VIBRATION ANALYSIS OF VARIABLE LIQUID FILLED STAINLESS STEEL THIN CYLINDRICAL SHELLS

P. KIRAN KUMAR

Assistant Professor, Department of Mechanical Engineering,
Chaitanya Bharathi Institute of Technology, Hyderabad, Telangana India

J. V. SUBRAHMANYAM

Rtd.Professor, Department of Mechanical Engineering,
Osmania College of Engineering, Hyderabad, India

ABSTRACT: In the present work Vibration behavior of empty and variable fluid levels, contained in stainless steel cylindrical shells of 2mm and 3mm subjected to horizontal accelerations are considered. Mathematical expressions, which shows the motion of cylinder were developed and modified by the method of small amplitude wave approximations, enabling equations for the various modes of vibrations and natural frequencies to be obtained. Also, expression for frequency is formulated by the consideration of fluid in the cylindrical shell. The results were compared by considering modal analysis of stainless steel shells, modelled and analysed using ANSYS. Natural frequencies for different mode shapes are developed for the both 2mm and 3mm stainless steel shells of empty and with variable water column by 20% from the base of the shell. Damping ratios were calculated using half power method. Natural frequencies predicted from analytical method correlated with ANSYS, were found to be in close agreement.

KEYWORDS: Thin Shells, Natural Frequency & Damping Ratio

1. INTRODUCTION

Shell structure with the presence of liquid tends to modify the dynamic properties of that structure, namely: natural frequencies, mode shapes of vibration and damping ratios. For thin-walled cylindrical structures such as liquid storage tanks and certain nuclear and offshore structures these effects are of a more complex nature than for structures with lesser flexibility. The change in the dynamic properties of the structure is generally attributed to an increase in the effective mass of that structure due to the liquid, known as the 'added-mass'. Some of the earlier works are as follows.

Vijayarachavan and Evan-Iwanowski [5] analyzed, both analytically and experimentally, the parametric instabilities of a circular shell under seismic excitation. The cylinder position was vertical and the base was axially excited by using a shaker. In this problem, the in-plane inertia is variable along the shell axis and, when the base is harmonically excited, it gives rise to a parametric excitation. Instability regions were found analytically and compared with experimental results.
Trotsenko and Trotsenko [58], studied vibrations of circular cylindrical shells, with attached rigid bodies, by means of a mixed expansion based on trigonometric functions and Legendre polynomials; they considered only linear vibrations.

Farshidianfar A. and FarshidianfarM.H.[6] were carried out both theoretical and experimental analyses on a long circular cylindrical shell. A total of 18 modes, consisting of all the three main mode groups (axisymmetric, beam-like and asymmetric) were found under a frequency range of 0–1000 Hz, by only applying acoustical excitation. Acoustical excitation results were compared with those obtained from contact excitation. It was discovered that if one uses contact methods, several exciting points are required to obtain all modes; whereas with acoustical excitation only one acoustical source location for the excitation is needed. Furthermore, acoustical excitation produced much better results, compared with contact excitation, in the frequency band 0–3200 Hz.

RaydinSalahifar[7] analysed cylindrical shells based on the variational form of Hamilton’s principle, the field equations and boundary conditions are formulated for circular cylindrical thin shells under general in-phase and out-of-phase harmonic loads. The resulting field equations are solved in a closed form for general loading and boundary conditions. Through Fourier decomposition of the body forces in the longitudinal direction, a technique for developing the particular solution for general harmonic loading was developed. Through four examples, the results based on the current formulation are shown to be in consistent agreement with those based on established shell models in Abaqus.

2. MATHEMATICAL FORMULATION

2.1 Mathematical formulation for determining the natural frequencies of the cylindrical shell.

From the structural standpoint, when writing the shell equations the axial component of motion is discarded and so are the axial variations of displacement field. Therefore radial and tangential Love equations are written as [1]:

$$\frac{E_s \cdot e}{1-\nu^2}\left\{\frac{U}{R^2} + \frac{e^2}{12\cdot R^2}\left(\frac{\partial^4 U}{R^2\partial\theta^4} - \frac{\partial^3 V}{R^2\partial\theta^3}\right) + \frac{\partial V}{R^2\partial\theta}\right\} + \rho_s \cdot e \cdot \frac{d^2 U}{dt^2} = p(R, \theta; t) \quad (1)$$

$$\frac{E_s \cdot e}{1-\nu^2}\left\{\left(1 + \frac{e^2}{12R^2}\right)\left(\frac{\partial^2 V}{R^2\partial\theta^2}\right) + \frac{\partial U}{R^2\partial\theta} - \frac{e^2}{12R^2}\cdot\frac{\partial^3 V}{R^2\partial\theta^3}\right\} - \rho_s \cdot e \cdot \frac{d^2 V}{dt^2} = 0 \quad (2)$$

Assuming hoop strain to be negligible, Love eqn. in radial direction is reduced to:

$$\frac{Es\cdot e^3}{12(1-\nu^2)R^4}\left(\frac{\partial^4 U}{\partial\theta^4} + \frac{\partial^2 U}{\partial\theta^2}\right) + \rho_s\cdot e\cdot\frac{d^2U}{dt^2} = p(R, \theta; t) \quad (3)$$

The mode shapes are of the following admissible type [1]:

$$\left\{u_n(\theta) = \alpha_n \cos(n\cdot\theta) + \beta_n \sin(n\cdot\theta); v_n(\theta) = \alpha_n \cos(n\cdot\theta) + b_n \sin(n\cdot\theta)\right\} \; n = 1, 2, \ldots\ldots \quad (4)$$

This can be conveniently split into two orthogonal families of mode shapes:

$$u_n^{\,1}(\theta) = \cos n\cdot\theta; v_n^{\,1}(\theta) = -\frac{1}{n}\sin n\theta;\;\; u_n^{\,2}(\theta) = \sin n\cdot\theta; v_n^{\,2}(\theta) = -\frac{1}{n}\cos n\theta; \quad (5)$$

The corresponding mass and stiffness coefficients per unit length are given by

$$m_s^{\,1,2}(n,n) = \rho s\cdot e\cdot\int_0^{2\pi}\left(1+\frac{1}{n^2}\right)\begin{Bmatrix}\cos^2 n\theta\\ \sin^2 n\theta\end{Bmatrix} R d\theta = e\cdot\pi\cdot R\left(1+\frac{1}{n^2}\right) \quad (6)$$

$$k_s^{\,1,2}(n,n) = \frac{E_s\cdot e^3\cdot n^2\cdot(n^2-1)}{12(1-\nu^2)R^3}\cdot\int_0^{2\pi}\left(1+\frac{1}{n^2}\right)\begin{Bmatrix}\cos^2 n\theta\\ \sin^2 n\theta\end{Bmatrix} R d\theta = \frac{E_s\cdot e^3\cdot\pi n^2\cdot(n^2-1)}{12(1-\nu^2)R^3} \quad (7)$$

The natural frequencies of the structure are given by[1]:

$$\omega_n = \sqrt{\frac{k_s(n,n)}{m_s(n,n)}} = \frac{n^2}{R}\left(\frac{e}{R}\right)\sqrt{\frac{Es}{12(1-\nu^2)}\frac{(n^2-1)}{(n^2+1)}} \quad (8)$$

Free and rigid in plane translations corresponds to n=1. The shapes constitute a subspace spanned by the two orthonormal vectors:

$$u_1 1 = \cos\theta; v_1 1 = -\sin\theta;\;\; u_2 1 = \sin\theta; v_1^{\,2} = \cos\theta; \quad (9)$$

The mode shapes n>1 corresponds to axial bending

2.2. CONSIDERING THE EFFECT OF FLUID IN THECYLINDER

To calculate the added mass coefficient of thefluid inside the cylinder, a strip model is used.The basic assumption of the strip model is toconsider a narrow strip between z and z+dz,located sufficiently far from the ends z=0 andz= H,We now turn our attention towards motion of thefluid forced by a radial vibration of the shell ofthe type:

$$U(\theta : t) = \sum_{n=1}^{\infty} q^1{}_n cosn\theta + q^2{}_n sinn\theta \quad (10)$$

Where eq1 and eq2 stand for modaldisplacements of the shellPressure is governed by the boundary valueproblem [1]:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p}{\partial\theta^2} = 0 \quad (11)$$

$$\frac{\partial p}{\partial r} = -\rho_f \Sigma\left(\ddot{q}^1{}_n cosn\theta + \ddot{q}^2{}_n sinn\theta\right);\ r = R \quad (12)$$

Splitting the above eqns. into sine and cosinefamilies:

$$\frac{\partial^2 p^1{}_n}{\partial r^2} + \frac{1}{r}\frac{\partial p^1{}_n}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p^1{}_n}{\partial \theta^2} = 0 \quad (13)$$

$$\frac{\partial p^1{}_n}{\partial r} = -\rho_f \ddot{q}_n{}^1 cosn\theta;\ r = R$$

$$\frac{\partial^2 p^2{}_n}{\partial r^2} + \frac{1}{r}\frac{\partial p^2{}_n}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p^2{}_n}{\partial \theta^2} = 0 \quad (14)$$

$$\frac{\partial p^2{}_n}{\partial r} = -\rho_f \ddot{q}_n{}^2 cosn\theta;\ r = R \quad (15)$$

Solving the above equations the pressure field can bedetermined as:

$$p^1{}_n(r, \theta, t) = -\frac{\rho_f H \ddot{q}_n{}^1}{nR^{n-1}} r^n cosn\theta \quad (16)$$

$$p^2{}_n(r, \theta, t) = -\frac{\rho_f H \ddot{q}_n{}^2}{nR^{n-1}} r^n sinn\theta \quad (17)$$

In the present problem, the addedmass matrix, as expressed in the structural modebasis is diagonal and the mode shapes of theshell are the same as in vacuum. The generalizedforce exerted by the fluid on a shell strip of unitlength is

$$Q_n = -\frac{\rho_f \ddot{q}_n{}^1}{n} HR^2 \int_0^{2\Pi} \vec{u}.\vec{u}.\cos^2 n\theta\, d\theta = -\frac{\rho_f . \Pi \cdot H \cdot R^2}{n} \ddot{q}_n{}^1 \quad (18)$$

Where $\vec{u}.$ is the unit normal vector in the radial direction, the added mass coefficients per unit length of the shell is given by:

$$M_a = \frac{\rho_f \pi \cdot HR^2}{n} \quad (19)$$

Where the mode shape is normalized by the condition max $u_n^{(1,2)}(\theta)=1$ and $m_f = \rho\pi R^2 H$, stands for the physical mass of the fluid contained in the shell of unit length.

1. ANSYS FORMULATION:

In modelling section cylinder is modelled as shell element both 2 and 3 mm thickness. Finite element model is shown in below figure for pre processing model. Boundary condition for cylinders for all models fixed at ground position and as well as cap as modelled as shell element as shown in figure. Inside fluid modelled as fluid element and for fluid structure interaction fluid velocity and density given for modal analysis and random analysis. Inside cylinder fluid element node to node connectivity is given to cylinder and fluid element.

.

Stainless steel hollow shell of 3mm thick, 100mm outer diameter, 300mm height is modeled with lid. The material properties of Aluminum with density (ρ) =7850kg/m^{3,} modulus of elasticity E =2.1x10^{11} N/m^2, Poisons ratio (ϑ) =0.29, element type SOLID185 Element is considered as shell element. Fluid with properties of Water density (ρ) =1000kg/m^{3,} sonic velocity=1498m/s^{2,} Viscosity= 0.00089 pa.s and element type for water as Fluid30 has been considered to prepare model using ANSYS17.0

The geometry is discretized into finite elements using solid 185 and fluid30 for structural and fluid respectively. Outer peripheral nodes of fluid domain and inner peripheral nodes of structural container are normally coupled. Boundary conditions are provided at the bottom end of the shell. With the ANSYS solver modal analysis carried out.

The below figures depicts mode shapes of hollow and water level varied 3mm stainless steel shell.

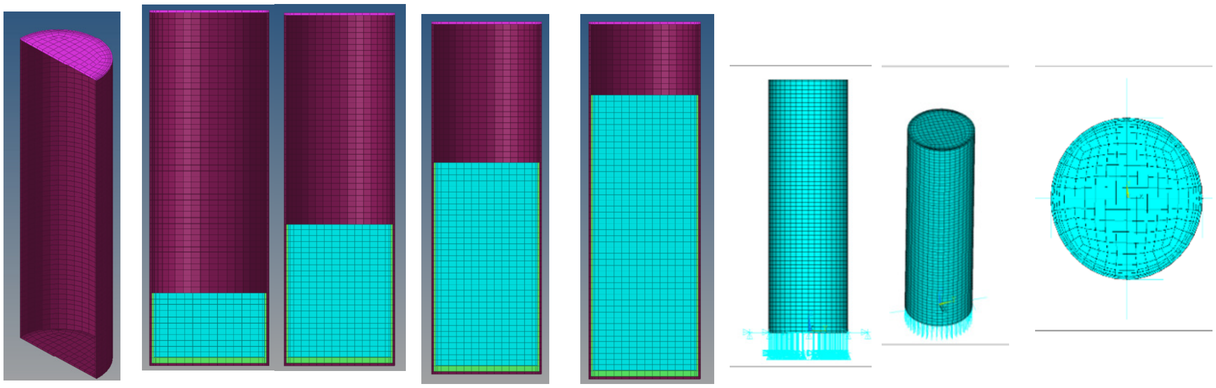

Figure1.: Empty and varying water level of shell model

2. RESULTS AND DISCUSSIONS:

Firstly, did modal analysis for above model and results are shown in below figure. Modal analysis performed for without water inside cylinder and with water 60,120,180,240 mm length from bottom of cylinder. Extracted first five mode shapes and frequencies in lanconze method shown in below figure.

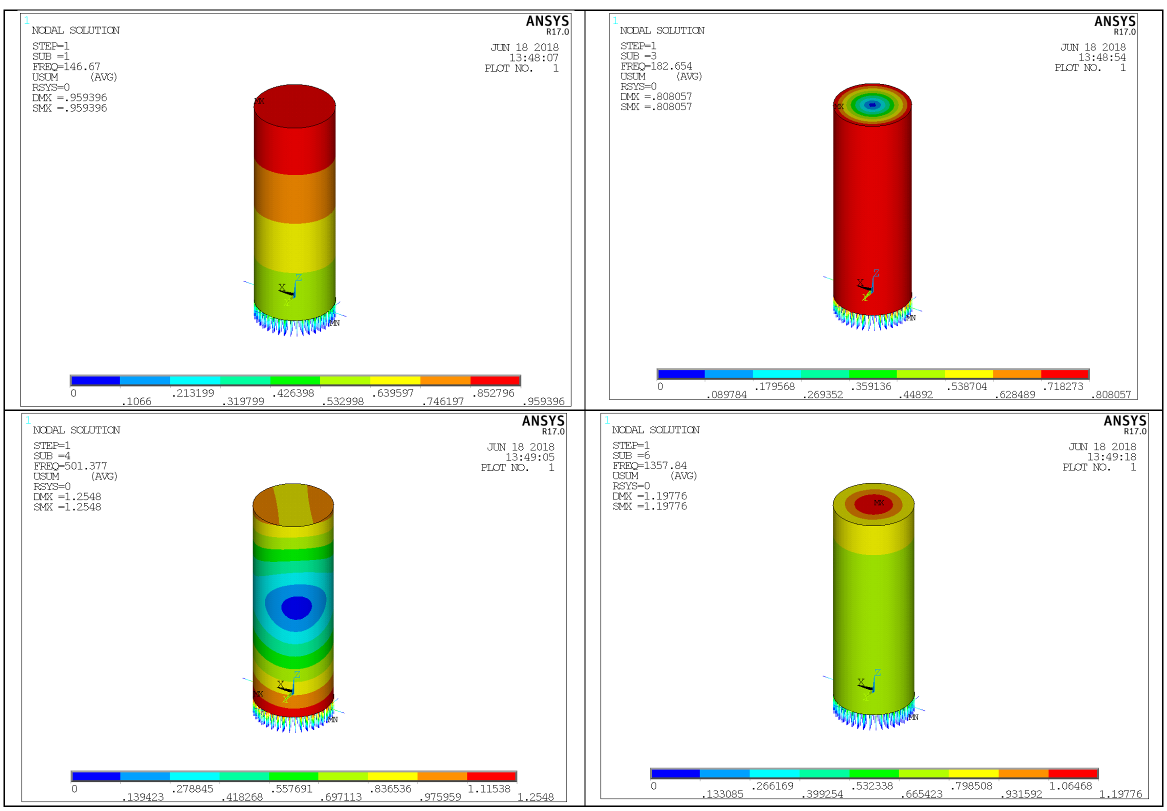

Figure 2: -First four mode shapes of cylinder no fluid inside.

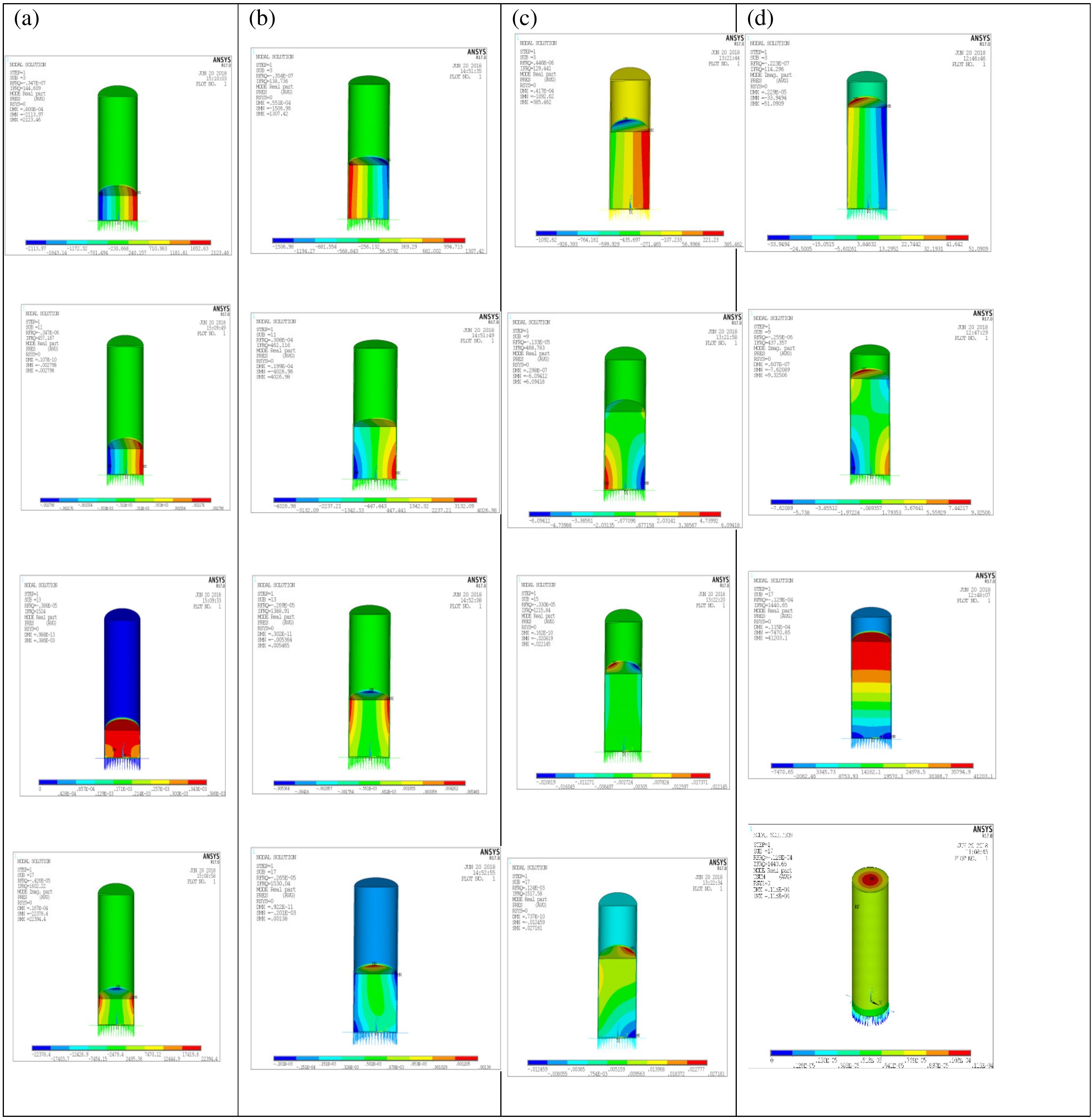

Figure 3:(a) 60 mm water mode shapes of cylinder, (b) 120 mm water mode shapes of cylinder, (c) 180 mm water mode shapes of cylinder. ,(d) 240 mm water mode shapes of cylinder.

Table 1: - Fundamental Frequencies and Damping ratios of Stainless Steel Cylinders of various thicknesses for different water column lengths

Thickness (mm)	Parameter	Water column length				
		0	60	120	180	240
2mm	1stNatural FrequencyHz (Analytical)	139.82	135.40	132.06	128.99	126.13
	1stNatural FrequencyHz(ANSYS)	146.7	144.6	138.7	129.4	114.3
	% of error	4.04	6.43	4.78	0.3	10.85
	1st mode Damping Ratio %	1.8336	3.0591	3.1381	3.1534	3.1571
3mm	1stNatural Frequency Hz(Analytical)	170.86	167.57	164.24	161.49	158.91
	1stNatural FrequencyHz(ANSYS)	175	155	165	160	152
	% of error	3.0	3.45	4.07	1.5	8.63
	1stmode Damping Ratio %	1.9802	2.1416	2.5597	2.5873	2.8523

Table 2: - 2nd natural Frequencies and Damping ratios of Stainless Steel Cylinders of various thicknesses for different water column lengths

Thickness (mm)	Parameter	Water column length				
		0	60	120	180	240
2mm	1stNatural Frequency Hz (Analytical)	540	531	520	489	470
	1stNatural Frequency Hz (ANSYS)	501.0	457.0	462.0	468.7	437.0
	% of error	4.65	3.30	4.41	5.38	7.02
	1st mode Damping Ratio %	1.5	3.15	3.20	3.38	3.45
3mm	1stNatural Frequency Hz (Analytical}	825	807	796	790	785
	1stNatural Frequency Hz (ANSYS)	830	855	800	805	802
	% of error	0.6	5.94	0.5	1.89	2.16
	1stmode Damping Ratio %	1.1	3.5	3.8	3.95	4.15

Table 3: - 3rdnatural Frequencies and Damping ratios of Stainless Steel Cylinders of various thicknesses for different water column lengths

Thickness (mm)	Parameter	Water column length				
		0	60	120	180	240
2mm	1stNatural Frequency Hz (Analytical	1400	1398	1280	1290	1100
	1stNatural Frequency Hz (ANSYS)	1357	1524	1389.0	1215.4	1128.0
	% of error					
	1st mode Damping Ratio %	1.85	3.6	4.8	7.8	9.1
3mm	1stNatural Frequency Hz (Analytical	1260	1360	1284	1300	1100
	1stNatural Frequency Hz (ANSYS)	1259	1359	1347	1339	1275
	% of error					
	1stmode Damping Ratio %	2.4	3.3	5.5	7.4	8.0

Table 4: - 4th natural Frequencies and Damping ratios of Stainless Steel Cylinders of various thicknesses for different water column lengths

Thickness (mm)	Parameter	Water column length				
		0	60	120	180	240
2mm	1stNatural Frequency Hz (Analytical	1600	1642	1630	1610	1605
	1stNatural Frequency Hz (ANSYS)	1647	1602	1620	1587	1560
	% of error	2.93	2.43	0.06	1.42	2.8
	1st mode Damping Ratio %	1.7	2.8	3.3	4.5	5.0
3mm	1stNatural Frequency Hz (Analytical	1722	1680	1500	1301	1320
	1stNatural Frequency Hz (ANSYS)	1733	1697	1540	1397	1310
	% of error	0.63	1.01	2.66	7.37	0.07
	1stmode Damping Ratio %	3.4	4.7	6.8	8.6	7.0

5.CONCLUSION

- The acceleration ratios are reduced for stainless steel thincylinder shell, the natural frequencies of the hollow cylinders increased with increase in thickness.
- The natural frequencies of the hollow cylinder is reduced as the length of the water column increases, due to the added mass effect of the water column and presence of the water column in the hollow cylinder, the damping ratios are also increased.
- The natural frequencies of the stainless steel thin cylinder were determined analytically and validated using numerically.

.

6.REFERENCES

1. Arthur P. Boresi and Richard J. Schmidt, AdvancedMechanics of Materials 6th Edition, Wiley,2002.
2. Francois Axisa and Jose Antunes – Modeling of Mechanical Systems Fluid Structure Interaction,Butterworth and Heinemann, Vol.3,2007.
3. Kubenko, V.D., Koval'chuk, P.S., 2009. Experimental studies of the vibrations and dynamic stability of laminated composite shells. International Applied Mechanics 45 (5), 514–533.
4. Mallon, N.J., 2008. Dynamic Stability if Thin-Walled Structures: A Semi-Analytical and Experimental Approach. PhD Thesis, Eindhoven University of Technology Library, ISBN 978-90-386-1374-1.
5. Vijayarachavan, A., Evan-Iwanowski, R.M., 1967. Parametric instability of circular cylindrical shells. Journal of Applied Mechanics 34, 985–990.
6. Farshidianfar A. and FarshidianfarM.H , 2011, Vibration analysis of long cylindrical shells using acoustical excitation. Journal of Sound and Vibration 330, 3381-3399.
7. .RaydinSalahifar, 2010, Analysis of circular cylindrical shells under harmonic forces. Thin-walled Structures 48,528-539
8. P.kirankumar, j.v.subrahmanyam., 2016, Base excited random vibration analysis of thin shells ,iccms 2016.
9. P.kirankumar, j.v.subrahmanyam Analytical And Numerical Approach To Vibration Analysis Of Liquid Filled AluminumThincylindrical Shells, International Journal of Mechanical and Production Engineering Research and Development ISSN(P): 2249-6890; ISSN(E): 2249-8001 Vol. 9, Issue 4, Aug 2019, 677-684.