

## Impact of Boundary roughness on magnetohydrodynamic Kelvin-Helmholtz instability

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### Abstract

The Kelvin-Helmholtz instability (KHI) occurs at the interface amongst two fluids, which are in relative motion with a common boundary. The growth rate of waves occurs whenever the relative velocity is greater as compared with the critical relative velocity. In the present paper, the influence of boundary roughness on KHI in presence of magnetic field in a couple-stress fluid layer bounded below by a rigid surface and above by a fluid saturated porous layer. Using suitable surface and boundary conditions, we have derived the dispersion relation and results are depicted graphically. As observed in presence of sharp interface, magnetic field exhibits stabilizing effect however, destabilizing effect is shown by the buoyancy force on KHI. Also noted that, the growth rate of interface reduces, as there is an increase in the value of roughness parameter.

**Keywords:** Kelvin-Helmholtz instability, boundary roughness, porous layer, Couple-stress fluids, magnetic field.

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### 1. Introduction

The basic instabilities affecting an interface between two-fluid systems, is KHI. One of the ideal situation is where one layer of heavier fluid underlying another of lighter fluid when both are moving horizontally with different velocities in same direction. Helmholtz [1] and Kelvin [2] developed this theory, which has developed as a standard foundation for fluid mechanics and basic theory of instabilities over the composite fluid layers as established in numerous studies. Lamb [3], Turner [5], Kundu [8] and Scorer [11] are few books to cite. Comparing to any other type of fluid instability, the investigations carried out into this instability is perhaps with far more depth.

A huge literature is generated, due to the significance of KHI in parallel flows occurring in laboratory, astrophysical or geophysical systems that were known many years back. In hydrodynamics, the study of KHI has a vast history especially with analysis of basic linear stability related to magnetohydrodynamic (MHD) KHI was considered from earlier [4]. Presently, there is a developing literature for nonlinear evolution magnetohydrodynamic KHI starting from various possible initial configurations, in two dimensions mostly in the earlier evolution stages. It is seen that strong magnetic fields due to their tension stabilize the KHI. Recently, it is emphasized that the significant potential for considerable weaker fields to transform the nonlinear instability i.e., to alter the consequent flow.

Malik and Singh [7] at the interface of two superposed Ferro fluids, which were moving parallel to the common interface with uniform speeds in presence of a tangential magnetic field, carried out the investigation of nonlinear KH properties of (2+1) dimensional wave propagating packets. Derived nonlinear equation manages the growth of the interface amplitude. KHI is very commonly observed among environmental fluids which are simultaneously subjected to the stabilization because of density stratification and destabilization due to velocity shear [9]. In the existence of a tangential magnetic field on nonlinear KHI, El-Dib [10] has analyzed the consequences of a time-dependent acceleration. By keeping the main objective as; the evolution of KHI can be reduced by using porous layer, a simple theory established on Stokes and lubrication approximations is applied, which follows Babchin et al., [6] and Rudraiah et al.,[12]. Bhatia and Sharma [13] examined the permeability effects due to porous medium and surface tension on KHI on a two-fluid system which are superposed and viscous in presence of uniform vertical magnetic field. The fluid considered in the above studies is Newtonian fluid.

Electrorheological KHI of a fluid sheet by considering the gravitational steadiness of an electrified Maxwellian fluid is deliberated by El-Dib and Matoog [14]. On the interfaces of the fluid sheet, the surface charges were produced by the field. In this study, the weak effects of viscoelastic fluids are considered for mathematical simplification due to the complexity of the problem.

In recent years, significant effort is put to understand the occurrence of couple-stress effects with fluid saturated porous media in non-Newtonian fluid flow. This field is of special interest as there are many applications in various fields. Such as, to improve oil recovery competency from water flooding projects in oil reservoir with displacing non-Newtonian fluids by mobility control. Understanding the effect of couple-stress in case of displaced and displacing non-Newtonian fluids in an oil displacement mechanism has become essential.

The parallel flow through porous media for the fluids with different viscosity, density and elasticity are involved in most of the technological processes. These flows are observed in the chemical industry particularly in packed bed reactors, petroleum engineering, boiling in porous media etc. The interface may become unstable due to a significant rise in the resistance on the flow, resulting into dry out in boiling porous media due to flooding in counter current packed chemical reactors. Similar to this, in petroleum production these discontinuities yield to emulsion formation. Therefore, in above processes to analyze the limiting operation situations we should have the knowledge of onset of instability conditions.

In the two-dimensional system immersed in presence of uniform horizontal magnetic field the KHI in two superposed highly viscous conducting fluids of uniform densities is discussed by Khan et al [15] by taking into deliberation of surface tension effects. The porosity, viscosity and surface tension effects are found to have stabilizing impact on the evolution of the unstable mode, whereas a destabilization on the system by streaming velocity is exhibited.

With the help of two simple models related to energy exchange occurring amongst the superposed fluids, Joshi et al.,[16] investigated the root cause of KHI. In determination of minimal relative speed causing the instability is due to surface tension and density of the fluids. In case of dielectric and ferrofluids the volume force employed by magnetic and electric field gradients is also discussed. Chavaraddi et al.,[17] studied the impact of boundary roughness on KHI in couple-stress fluid layer. The surface roughness effects in the existence of magnetic field on KHI are investigated [18].

Discontinuity of a two-fluid system which are immiscible, non-viscous, superposed, counter streaming, electrically conducting with lower fluid layer heavier than the upper fluid layer in presence of magnetic field is numerically investigated by Veena et al.,[19]. Using linear theory by normal mode analysis, the exact solutions for stress free bounding surfaces related to eigen-value problem were obtained. On the evolution of the unsteady perturbation of the physical system, the significance of velocity and magnetic field of streaming fluids is examined. Magnetic field has found to have insignificant stabilizing effect whereas, a great extent of stabilization is due to velocity of the counter-streaming fluids. The magnetic field effects on KHI in a couple-stress fluid layer bounded below by a rigid surface and above by a porous layer is analyzed by Chavaraddi et al.,[20].

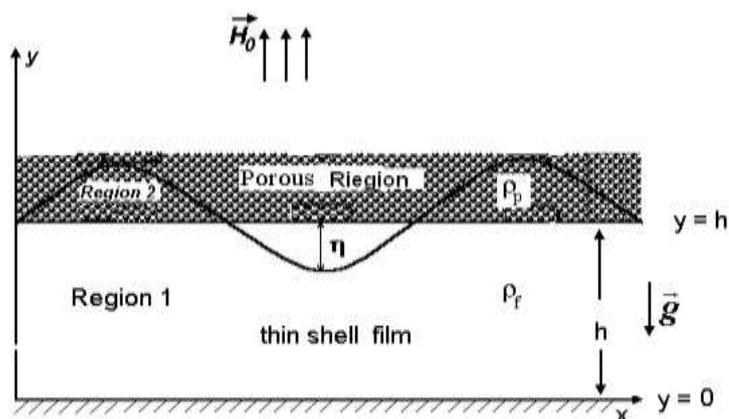
KHI of a parallel shear flow system consisting of hyperbolic tangent velocity profile at high Reynolds number is simulated Ziu et al [21]. In the study, fluid with low viscosity,

which is perfectly conducting is considered. There is a variation in applied magnetic field strength from weak to strong. On the shear flow with short wave perturbations, it is observed that magnetic field has a stabilizing effect, due to two parts: the splitting effect arising from transverse magnetic pressure and the anti-bonding magnetic tension effect.

In the present paper, the influence of boundary roughness on KHI in existence of magnetic field in couple-stress is studied. The paper is organized as: In Section 2, the basic equations along with Maxwell's equations are established. In Section 3, equations are non-dimensionalized by following Stokes and lubrication approximations. By applying suitable surface and boundary conditions the dispersion relation is derived in Section 4. In addition, the cutoff wave number, maximum wave number as well as corresponding maximum growth rate are found. As observed in the graphical representations, some important conclusions are discussed in last Section.

## 2. Formulation of the Problem

The physical configuration of the problem is represented in Figure-1. A thin target shell is considered in the form of a thin film with  $h$  as unperturbed thickness (Region 1) occupied with an incompressible, poorly



**Figure 1: Physical Configuration**

electrically conducting, viscous, light fluid of density  $\rho_f$  bounded below by a rigid surface at  $y=0$ . An incompressible poorly conducting viscous heavy fluid bounded above the light fluid with density  $\rho_p$  saturating a dense porous layer of large extent in comparison with  $h$  the shell thickness. The  $x$ - co-ordinate and  $y$  - co-ordinate spans respectively the horizontal and vertical directions. The interface existing at  $y=h$  is symbolized by  $\eta(x, t)$ . Obviously, when the interface is flat then  $\eta = 0$  at  $y=h$ . The fluid is assumed to be non-Newtonian, electrically

conducting, viscous and incompressible with the velocity vector  $\vec{q}_1 = (u, v)$ . The slip parameter at the interface is  $\alpha_p$  and  $\mu_f$  ( $\mu_p$ ) represents viscosity of fluid (porous medium). The stress gradient is  $\delta = g(\rho_p - \rho_f)$  which is related to the gravitational acceleration. The perturbed interface  $\eta(x, t)$  is along the y direction.

Basic equations used for clear fluid layer (Region 1) and those related to porous layer (Region 2) are:

**Region-1:**

$$\nabla \cdot \vec{q}_1 = 0 \quad (2.1)$$

$$\rho \left( \frac{\partial \vec{q}_1}{\partial t} + (\vec{q}_1 \cdot \nabla) \vec{q}_1 \right) = -\nabla p_1 + \mu_f \nabla^2 \vec{q}_1 - \lambda_f \nabla^4 \vec{q}_1 + \mu_0 (\vec{J}_1 \times \vec{B}_1) \quad (2.2)$$

**Maxwell's Equations:**

$$\nabla \cdot \vec{E}_1 = 0, \quad \nabla \cdot \vec{H}_1 = 0, \quad \nabla \times \vec{E}_1 = -\frac{\partial \vec{B}_1}{\partial t}, \quad \nabla \times \vec{H}_1 = \vec{J}_1 + \frac{\partial \vec{D}_1}{\partial t} \quad (2.3)$$

Auxiliary equations

$$\vec{D}_1 = \epsilon_0 \vec{E}_1, \quad \vec{B}_1 = \mu_0 \vec{H}_1, \quad \vec{J}_1 \times \vec{B}_1 = \sigma [\vec{E}_1 + \vec{q}_1 \times \vec{B}_1] \times \vec{B}_1 \quad (2.4)$$

**Region-2:**  $\vec{Q}_1 = -\frac{k}{\mu_f} \frac{\partial p_1}{\partial x} \quad (2.5)$

where  $\vec{E}_1$  is electric field,  $\vec{J}_1$  is current density,  $\vec{H}_1$  is magnetic field,  $\vec{B}_1$  is magnetic induction,  $\vec{D}_1$  is dielectric field,  $\sigma$  is electrical conductivity,  $p_1$  is pressure,  $\mu_0$  is magnetic permeability,  $\lambda$  is couple-stress parameter,  $k$  is permeability of the porous medium,  $\vec{Q}_1 = (Q_1, 0, 0)$  is uniform Darcy velocity.

To simplify basic equations Stokes and lubrication assumptions and electro hydrodynamic approximations (Rudraiah et al [13]) are considered as follows:

- i) For the liquid considered, the electrical conductivity  $\sigma$  is insignificantly small, i.e.,  $\sigma \ll 1$ .
- ii) The film thickness  $h$  is much smaller in comparison with the thickness  $H$  of the dense fluid above the film.  
i.e.,  $h \ll H$ .
- iii) The surface elevation  $\eta$  is assumed to be small as compared with the film thickness  $h$ .  
i.e.,  $\eta \ll h$ .

iv) The Strauhal number  $S$  in Eq. (2.2), is negligibly small.

$$\text{i.e., } S = \frac{L}{TU} \ll 1$$

where,  $L = \sqrt{\frac{\gamma}{\delta}}$  is characteristic length,  $U = \frac{v}{L}$  is characteristic velocity,

$$T = \frac{\mu\gamma}{h^3\delta^2} \text{ is characteristic time.}$$

The above equations are non-dimensionalized using the following,

$$u^* = \frac{u}{\delta h^2 / \mu_f}, v^* = \frac{v}{\delta h^2 / \mu_f}, p_1^* = \frac{p_1}{\delta h}, Q_1^* = \frac{Q_1}{\delta h^2 / \mu_f}, t^* = \frac{t}{\delta h / \mu_f}, x^* = \frac{x}{h}, y^* = \frac{y}{h} \quad (2.6)$$

By using the assumptions and approximations as stated above, also by assuming that the heavy fluid in the porous layer is practically stationary caused by creeping flow approximation and using Eq. (2.6) in the Eqs (2.1) and (2.2), we get,

**Region 1:**

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad (2.7)$$

$$0 = -\frac{\partial p_1}{\partial x} + \frac{\partial^2 u}{\partial y^2} - M_0^2 \frac{\partial^4 u}{\partial y^4} - M^2 u \quad (2.8)$$

$$0 = -\frac{\partial p_1}{\partial y} \quad (2.9)$$

where

$$M_0^2 = \frac{\lambda_f}{\mu_f h^2} \text{ is couple-stress parameter, } M^2 = \frac{\mu_h^2 H_0^2 \sigma_f h^2}{\mu_f} \text{ is Hartmann number.}$$

**Region 2 :**

$$Q_1 = -\frac{1}{\sigma_p^2} \frac{\partial p_1}{\partial x} \quad (2.10)$$

where  $\sigma_p = \frac{h}{\sqrt{k}}$  is porous parameter.

### 3. Dispersion Relation

Firstly, the velocity distribution is obtained from Eq. (2.8) to find the dispersion relation, by using the boundary and surface conditions given as follows:

(i) Roughness condition:

$$-\beta \frac{\partial u}{\partial y} = u \text{ at } y = 0 \quad (3.1)$$

(ii) Beaver-Joseph slip condition:

$$\frac{\partial u}{\partial y} = -\sigma_p \alpha_p (u_B - Q_1) \text{ at } y = 1 \quad (\text{also } y=1, u = u_B) \quad (3.2)$$

(iii) Kinematic condition:

$$v = \frac{\partial \eta}{\partial t} \text{ at } y = 1 \quad (3.3)$$

(iv) Dynamic condition :

$$p_1 = -\eta - \frac{1}{B_1} \frac{\partial^2 \eta}{\partial x^2} \text{ at } y = 1. \quad (3.4)$$

Here  $\eta = \eta(x, y, t)$  is interface elevation,  $B_1 = \frac{\delta h^2}{\gamma}$  is Bond number.

The solution of equation(2.8) using the above surface and boundary conditions is

$$u = \left[ C_1 \text{Cosh}(\alpha_1 y) + C_2 \text{Sinh}(\alpha_1 y) + C_3 \text{Cosh}(\alpha_2 y) + C_4 \text{Sinh}(\alpha_2 y) - \frac{P}{M^2} \right] \quad (3.5)$$

where  $P = \frac{\partial p_1}{\partial x}$ ,

$$\alpha_1^2 = \frac{1 + \sqrt{1 - 4M^2 M_0^2}}{2M_0^2}, \quad \alpha_2^2 = \frac{1 - \sqrt{1 - 4M^2 M_0^2}}{2M_0^2},$$

$$a_1 = \alpha_1 \text{Sinh}(\alpha_1) + \alpha_p \sigma_p \text{Cosh}(\alpha_1), \quad a_2 = \alpha_1 \text{Cosh}(\alpha_1) + \alpha_p \sigma_p \text{Sinh}(\alpha_1),$$

$$a_3 = \alpha_2 \text{Sinh}(\alpha_2) + \alpha_p \sigma_p \text{Cosh}(\alpha_2), \quad a_4 = \alpha_2 \text{Cosh}(\alpha_2) + \alpha_p \sigma_p \text{Sinh}(\alpha_2),$$

$$a_5 = \alpha_1^2 \text{Cosh}(\alpha_1), \quad a_6 = \alpha_1^2 \text{Sinh}(\alpha_1), \quad a_7 = \alpha_2^2 \text{Cosh}(\alpha_2), \quad a_8 = \alpha_2^2 \text{Sinh}(\alpha_2),$$

$$b_1 = \frac{1}{M^2}, \quad b_2 = \frac{1}{M^2} - \frac{\alpha_p}{\sigma_p},$$

$$C_1 = \frac{P \alpha_2^2 [b_1 (a_2 a_8 - a_4 a_6) - b_2 \beta (\alpha_1 a_8 - \alpha_2 a_6)]}{\beta [\alpha_1^3 (a_3 a_8 - a_4 a_7) + \alpha_1^2 \alpha_2 (a_2 a_7 - a_3 a_6) + \alpha_1 \alpha_2^2 (a_4 a_5 - a_1 a_8) + \alpha_2^3 (a_1 a_6 - a_2 a_5)] + (\alpha_1^2 - \alpha_2^2) (a_4 a_6 - a_2 a_8)},$$

$$C_2 = \frac{P\{b_1[\alpha_1^2(a_3a_8 - a_4a_7) - \alpha_2^2(a_1a_8 - a_4a_5)] + b_2[\beta(\alpha_1^2\alpha_2a_5 - \alpha_2^3a_5) - (\alpha_1^2 - \alpha_1^2)a_8]\}}{\beta[\alpha_1^3(a_3a_8 - a_4a_7) + \alpha_1^2\alpha_2(a_2a_7 - a_3a_6) + \alpha_1\alpha_2^2(a_4a_5 - a_1a_8) + \alpha_2^3(a_1a_6 - a_2a_5)] + (\alpha_1^2 - \alpha_2^2)(a_4a_6 - a_2a_8)},$$

$$C_3 = \frac{-P\alpha_1^2[b_1(a_2a_8 - a_4a_6) - b_2\beta(\alpha_1a_8 - \alpha_2a_6)]}{\beta[\alpha_1^3(a_3a_8 - a_4a_7) + \alpha_1^2\alpha_2(a_2a_7 - a_3a_6) + \alpha_1\alpha_2^2(a_4a_5 - a_1a_8) + \alpha_2^3(a_1a_6 - a_2a_5)] + (\alpha_1^2 - \alpha_2^2)(a_4a_6 - a_2a_8)},$$

$$C_4 = \frac{P\{b_1[\alpha_1^2(a_2a_7 - a_4a_6) + \alpha_2^2(a_1a_6 - a_2a_5)] - b_2[\beta(\alpha_1^3a_7 - \alpha_1\alpha_2^2a_5) - (\alpha_1^2 - \alpha_1^2)a_6]\}}{\beta[\alpha_1^3(a_3a_8 - a_4a_7) + \alpha_1^2\alpha_2(a_2a_7 - a_3a_6) + \alpha_1\alpha_2^2(a_4a_5 - a_1a_8) + \alpha_2^3(a_1a_6 - a_2a_5)] + (\alpha_1^2 - \alpha_2^2)(a_4a_6 - a_2a_8)}.$$

By integrating Eq. (2.7) from  $y = 0$  to  $y = 1$  and using Eq. (3.5), we obtain

$$v(1) = \left[ \frac{\partial^2 \eta}{\partial x^2} + \frac{1}{B_1} \frac{\partial^4 \eta}{\partial x^4} \right] \Delta_1 \quad (3.6)$$

$$\text{where } \Delta_1 = \frac{C_1}{\alpha_1} \text{Sinh}(\alpha_1) + \frac{C_2}{\alpha_1} [\text{Cosh}(\alpha_1) - 1] + \frac{C_3}{\alpha_2} \text{Sinh}(\alpha_2) + \frac{C_4}{\alpha_2} [\text{Cosh}(\alpha_2) - 1] - \frac{1}{M^2}.$$

Using Eqs. (3.6) and (3.4), then Eq. (3.3), reduces to

$$\frac{\partial \eta}{\partial t} = \left[ \frac{\partial^2 \eta}{\partial x^2} + \frac{1}{B_1} \frac{\partial^4 \eta}{\partial x^4} \right] \Delta_1. \quad (3.7)$$

Let us consider the solution of Eq. (3.7) in the following form given by (3.8) to examine  $n$  i.e. growth rate of the interface with periodic perturbation.

$$\eta = \eta(y)e^{i\ell x + nt} \quad (3.8)$$

where  $\eta(y)$  is perturbation amplitude of the interface and  $\ell$  is the wave number.

Let us substitute Eq. (3.8) in Eq. (3.7), the dispersion relation is obtained in the form

$$n = \ell^2 \left( 1 - \frac{\ell^2}{B_1} \right) \Delta. \quad (3.9)$$

where  $\Delta = -\Delta_1$

Also, Eq. (3.9) is expressed as

$$n = n_b - \ell \beta v_a \quad (3.10)$$

$$\text{where } n_b = \frac{\ell^2}{3} \left[ 1 - \frac{\ell^2}{B_1} \right], \quad \beta = \Delta \ell \left[ 1 - \frac{\ell^2}{B_1} \right], \quad v_a = \left( \frac{1-3\Delta}{3\Delta} \right) \left( 1 - \frac{\ell^2}{B_1} \right).$$

By setting  $n = 0$  in Eq. (3.9), the cut-off wavenumber  $\ell_{ct}$  is obtained as

$$\ell_{ct} = \sqrt{B_1} \quad (3.11)$$

since  $\ell$  and  $\Delta$  are different from zero.

From Eq. (3.9), by setting  $\frac{\partial n}{\partial \ell} = 0$  the value of  $\ell_m$  i.e., maximum wavenumber is established as

$$\ell_m = \sqrt{\frac{B_1}{2}} = \frac{\ell_{ct}}{\sqrt{2}} \quad (3.12)$$

The value of  $n_m$  i.e. corresponding maximum growth rate is expressed as

$$n_m = \frac{B_1}{4} \Delta \quad (3.13)$$

In similar way by using  $\ell_m = \sqrt{\frac{B_1}{2}}$ , we get

$$n_{bm} = \frac{B_1}{12} \quad (3.14)$$

Therefore maximum growth rate is given by

$$G_m = \frac{n_m}{n_{bm}} = 3\Delta. \quad (3.15)$$

The growth rate represented by Eq. (3.9) is calculated for various parameter values. The results are depicted through graphs in **Figures 2-6**.

#### 4. Results and Discussion

In this present work, we have analyzed the influence of boundary roughness on KHI in presence of magnetic field which is occurring in a couple-stress fluid layers bounded below by a rigid boundary and above by a porous layer. Computations to investigate the growth rate at the interface were executed for various fluid properties corresponding to Hartmann number  $M$ , roughness parameter  $\beta$ , couple-stress parameter  $M_0$ , porous parameter  $\sigma_p$  and Bond number  $B_1$  at different wavenumbers. Graphs were plotted for non-dimensionlized growth rate  $n$  of the perturbation against  $\ell$  the non-dimensionlized wavenumber, only for certain cases. In agreement with the dispersion relation, all perturbed values grow exponentially in case of linear stage given by Eq. (3.9). The boundary between the layers at this stage attains a sinusoidal shape with small amplitude.

On the two-layer channel flow problem, the role of magnetic field is studied and it is demonstrated that either stabilization or destabilization can be obtained. For an increase in the values of Hartmann number  $M$  it represents growth rates in conditions for which magnetic field is stabilizing over a broad range of wavenumbers as depicted in Figure 2, where  $\alpha_p = 0.1$ ,  $\sigma_p = 4$ ,  $B_1 = 0.02$ ,  $\beta = 3.3 \times 10^{-3}$ ,  $M_0 = 0.3$ . Noted that, as the values of  $M$  are increased from 5 to 50 there is a decrease in maximum growth rate.

Growth rate versus the wavenumber is depicted in Figure 3 with fixed the values  $\alpha_p = 0.1$ ,  $\sigma_p = 4$ ,  $B_1 = 0.02$ ,  $M = 5$ ,  $\beta = 3.3 \times 10^{-3}$  for different values of  $M_0$ . An increase in the value of couple-stress parameter results into decrease in maximum growth rate this is due to the action of the body couples on the system.

We have considered  $\alpha_p = 0.1$ ,  $M_0 = 0.3$ ,  $\sigma_p = 4$ ,  $\beta = 3.3 \times 10^{-3}$ ,  $M = 5$  in our sample calculations, and variations in Bond number  $B_1$ . It is observed from Figure 4 that as Bond number  $B_1$  is decreased from 0.04 to 0.01, there is a decrease in critical wavenumber and maximum growth rate. As Bond number and surface tension are inversely related so, an increase in surface tension results into decrease in the growth rate and therefore interface tends to attain stability.

The effects of porous properties on the instability are investigated by using following input values. The values of the parameters  $M_0 = 0.3$ ,  $\alpha_p = 0.1$ ,  $B_1 = 0.02$ ,  $\beta = 3.3 \times 10^{-3}$ ,  $M = 5$  are fixed and vary the value of  $\sigma_p$ . Figure 5 depicts the consequences of our calculations,

shows that as value of porous parameter  $\sigma_p$  is increased from 20 to 100, corresponds to an increase in the critical wavelength and decrease in the maximum growth rate, hence exhibits stabilization. This occurs due the resistance offered to the fluid by the solid particles of the porous layer.

In order to analyze the boundary roughness effects, the other parameter values are fixed as  $\alpha_p = 0.1$ ,  $\sigma_p = 4$ ,  $B_1 = 0.02$ ,  $M_0 = 0.3$ ,  $M = 5$  and ratios of roughness parameter  $\beta$  is varied. We note that an increase in the value of  $\beta$  corresponds to decrease in the evolution of the interface. This occurs as to overcome the resistance offered by the boundary roughness a part of kinetic energy is transformed into potential energy.

## 5. Conclusions

Linear stability analysis of a two-fluid flow system in a channel is carried out for the fluids which are non-Newtonian possessing various fluid properties. The system under consideration is subjected to magnetic field normal to their interface. The equations of motion were derived and then linearized as there occurs an interaction between the couple-stress and hydrodynamic problems at the fluid interface through the stress balance. By the normal mode technique, we calculated the growth rate of the perturbation and analyzed the changes in growth rate as a function of dimensionless parameters  $M, M_0, \sigma_p, B_1, \beta$ . From our study we conclude that an increase in the values of  $M, M_0, \sigma_p, \beta$  the evolution of the interface reduces and the perturbed system tends to attain stability. However, by decreasing the value of the Bond number  $B_1$  the growth rate decreases and system becomes more stable.

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Figure 2: Variations of Hartmann Number

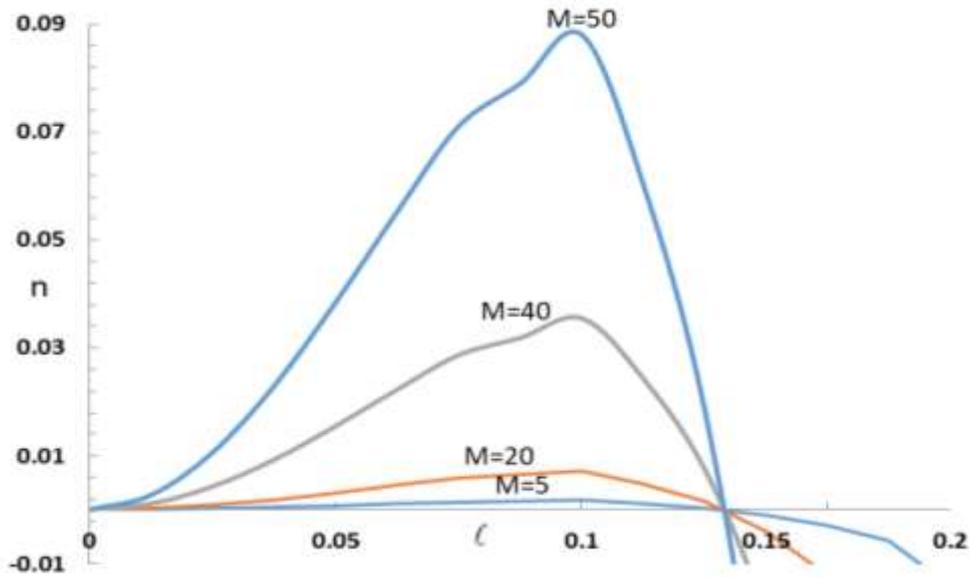


Figure 3: Variations of Couple-Stress parameter

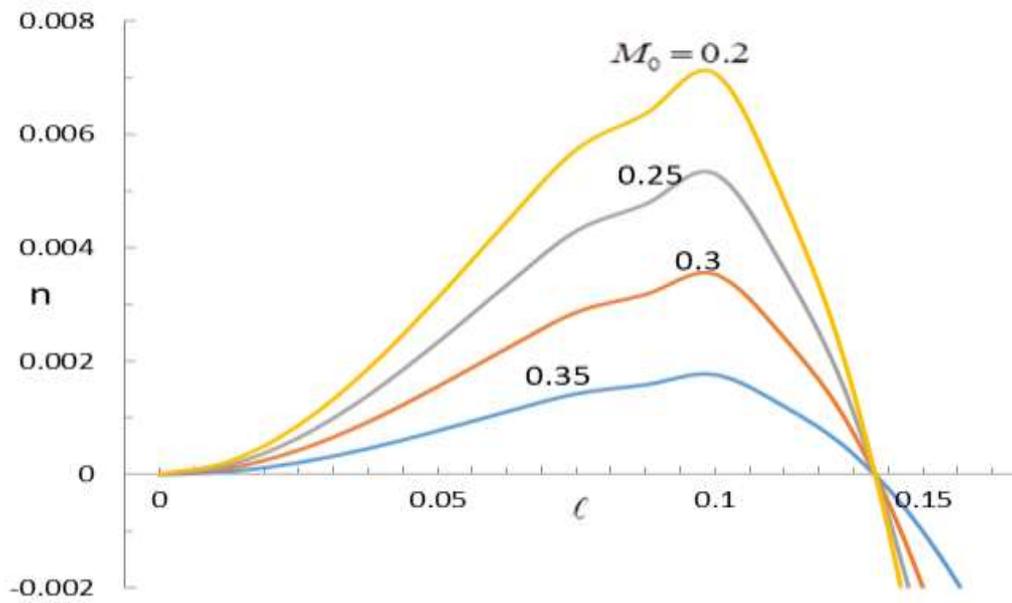


Figure 4: Variations of Bond number

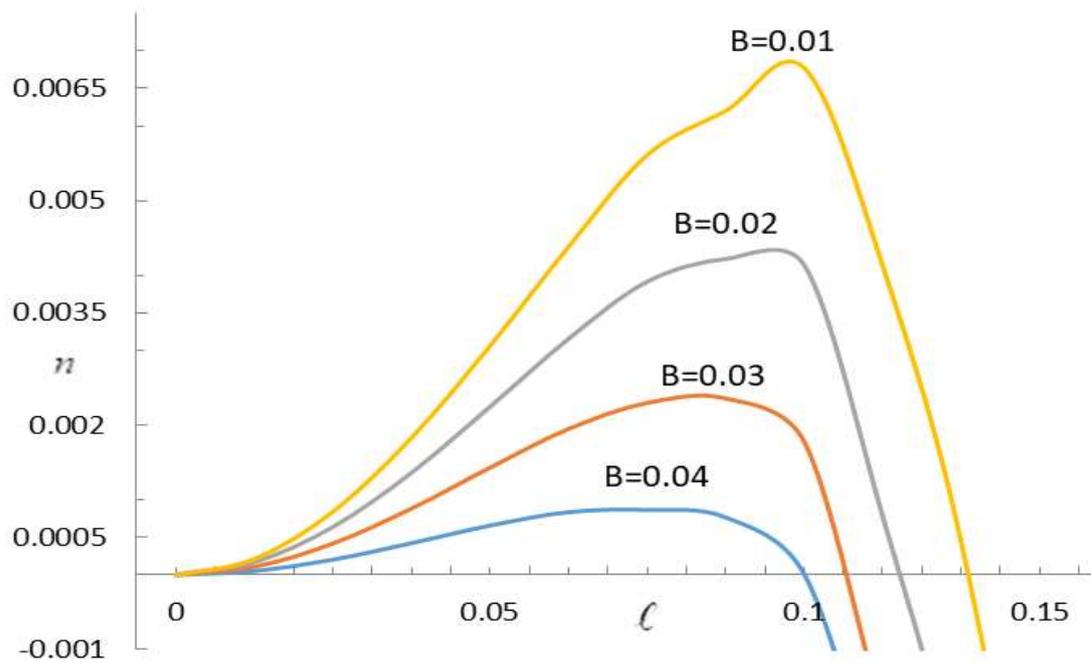
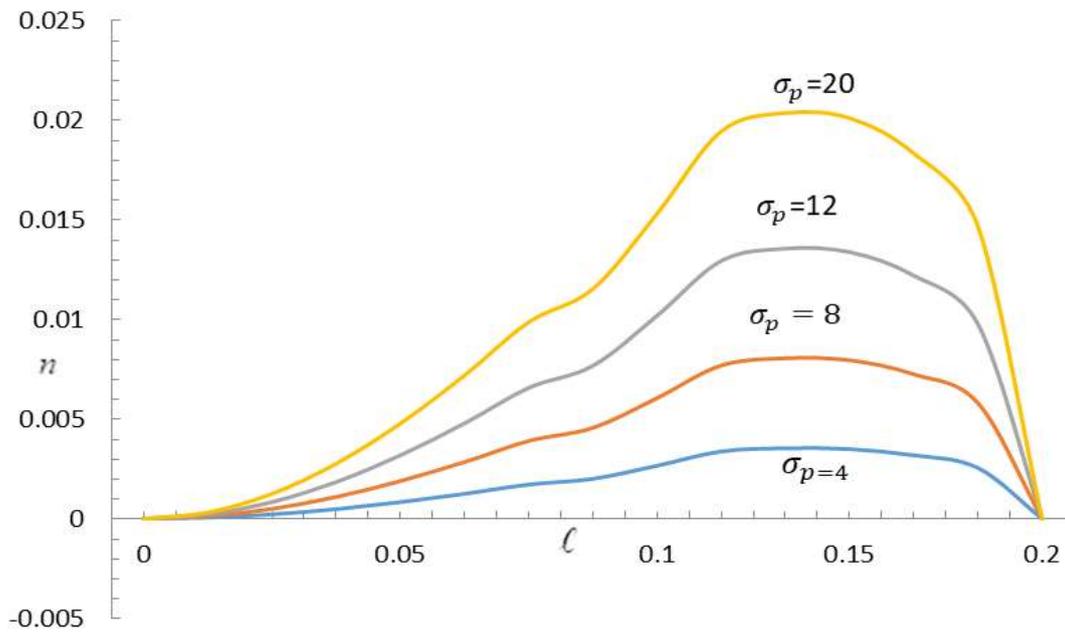
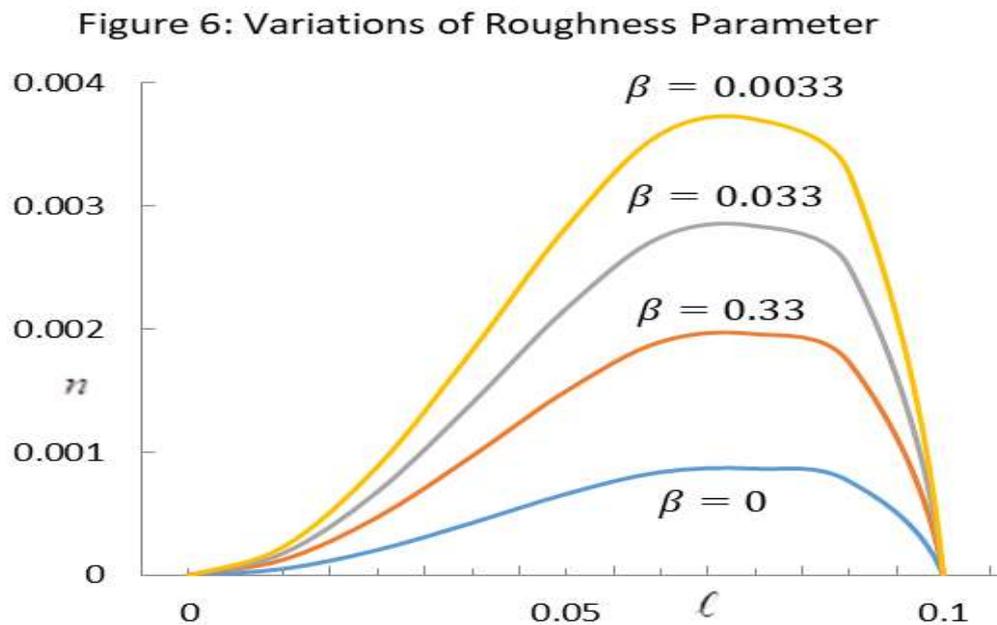


Figure-5: Variations of Porous Parameter





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