BIANCHI TYPE-III SPACE-TIME COSMOLOGICAL MODEL WITH ELECTROMAGNETIC FIELD IN GENERAL RELATIVITY

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ABSTRACT

In this paper, we have investigated bianchi type-III space time cosmological model in presence of electromagnetic field in general relativity. Here we assume that F_{23} is only non-zero component of electromagnetic field tensor F_{ij}. To get the deterministic solution of the field equations we assume the expansion in the model is proportional to the shear, which gives $C = A^n$. the metric potentials are functions of t only and n is constant. This model has point type singularity. The behavior of the model together with physical and geometrical aspects is discussed in the presence of electromagnetic field.

Keywords: Bianchi type-III; Electromagnetic field; Strings; General relativity.

1. INTRODUCTION

When we study the Bianchi type models, we detect that accessible theories for the constitution of the universe fall in to two categories, based either leading the amplification of quantum fluctuations in a scalar field during price rises or leading equilibrium breaking phase conversion in the early universe, which leads to the formation of topological defects such as domain walls, monopoles, cosmic strings textures and other creatures. Space time admitting three parameter groups of automorphisms are important in discussing the cosmological models. The case where the group is purely transitive above the three dimensional, particularly constant time subspace is useful for two reasons. First, Bianchi has shown that there are only nine sets of distinct structure constants for groups of this type. So for the classification of homogeneous space time we can use algebra and other reason for importance of Bianchi type space-time is the simplicity of the field equations.

Cosmic strings are observing topologically stable objects, which might be found during a segment change in the premature cosmos. It is believed that the existence of strings in the universe give rise to the density fluctuation, which leads to the formation of galaxies. These string have stress energy and couple to gravitational field. So it is interesting to study the gravitation effects of strings. The general relativistic treatment of strings was initiated by Letelier [1-2] and Stachel [3]. find the solution to einstiens field eqations for a cloud of strings with spherical, plane and cylindrical symmetry. Then in 1983, he solved Einsteins field equations for a cloud of massive strings.

An axially symmetric Bianchi type-I string dust cosmological model with and without magnetic field by Banerjee [4]. Lorenz [5] has presented tilte electromagnetic Bianchi type-III cosmological model. Tikekar and Patel [6] have discussed some exact solution in Bianchi type-VI_0 string cosmology. Patel and Maharaj [7] discussed stationary rotating world model with magnetic field. Bali and Jain [8] have investigated bianchi type-III non-static magnetized cosmological model for perfect fluid distribution in general relativity. Bali et al.[9-12] have discussed bianchi type I and IX string cosmological models in general relativity.

The presence of primordial magnetic field in the early stages of the evolution of the universe has been discussed by several authors,[13-22]. Electric current exist due to the occurrence of magnetic field and strong magnetic field can be created due to adiabatic compression in clusters of galaxies. Bali and Tyagi[23] and Pradhan et al.[24-25] have discussed cylindrically symmetric in homogeneous cosmological models in presence of electromagnetic field. Pradhan, Lata and Amirhashchi [26] have investigated massive string cosmology in bianchi type-III space-time with electromagnetic field. Pradhan and Amirhaschi H. Zainuddin [27] have studied dark energy model in anistropic bianchi type-III space time with variable EoS parameter.

Bali, Pradhan and Rai [28] are discussed string cosmological model in cylindrically symmetric inhomogeneous universe with electromagnetic field. Deo, Punwatkar and Patil [29] are investigated bianchi type-III cosmological model electromagnetic field with cosmic string in general theory of relativity. In this paper, we discussed bianchi type-III space time cosmological model with electromagnetic field. Here we assume that F_{23} is only non vanishing component of electromagnetic field tensor F_{ij}. To get the deterministic solution of the field equations we assume the expansion in the model is proportional to the shear, which gives $A = C^n$. the metric potentials are functions of t only and n is constant. We find out that This model has point type singularity. The behavior of the model together with physical and geometrical aspects is discussed in the presence of electromagnetic field.

2. THE METRIC AND FIELD EQUATIONS

We consider spatially homogeneous space time of Bianchi type-III can be written in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2ax}dy^2 + C^2dz^2 \tag{2.1}$$

$$(x^1 = x, x^2 = y, x^3 = z, x^4 = t)$$

Here A, B and C are function of t and a is non zero constant.

The volume element is given as for the model (2.1)

$$V = \sqrt{-g} = ABCe^{-ax} \tag{2.2}$$

The energy momentum tensor with electromagnetic field for the string is given by

$$T_i^j = \rho u_i u^j - \lambda x_i x^j + E_i^j \tag{2.3}$$

Where co-moving are as $u^4 = 1, u^1 = u^2 = u^3 = 0$ and we choose the direction of string parallel to x-axis so $x^1 = \frac{1}{A}, x^2 = x^3 = x^4 = 0$

i.e. $u^i u_i = -x^i x_i = -1 \; and \; u^i x_i = 0$ (2.4)

In equation (2.3), electromagnetic energy tensor E_i^j is given by Lichnerowicz[30]

$$E_i^j = \bar{\mu}\left[h_l h^l \left(u_i u^j + \frac{1}{2} g_i^j\right) - h_i h^j\right] \tag{2.5}$$

Where h_i is the magnetic flux vector and $\bar{\mu}$ the magnetic permeability defined as

$$h_i = \frac{1}{\bar{\mu}} * F_{ji} u^j \tag{2.6}$$

Here $* F_{ij}$ dual electromagnetic field tensor is defined by Syange [31]

$$* F_{ij} = \frac{\sqrt{-g}}{2} \epsilon_{ijkl} F^{kl} \tag{2.7}$$

Where F_{ij} is the electromagnetic field tensor and ϵ_{ijkl} is the Levi- civita tensor density.

We consider that $F_{23}$is the only non vanishing component of F_{ij}.for the line-element (2.1) non vanishing component of the electromagnetic energy tensor E_i^j are obtained as

$$E_1^1 = E_4^4 = \frac{-(F_{23})^2 e^{2ax}}{2\bar{\mu} B^2 C^2}$$

$$E_2^2 = E_3^3 = \frac{(F_{23})^2 e^{2ax}}{2\bar{\mu} B^2 C^2} \tag{2.8}$$

And the corresponding component of energy momentum tensor T_i^j are obtained as

$$T_1^1 = -\lambda - \frac{(F_{23})^2 e^{2ax}}{2\bar{\mu} B^2 C^2} \tag{2.9}$$

$$T_2^2 = \frac{(F_{23})^2 e^{2ax}}{2\bar{\mu} B^2 C^2} \tag{2.10}$$

$$T_3^3 = \frac{(F_{23})^2 e^{2ax}}{2\bar{\mu} B^2 C^2} \tag{2.11}$$

$$T_4^4 = -\rho - \frac{(F_{23})^2 e^{2ax}}{2\bar{\mu} B^2 C^2} \tag{2.12}$$

The Einstein's field equation

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi k T_i^j \tag{2.13}$$

Where is R_i^j Ricci tensor and $R = g^{ij} R_{ij}$ is the Ricci scalar and T_i^j is energy momentum tensor

For the line element (2.1), Einstein's field equation (2.13) lead to the following system of equations.

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = 8\pi k \left[\lambda + \frac{(F_{23})^2 e^{2ax}}{2\bar{\mu} B^2 C^2}\right] \tag{2.14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi k \left[\frac{(F_{23})^2 e^{2ax}}{2\bar{\mu} B^2 C^2}\right] \tag{2.15}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{a^2}{A^2} = -8\pi k \left[\frac{(F_{23})^2 e^{2ax}}{2\bar{\mu} B^2 C^2}\right] \tag{2.16}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} = 8\pi k \left[\rho + \frac{(F_{23})^2 e^{2ax}}{2\bar{\mu} B^2 C^2}\right] \tag{2.17}$$

$$a\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) = 0 \tag{2.18}$$

From equation (2.18) become

$$\frac{\dot{B}}{B} = \frac{\dot{A}}{A} \tag{2.19}$$

Integrating, we get

$$B = LA \tag{2.20}$$

Where L is constant of integration. We consider $B = A$, taking L=1 without loss of generality then the field equations (2.14) to (2.17) reduce to

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = 8\pi k \left[\lambda + \frac{(F_{23})^2 e^{2ax}}{2\bar{\mu}A^2C^2}\right] \tag{2.21}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi k \left[\frac{(F_{23})^2 e^{2ax}}{2\bar{\mu}A^2C^2}\right] \tag{2.22}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{a^2}{A^2} = -8\pi k \left[\frac{(F_{23})^2 e^{2ax}}{2\bar{\mu}A^2C^2}\right] \tag{2.23}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} = 8\pi k \left[\rho + \frac{(F_{23})^2 e^{2ax}}{2\bar{\mu}A^2C^2}\right] \tag{2.24}$$

The expansion scalar ($\boldsymbol{\theta}$) for the model (2.1) is given by

$$\theta = u^i_{;i} = \left(2\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) \tag{2.25}$$

The shear tensor is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[2\left(\frac{\dot{A}}{A} - \frac{\theta}{3}\right)^2 + \left(\frac{\dot{C}}{C} - \frac{\theta}{3}\right)^2\right] \tag{2.26}$$

3. SOLUTIONS OF THE FIELD EQUATIONS

The field equations (2.21) to (2.24) are system of four non linear differential equations with five unknown variables A, C, ρ, λ and F_{23}. Therefore one physically reasonable condition between these unknowns is required to obtain explicit solution of the field equations. So let us assume that the expansion scalar (θ) is proportional to the shear tensor (σ) this condition leads to

$$C = A^n \tag{3.1}$$

Where (n) is a constant such that $n \neq 1$

Equations (2.22) and (2.23) lead to

$$\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{a^2}{A^2} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} = 0 \tag{3.2}$$

Using equation (3.1) in (3.2), we get

$$\frac{\ddot{A}}{A} + (1+n)\frac{\dot{A}^2}{A^2} = \frac{a^2}{(1-n)A^2} \tag{3.3}$$

Now let $\dot{A} = f(A)$, then equation (3.3) reduce in the form

$$\frac{d(f^2)}{dA} + \frac{2(n+1)f^2}{A} = \frac{2a^2}{(1-n)A^2} \tag{3.4}$$

Integrating equation (3.4), we get

$$f^2 = \frac{a^2}{(1-n^2)} + c_1 A^{-2(n+1)} \tag{3.5}$$

Where c_1 is constant of integration

The Bianchi type-III model in this case reduce to

$$ds^2 = -\left(\frac{dt}{dA}\right)^2 dA^2 + A^2\left(dx^2 + e^{-2ax}dy^2\right) + A^{2n}dz^2 \tag{3.6}$$

After using suitable transformation of coordinates model reduce to

$$ds^2 = -\left[\frac{dT^2}{\frac{a^2}{(1-n^2)} + c_1 T^{-2(n+1)}}\right] + T^2\left(dX^2 + e^{-2ax}dY^2\right) + T^{2n}dZ^2 \tag{3.7}$$

Where $A = T, x = X, y = Y, z = Z$

4. PHYSICAL AND GEOMETRICAL ASPECTS OF THE MODEL

The electromagnetic field (F_{23}), the energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ)and shear tensor (σ). For the model (3.7) physical and geometrical properties by using equations (3.4) and (3.5) in equations (2.21) to (2.24)

$$(F_{23})^2 = \frac{\bar{\mu} e^{-2ax}}{4\pi k}\left[\frac{c_1(2n+1)}{T^2} - \frac{a^2n^2T^{2n}}{(1-n^2)}\right] \tag{4.1}$$

The string tension density

$$\lambda = \frac{1}{4\pi k}\left[\frac{a^2n^2}{(1-n^2)T^2} - \frac{c_1(2n+1)}{T^{2n+4}}\right] \tag{4.2}$$

The energy density

$$\rho = \frac{a^2 n}{4\pi k(1-n)} \tag{4.3}$$

The particle density

$$\rho_p = \rho - \lambda = \frac{1}{4\pi k}\left[\frac{a^2n}{(1-n^2)T^2} + \frac{c_1(2n+1)}{T^{2n+4}}\right] \tag{4.4}$$

The expansion

$$\theta = n(2n+1)\left[\frac{a^2}{(1-n^2)T^2} + \frac{c_1}{T^{2n+4}}\right]^{1/2} \tag{4.5}$$

The shear tensor

$$\sigma^2 = \frac{n^2(1-n)^2}{3T^2}\left(\frac{a^2}{(1-n^2)} + \frac{c_1}{T^{2n+2}}\right) \tag{4.6}$$

From equation (4.5) and (4.6), we find

$$\frac{\sigma^2}{\theta^2} = \frac{(1-n)^2}{3(2n+1)^2} = constant \tag{4.7}$$

5. CONCLUSION

In this paper we have investigated a new exact solution of Einstein's field equations for Bianchi type-III in presence of electromagnetic field. This model starts expanding with big bang at T=0, and the expansion of the model decreases as time increases if $n(1+2n)>0$. The ratio of the shear scalar σ and expansion θ tends to finite value $i.e.\ \frac{\sigma}{\theta} \to finite$. Since $\lim_{t\to\infty}\frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large value of t. we also observed that ρ, λ, ρ_p tends to ∞ when T→0. The model has point type singularity at T=0. The geometrical and physical behavior of the model is also discussed.

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