Chaos in Fractional Order Hyperchaotic Voltas System

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Abstract- The paper talks about the chaotic behaviour of fractional order hyperchaotic Voltas's system. The chaotic behaviour has been ascertained with the help of phase portraits and state evolution. The chaotic patterns obtained for different values of fractional order have been obtained and ensure the chaotic nature of the system. The results for various values of system parameters have also been presented.

Keywords – Fractional order, chaotic systems, Volta's system, hyperchaotic systems

I. INTRODUCTION

Fractional derivative and integration have found wide applications in the past two decades. Control system has become one of the areas of application of fractional calculus. Nonlinear systems especially chaotic systems find applicability in almost every area of engineering and science. The fractional order version of the chaotic systems also called as fractional order chaotic systems (FOCS) have become the point of discussion in the past 15 years and various researchers have put forward the analysis and control of different FOCS [1], [2]. A number of techniques which have been employed for control of integer order chaotic systems (IOCS) [3], [4], have been extended for FOCS also [5], [6].

One of the main advantages of going for fractional order chaotic systems is that such systems display chaos for a range of values of fractional order and hence can be used for different applications. Fractional order version of various IOCS has been studied in literature. Some prominent contributions can be found in [7], [8] etc.

Usage of fractional calculus for nonlinear dynamical systems started with the fractional order version of Chua's circuit (Hartley et al., 1995) which was proposed by Hartely in 1995. This was the first paper to introduce the idea of fractional derivative to dynamical chaotic systems. The authors suggested that the traditional idea of order should be modified in case of fractional order systems. It was also validated that chaos can occur in the continuous-time systems with mathematical order less than three. This paper started a new area of fractional order chaotic systems and motivated various researchers to come up with new control and synchronization schemes for various FOCS. Investigation of bifurcation and chaos in fractional order CNN (Cellular Neural Network) attractor has been carried out in (Arena et al., 1998). In this work, chaotic patterns were investigated for different values of fractional order and Lyapunov exponents were calculated to confirm the chaotic behavior. In the first decade of 21st century, fractional order models of various integer order chaotic system have been proposed, such as, Chen system (Peng, 2004; Wang et al., 2006), Lorenz system (Grigorenko and Grigorenko, 2003), Rössler's system (Li and Chen, 2004), Lu system (Lu, 2006), etc.

The main contribution of the paper is to propose the hyperchaotic version of existing fractional order Volta's system as proposed in [1]. The hyperchaotic nature has been verified with the help of phase portraits and state evolution. The proposed system can be further controlled via several techniques proposed in literature and then can be utilized for different real time applications.

The rest of the paper is organized as follows. Proposed system and its dynamics are explained in section II. Analysis of chaotic behaviour is presented in section III. Concluding remarks are given in section IV.

II. PROPOSED SYSTEM

Volta's system developed by Volta is a prominent chaotic system. Integer order Volta's system can be generalized as following description [34]:

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) - ax_2(t) - x_3(t)x_2(t) \\ \dot{x}_2(t) &= -x_2(t) - bx_1(t) - x_3(t)x_1(t) \\ \dot{x}_3(t) &= cx_3(t) + x_1(t)x_2(t) + 1 \end{aligned} \tag{1}$$

The above system exhibits chaotic behavior with parameters $(a, b, c) = (5, 85, 0.5)$ and the corresponding Lyapunov exponents are 0.064979, -1.0708 and -1.4936 for initial conditions given as $(8, 2, 1)$.

On the similar grounds the non-integer order counterpart of Volta's system can also be defined as below:

$$\begin{aligned} D_t^{q_1} x_1(t) &= -x_1(t) - ax_2(t) - x_3(t)x_2(t) \\ D_t^{q_2} x_2(t) &= -x_2(t) - bx_1(t) - x_3(t)x_1(t) \\ D_t^{q_3} x_3(t) &= cx_3(t) + x_1(t)x_2(t) + 1 \end{aligned} \tag{2}$$

where q_1, q_2 and q_3 are the orders of derivatives.

Also, for $q_1 = q_2 = q_3 = q$ one can have a commensurate order system. The above system exhibits chaotic character as shown in Fig.1-3 for parameters $(a, b, c) = (19, 11, 0.73)$, order $q = 0.98$ and initial conditions $\big(x_1(0), x_2(0), x_3(0)\big) = (8, 2, 1)$. The simulation time $T_{sim} = 20\ s$ and time step $h = 0.0005$. These figures depict phase portrait for different combinations of states of Volta's system.

The stability and chaotic behavior of fractional order Volta's system is explained in [26]. For the set of parameters given above, it has three equilibrium points. The points along with corresponding eigen values are as follows:

$E_1^0 = (0, 0, -1.3698)$: $\lambda_1 = 12.02998, \lambda_2,\ \lambda_3 = 0.36499 \pm 10.33133i$

$E_2^- = (-1.26310, -10.26032, , -19.12310)$: $\lambda_1 = -14.02998, \lambda_2, \lambda_3 = 0.36499 \pm 10.33133i$

$E_3^+ = (1.26310, 10.26032, , -19.12310)$: $\lambda_1 = 0.73000, \lambda_2 = \lambda_3 = -2.0000$

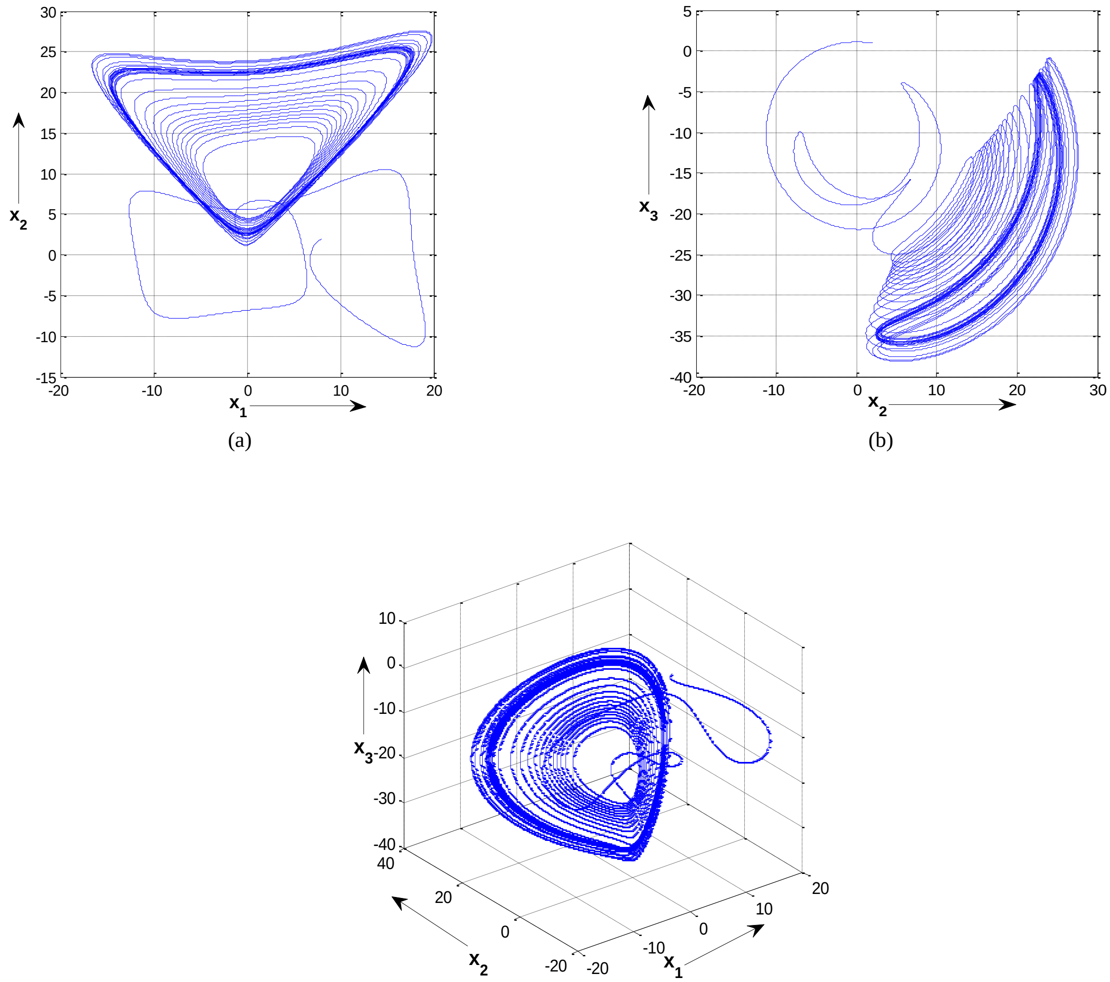
It can be noted that equilibria E_2^- and E_3^+ are saddle points symmetrically placed with respect to the x_3 axis. As given in [26], for equilibrium E_3^+, following conditions can be specified:

$$|\arg(\lambda_1)| < q\pi/2$$

and for E_2^-, we have

$$\left|\arg(\lambda_{2,3})\right| < q\pi/2$$

As the eigen values are in unstable region, it can be confirmed that, for initial conditions $\big(x_1(0), x_2(0), x_3(0)\big) = (8, 2, 1)$, Volta's system shows chaotic behavior.



(c)

Fig. 1 (a) Phase portrait for states $x_1 - x_2$ of Volta's system; (b) Phase portrait for states $x_3 - x_2$ of Volta's system (c) Phase portrait for states x_1, x_2, x_3 of Volta's system

III. Fractional Order Hyperchaotic Volta's System

The novel proposed fractional order hyperchaotic Voltas's system can be represented as below:

$$
\begin{aligned}
D_t^q x_1(t) &= -x_1(t) - a x_2(t) - x_3(t) x_2(t) + x_4(t) \\
D_t^q x_2(t) &= -x_2(t) - b x_1(t) - x_3(t) x_1(t) \\
D_t^q x_3(t) &= c x_3(t) + x_1(t) x_2(t) + 1 \\
D_t^q x_4(t) &= -d x_4(t) + x_2(t) x_3(t)
\end{aligned} \tag{3}
$$

Here, x_1, x_2, x_3 *and* x_4 are the states of the systems. *a, b, c and d* are the system parameters. The above system shows chaotic behaviour for different values of fractional order *q*. The Phase portraits between different states are shown in figure 2 for fractional order *q=0.99*. The simulation time is 500 seconds and the step size is of 0.005. The phase portraits confirm the chaotic behaviour of the system.

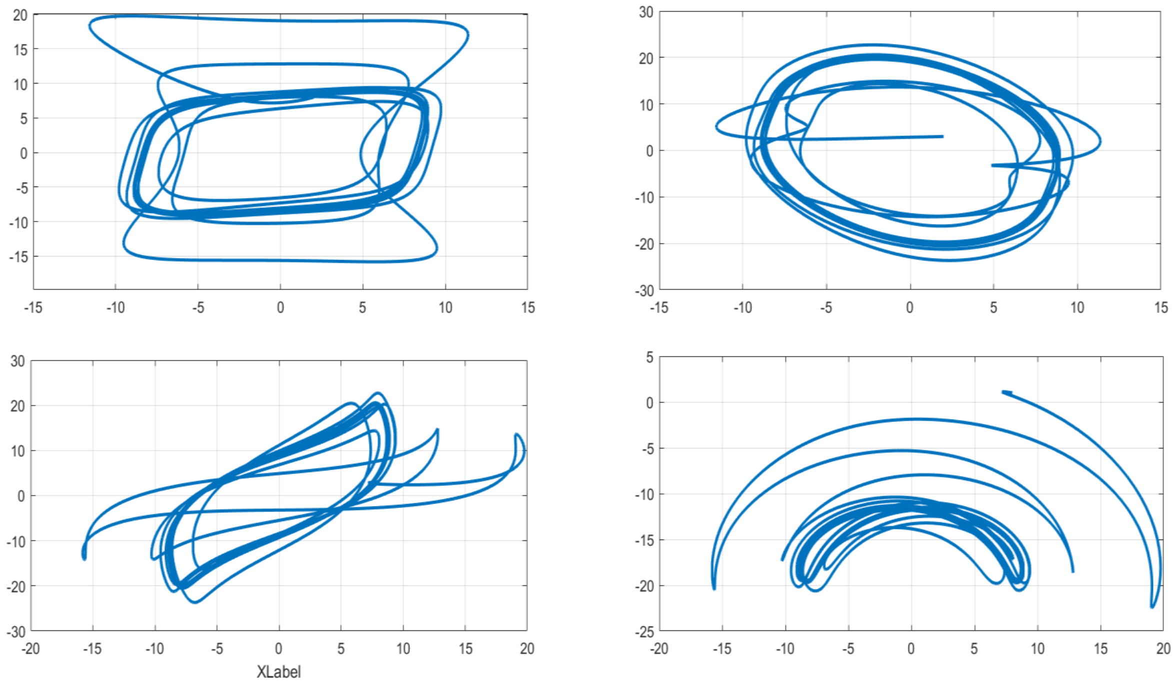


Fig. 2: Phase portraits of fractional order hyperchaotic Volta's system for q=0.99 and *a=19, b=11, c=0.7 and d=2.*

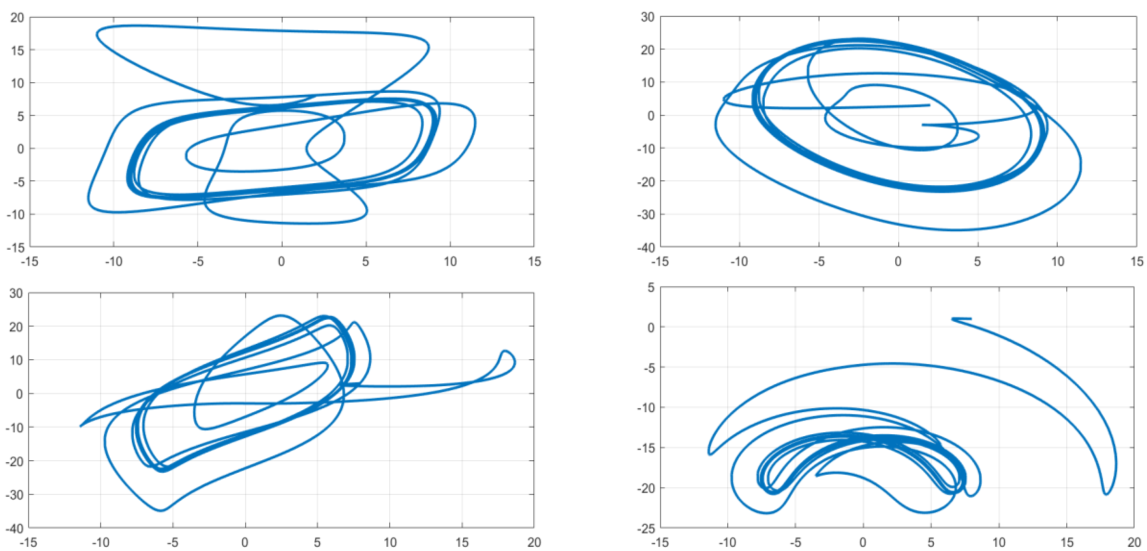
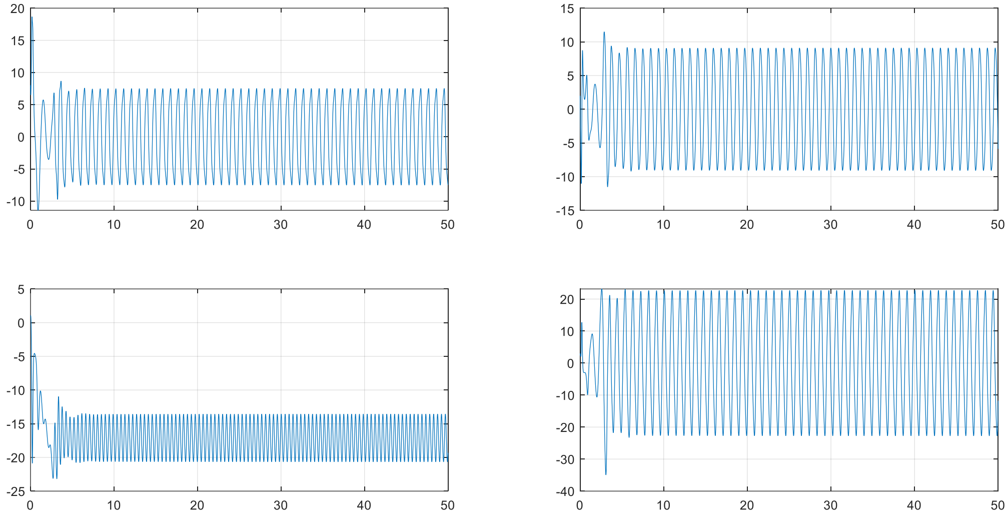


Fig 3. Phase portraits of fractional order hyperchaotic Volta's system for q=0.95 and *a=19, b=11, c=0.7 and d=2.*

Further the state evolution with respect to time is shwon below.



(c) (d)

Figure 4. (a) Original image (b) DDNT watermark Image (c) Watermarked image (d) Recovered watermark Image

IV.CONCLUSION

The chaotic behaviour of fractional order hyperchaotic Voltas's system has been presented here. The Phase portraits and the state evolution diagram verifies the chaotic behaviour.

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