C,so there are vague -τ_2-open set V and vague -τ_2-open set W such that z ϵ V,C⊆ W and V∩W= ϕ.

But V∩ Z and W∩ Z are $(\tau_2)_z$-open set and $(\tau_2)_z$-open set in Z,respectively.Furthermore, z ϵ V∩ Z, F⊆ $W \cap Z$ and (V∩ Z) ∩ (W∩ Z) = ϕ which concludes the proof.

Definition :5.8

A vague bitopological space (X, τ_1, τ_2) is known as pairwise normal if given a vague -τ_1-closed set B and a vague -τ_2-closed set C with B∩C= ϕ,there exist a vague -τ_2-open set V and a vague -τ_1-open set W such that z ϵ V,C⊆ W and V∩W= ϕ.

Equivalently, (X, τ_1, τ_2) is pairwise normal,if given a vague -τ_2-closed set D and a vague -τ_1-open set E such that D ⊆ E,there are a vague -τ_1-open set F and a vague -τ_2-closed set G such that D⊆ F⊆ G⊆E.

Example :5.9

(X, τ_1, τ_2) is a vague bitopological space.Assume X = {y}. τ_1={0,M_1,1}, τ_2={0,M_2,1},where M_1={<[0.5,0.7]>}, M_2={<[0.5,0.5]>}.D ={<[0.5,0.5]>}and E ={<[0.5,0.7]>}.Assume F = {<[0.5,0.5]>}and G ={<[0.5,0.5]>}.Here (X, τ_1, τ_2) is vague pairwise normal.

Theorem :5.10

A vague bitopological space (X, τ_1, τ_2) is pairwise normal iff given a vague -τ_2-closed set D and a vague -τ_1-open set E such that D⊆ E, there exist a vague -τ_1-open set F and a a vague -τ_2-closed set G such that D⊆ F⊆ G⊆E.

Proof:

Necessity:

If (X, τ_1, τ_2) is pairwise normal.Assume D to be vague -τ_2-closed set and E to be vague -τ_1-open set such that D⊆ E.Then L = E^c is a vague -τ_1-closed set with L∩D = ϕ. As (X, τ_1, τ_2) is pairwise normal,there is a vague -τ_2-open set V and a vague -τ_1-open set W such that L⊆ V, D⊆ F and V∩F = ϕ.Thus F⊆ V^c ⊆ L^c = E.Hence D⊆ F⊆ V^c⊆E and the result follows by considering V^c = G.

Sufficiency:

If the condition holds.Assume B to be a vague -τ_1-closed set and C to be a vague -τ_2-closed set with B∩C = ϕ.Then E = B^c is a vague -τ_1-open set with C⊆ E.By hypothesis,there is a vague -τ_1-open set F and a vague -τ_2-closed set G such that C⊆ F⊆ G⊆E.It follows that B = E^c ⊆ G^c, C⊆ F and G^c∩F = ϕ,where G^c is vague -τ_2-open set and F is vague -τ_1-open set which completes the proof.

Theorem :5.11

A vague bitopological space (X, τ_1, τ_2) is pairwise normal iff given a vague -τ_1-closed set D and a vague -τ_2-open set E such that D⊆ E, there are a vague -τ_2-open set V and a vague -τ_1-closed set G such that D⊆ V⊆ G⊆E.

Proof:

Necessity:

If (X, τ_1, τ_2) is pairwise normal.Assume D to be vague -τ_1-closed set and E to be vague -τ_2-open set such that D⊆ E.Then L = E^c is a vague -τ_2-closed set with D∩L = ϕ. As (X, τ_1, τ_2) is pairwise normal,there is a vague -τ_2-open set V and a vague -τ_1-open set W such that D⊆ V, L⊆ W and V∩W = ϕ.Thus V⊆ W^c ⊆ L^c = E.Hence D⊆ V⊆ W^c⊆E and the result follows by considering W^c = G.

Sufficiency:

If the condition holds.Assume B to be a vague -τ_1-closed set and C to be a vague -τ_2-closed set with B∩C = ϕ.Then E = C^c is a vague -τ_2-open set with B⊆ E.By hypothesis,there is a vague -τ_2-open set V and a vague -τ_1-closed set G such that B⊆ V⊆ G⊆E.It follows that C = E^c ⊆ G^c, B⊆ V and G^c∩V = ϕ,where G^c is vague -τ_2-open set and V is vague -τ_2-open set which completes the proof.

Definition :5.12

A Vague bitopological space (X, τ_1', τ_2') is known as P_1 normal if given C and D are Vague - closed sets with C'∩ D '= ∅,there exist a Vague - τ_2'-open set U ' and a Vague - τ_1'-open set V ' such that C '⊆ U ', D '⊆ V 'and U'∩ V '= ∅.

Example :5.13

(X, τ_1', τ_2') is a vague bitopological space.Assume X = {a}. τ_1={0,M_1,1}, τ_2={0,M_2,1},where M_1={<[0.7,0.9]>}, M_2={<[0.6,0.8]>}. C' ={<[0,0]>}and D' ={<[0.2,0.4]>}.Assume U' = {<[0,0]>}and V' ={<[0.5,0.6]>}.Here (X, τ_1', τ_2') is P_1 normal.

Theorem :5.14

A Vague bitopological space (X, τ_1', τ_2') is P_1 normal iff given a Vague-closed set C ' and Vague open set D ' such that C '⊆ D ',there exist a Vague - τ_i'-open set G ' and a Vague- τ_j'-closed set F ' such that C '⊆ G '⊆ F '⊆ D '.

Proof:

If (X, τ_1', τ_2') is P_1 normal.Assume C ' to be a Vague closed set and D ' to be a Vague open set such that C '⊆ D '.So then K' = X\ D ' which is a Vague closed set having K'∩ C '= ∅.As (X, τ_1', τ_2') is P_1 normal,there exist a Vague - τ_j'-open set U ' and a Vague - τ_i'-open set G ' such that K' ⊆ U ', C '⊆ G ' and U'∩ G '= ∅.Thus G '⊆ X\ U '⊆ X\ K ' = D.Hence C '⊆ G '⊆ X\U '⊆ D ' and the result follows by assuming X\U ' = F '.

Conversely,If the condition holds.Assume A and B to be a Vague closed sets having A'∩ B '= ∅.So then D' = X\ A ' is a Vague open set with B '⊆ D '.By hypothesis,there are a Vague - τ_i'-open set G ' and a Vague - τ_j' -closed set F ' such that B '⊆ G '⊆ F '⊆ D '.

It follows that A ' = X\ D '⊆X\ F ', B '⊆ G ' and (F $'^c$) ∩ G '= ∅ where F $'^c$ is a Vague - τ_j'-open set and G ' is a Vague - τ_i' -open set which concludes the proof.

It is obvious that every pairwise normal space is P_1 normal.The converse is not true in general.

Theorem :5.15

Every closed subspace of a pairwise normal space is pairwise normal in vague bitopological space.

Proof:

Assume (X, τ_1', τ_2') to a pairwise normal vague bitopological space and assume (Y, σ_1', σ_2') to be a Vague closed subspace of X.If C and D are disjoint subsets of Y such that C is Vague- σ_1'-closed set and D is Vague- σ_2'-closed ,then C is Vague - τ_1'-closed in X and D is Vague - τ_1'-closed in X and since X is is pairwise normal,there are U ' which is Vague - τ_2'-open set and V ' which is Vague - τ_1'-open set such that U'∩ V '= ∅,where C⊂ U' and D⊂ V'.Then U'∩Y and V'∩Y are disjoint Vague- σ_2'-open and Vague- σ_1'-open sets respectively.Also C⊂ U'∩Y and D⊂ V'∩Y.Hence Y is pairwise normal in vague bitopological space.

Theorem: 5.16

Every closed subspace of a P_1 normal space is P_1 normal in vague bitopological space.

Proof:

Proof is obvious from the previous theorem.

VI.Vague Pairwise semi-H_i-Spaces and Vague Pairwise semi-U_i-Spaces:

The bitopological analogues of semi-H_i-spaces (i = 0, 1, 2) and semi-U_i-spaces are introduced here.

Definition:6.1

A Vague bitopological space (X, τ_1, τ_2) is said to be **pairwise semi-H_0** if for every pair of points r and s such that r ∉ vague-τ_i -scl {s} there exists a vague-τ_i semi-open set R and a vague-τ_j semi-open set V such that r ϵ R, s ϵ S, R ∩ S = ∅.; i ≠j, i, j = 1, 2.

Definition :6.2

A vague bitopological space (X, τ_1, τ_2) is said to be **pairwise semi-H_1** if for every pair of points r and s such that vague-τ_i -scl {r} ∩ vague-τ_j -scl {s} = ∅, there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that r ϵ R, s ϵ S, R ∩ S = ∅; i ≠ j, i, j = 1, 2.

Definition :6.3

A vague bitopological space (X, τ_1, τ_2) is said to be **pairwise semi-H_2** if for every vague-τ_i -semi-closed set B and a point r such that vague-τ_i -scl {r} ∩ B = ∅, there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that r ϵ R, B ⊆ S, R ∩ S= ∅; i ≠ j, i, j = 1, 2.

Definition :6.4

A vague bitopological space (X, τ_1, τ_2) is said to be **pairwise semi-U_0** if for every pair of points r and s such that r ∉ vague-τ_i -scl{s}, there exists a vague-τ_i -semi-open set R and a vague-τ_j semi-open set S such that r ϵ R, s ϵ S, vague-τ_j -scl R ∩ vague-τ_i -scl S = ∅; i ≠ j, i, j = 1, 2.

Definition :6.5

A vague bitopological space (X, τ_1, τ_2) is said to be **pairwise semi-U_1** if for every pair of points r and s such that vague-τ_i -scl {r} ∩ vague-τ_j -scl {s} = ∅, there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that r ϵ S, s ϵR, vague-τ_j -scl R ∩ vague-τ_i - scl S = ∅.

Theorem :6.6

Every pairwise irresolutely-normal space is pairwise semi-H_2 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) to be pairwise irresolutely-normal space. Let $r \epsilon X$ and let B be a vague-τ_j -semi-closed set such that vague-τ_i -scl $\{x\} \cap B = \emptyset$. By pairwise irresolutely-normality of (X, τ_1, τ_2),there exists a vague-τ_i -semi-open set R and a vague-τ_j semi-open set S such that vague-τ_i -scl $\{r\} \subseteq S$, $B \subseteq R$, $R \cap S = \emptyset$. Therefore $r \epsilon S$, $B \subseteq R$, $R \cap S = \emptyset$. Hence (X, τ_1, τ_2) is pairwise semi-H_2 in vague bitopological space.

Theorem :6.7

Every pairwise semi-H_2 space is pairwise semi-H_1 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) to be pairwise semi-H_2 space. Let r and s be two distinct points of X such that vague-τ_i -scl $\{r\} \cap$ vague-τ_j -scl $\{s\} = \emptyset$. As X is pairwise semi-H_2, therefore, there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that $r \in S$, vague-τ_j -scl $\{s\} \subseteq R$, $R \cap S = \emptyset$. Thus $r \epsilon S$, $s \epsilon R$, $R \cap S = \emptyset$. Hence, (X, τ_1, τ_2) is pairwise semi- H_1 in vague bitopological space.

Theorem :6.8

Every pairwise semi-H_0 space is pairwise semi-H_1 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) to be pairwise semi-H_0 space. Let r and s be two distinct points of X such that vague-τ_i -scl $\{r\} \cap$ vague-τ_j -scl $\{s\} = \emptyset$.Hence $r \notin$vague-τ_j -scl $\{s\}$. As X is pairwise semi-H_0, therefore there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that $r \epsilon S$, $s \epsilon R$, $R \cap S = \emptyset$.Hence, (X, τ_1, τ_2) is pairwise semi-H_1 in vague bitopological space.

Theorem :6.9

Every pairwise semi-R_1 space is pairwise semi-H_0 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) to be pairwise semi-R_1 space. Let $r \notin$ vague-τ_i -scl $\{s\}$. Then vague-τ_j -scl$\{r\} \neq$ vague-τ_i-scl $\{s\}$. Thus there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that $r \epsilon R$,$s \epsilon S$, $R \cap S = \emptyset$.Hence, (X, τ_1, τ_2) is pairwise semi-H_0 in vague bitopological space.

Theorem :6.10

Every pairwise semi-R_1 space is pairwise semi-H_1 in vague bitopological space.

Proof:

Follows in view of theorem 4.8 and 4.9.

Theorem :6.11

Every pairwise semi-H_0 space is vague pairwise semi-R_0 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) to be pairwise semi-H_0 space. Let $r \epsilon G \epsilon SO(\tau_i)$ and let $s \epsilon G^c$. Then $r \notin$vague-τ_i -scl $\{s\}$. Since X is pairwise semi-H_0, there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that $r \epsilon R$, $s \epsilon S$, $R \cap S = \emptyset$. Thus $\{r\} \cap S = \emptyset$. so that $s \notin$ vague-τ_j -scl $\{r\}$. Hence, $G^c \subseteq X -$ vague-τ_j -scl $\{r\}$ or vague-τ_j -scl $\{r\} \subseteq G$. Thus X is pairwise semi-R_0.

Theorem :6.12

Every pairwise strongly s-regular space is pairwise semi-H_2 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) is pairwise strongly s-regular space. Let r ϵ X and let B be a vague-τ_j -semi-closed subset of X such that vague-τ_i -scl {r} ∩ B = ∅. Then r ∉ B. By pairwise strongly s-regularity of (X, τ_1, τ_2), there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi- open set S such that r ϵ S, B ⊆ R, R ∩ S = ∅.Hence (X, τ_1, τ_2) is pairwise semi- H_2.

Theorem :6.13

A vague bitopological space (X, τ_1, τ_2) is pairwise semi-T_2 if and only if it is pairwise semi-T_0 and pairwise semi-H_0.

Proof:

Assume (X, τ_1, τ_2) is pairwise semi-T_2 space. Clearly X is pairwise semi-T_0. Assume r, s ϵ X such that r ∉ vague-τ_i scl {s}. Then r ≠ s, since X is pairwise semi-T_2, there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S, such that r ϵ R, s ϵ S, R ∩ S = ∅. Hence X is pairwise semi-H_0.

Conversely, assume (X, τ_1, τ_2) to be pairwise semi-T_0 and pairwise semi-H_0. Let r, s be two distinct points of X. Since X is pairwise semi-T_0, there exists a vague-τ_i semi-open set R or a vague-τ_j -semi-open set S such that r ϵ R, s ∉ S or r ∉ R, s ϵ S. Thus r ∉ vague-τ_i - scl {s} or s ∉vague-τ_j -scl {r}. Let r ∉ vague-τ_i - scl {s}. As the space is semi-H_0, there exists a vague-τ_i -semi-open set P and a vague-τ_j -semi-open set Q such that r ϵ P, s ϵ Q, P∩ Q = ∅.Similarly, the result follows in case y ∉ T_j-scl {x}. Hence X is pairwise semi-T_2.

Theorem :6.14

A vague bitopological space (X, τ_1, τ_2) is pairwise semi-T_2 if and only if it is pairwise semi-T_1 and pairwise semi-H_1.

Proof:

Assume (X, τ_1, τ_2) to be pairwise semi-T_2 space. Clearly X is pairwise semi-T_1. Let r, s ϵ X such that vague-τ_i -scl {r} ∩ vague-τ_j -scl {s} = ∅.Then r, s are distinct points of X so that there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S, such that r ϵ S, s ϵ R, R ∩ S = ∅.Hence X is pairwise semi-H_1.

Conversely, assume (X, τ_1, τ_2) to be pairwise semi-T_1 and pairwise semi-H_1. Let r, s be two distinct points of X. As X is pairwise semi-T_1, therefore {r} and {s} are bi- semi-closed sets. Hence vague-τ_j -scl {r} ∩ vague-τ_i -scl{s} = {r} ∩{s} = ∅.As X is pairwise semi-H_1, there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that r ϵ R, s ϵ S, R ∩ S = ∅.Hence X is pairwise semi-T_2.

Theorem :6.15

Every pairwise strongly s-regular space is pairwise semi-U_0 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) to be pairwise strongly s-regular space. Let r, s ϵ X such that r ∉ vague-τ_i -scl{s}. As the space is pairwise strongly s-regular, there exists a vague-τ_i -semi-open set R and a vague-τ_j -semi-open set S such that r ϵ R, vague-τ_i scl{s} ⊆ S, vague-τ_j -scl R ∩ vague-τ_i -scl S = ∅.Hence r ϵ R, s ϵ S, vague-τ_j -scl R ∩ vague-τ_i -scl S = ∅ and hence the space is pairwise semi- U_0.

Theorem :6.16

Every pairwise semi-U_0 space is pairwise semi-H_0 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) to be pairwise semi-U_0 space. Let r, s ϵ X such that r $\notin$ vague-τ_i -scl{s}. As X is pairwise semi-U_0, there exists a vague-τ_i - semi-open set R and a vague-τ_j -semi- open set S such that r ϵ R, s ϵ S, vague-τ_j -scl R $\cap$ vague-τ_i -scl S = $\emptyset$.Thus r ϵ R, s ϵ S, R $\cap$ S= $\emptyset$ and hence X is pairwise semi-H_0.

Theorem :6.17

Every pairwise semi-U_1 space is pairwise semi-H_1 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) to be pairwise semi-U_1 space. Let r, s ϵ X such that vague-τ_i -scl {r}$\cap$ vague-τ_j -scl {s} = $\emptyset$. As X is pairwise semi-U_1, there exists a vague-τ_i semi-open set R and a vague-τ_j -semi-open set S such that r ϵ S, s ϵ R, vague-τ_j -scl R $\cap$ vague-τ_i -scl S = $\emptyset$. Hence r ϵ S,s ϵ R, R $\cap$ S = $\emptyset$. and thus X is pairwise semi-H_1.

Theorem :6.18

Every pairwise irresolutely-normal space is pairwise semi-U_1 in vague bitopological space.

Proof:

Assume (X, τ_1, τ_2) to be pairwise irresolutely-normal space. Let r, s ϵ X such that vague-τ_i -scl {r} $\cap$ vague-τ_j -scl {s} = $\emptyset$.As X is pairwise irresolutely-normal, there exists a vague-τ_i - semi-open set R and a vague-τ_j semi-open set S such that vague-τ_i scl {r} $\subseteq$ S, vague-τ_j -scl {s} $\subseteq$ R, vague-τ_j -scl R $\cap$ vague-τ_i -scl S = $\emptyset$.Thus r ϵ S, s ϵ R, vague-τ_j -scl R $\cap$ vague-τ_i -scl S = $\emptyset$.Hence (X, τ_1, τ_2) is pairwise semi-U_1.

VII.Conclusion:

Hence the various concepts and properties of separation axioms in vague bitopological space based on semi set difference set ,T_0, T_1 and T_2, pairwise regular and pairwise normal space,the pairwise semi-H_i-spaces and pairwise semi-U_i-spaces are studied.

VIII.References:

(1)Arafa Nasef and Roshdey Mareay,"On some bitopological separation axioms",Journal of New theory,No.12,2016,pg:44-50.

(2)Harjot Singh, "Topological Invariance of separation properties in bitopological spaces",IOSR journal of Engineering,vol.8,Issue 10,October 2018,pg.48-50.

(3)Harjot Singh, "Pairwise separated sets in bitopological spaces", IOSR journal of Engineering,vol.8, Issue 8,August 2018,pg.22-24.

(4) Harjot Singh, "Weak separation properties as Hereditary properties",International journal of applied engineering research, vol. 13,No.20,2018,pg.14679-14681.

(5)T.Indira , "Some pairwise maps on bitopological spaces",Ph.D Thesis,Alagappa University,September 2004.

(6)R.N.Majeed, "On pairwise semi-p-separation axioms in bitopological spaces",IBN Al-Haitham Journal for pure and applied sciences,vol.24(3),2011.

(7)Miguel Caldas, "A separation axiom between semi T_0 and T_1^{*}",pro mathematica:vol.XI,1997,Nos:21-22.

(8)T.M.Nour, "A note on five separation axioms in bitopological spaces" , Indian .J.pure appl.math.,26(7):July 1995,pg:669-674.

(9)P.Padma ,S.Udaya kumar and K.Chandrasekhara rao, "Pairwise Q* separation axioms in bitopological spaces",International journal of Mathematical Archieve -3(12),2012,pg:4959-4971.

(10)Rupaya Roshmi and M.S.Hossain, "Properties of separation axioms in bitopological spaces,J.Bangladesh Acad.Sci,Vol.43,No.2,2019,pg.191-195.

(11)Yogesh kumar “Separation and covering axioms in bitopological spaces”, Ph.D Thesis,Chaudhary Charan Singh University,Meerut,2009.

(12)Trishla Garg and Krishna Singhal, “ Study of some mappings in bitopological spaces”,Global Journal of Mathematical Sciences,vol.5,No.1,2013,pg.69-85.