On several properties of fuzzy magnified Boolean algebra

Sanjib Kumar Datta[1], Saral Datta[2], Tandra Sarkar[3] and Aditi Biswas[4]
[1]Department of Mathematics, University of Kalyani
P.O.: Kalyani, Dist.: Nadia, PIN :741235, West Bengal, India.
E-mail id: sanjibdatta05@gmail.com
[2]Department of Mathematics,Brahmananda Keshab Chandra College
111/2 B. T. Road, Bonhooghly, Kolkata - 700108.
E-mail id: saraldutta@gmail.com.
[3]Department of Mathematics, University of Kalyani
P.O.: Kalyani, Dist.: Nadia, PIN :741235, West Bengal, India.
E-mail id: tandrasarkar073@gmail.com
and
[4]Department of Mathematics, Fakir Chand College, Diamond Harbour
P.O.:Diamond Harbour, South 24 Parganas, PIN:743331,West Bengal, India.
E-mail Id: biswasaditi.91.ab@gmail.com.

Abstract

In the paper we introduce a new kind of fuzzy Boolean algebra termed as Fuzzy Magnified Boolean Algebra and study some properties of it.

Keywords and Phrases: Boolean algebra, fuzzy magnification, atom, co-atom, cardinality.

AMS subject classification code: 03G05, 03B52, 06Exx.

1 Introduction, Definitions and Notations.

Fuzzy sets deal with objects which are "matter-of-degree". It specifies the degree of belongingness of an element to the set. In general, fuzzy sets cannot form a Boolean algebra, an algebraic system initiated by English Mathematician George Boole in 1854 as it does not satisfy the complementation laws i.e., if A is a fuzzy set and A' be its complement then $A \cup A' \neq I$,the universal set and $A \cap A' \neq \phi$,the empty set. By extending the definition of complementation of fuzzy sets, Dhar [3] has made hold the complementation laws in the system of fuzzy sets.

Majumder& Sardar [8] and Datta[2] used the concept of fuzzy magnified translation respectively in fuzzy group and ring theory. Following the concepts of relationship between atoms and co-atoms in fuzzy Boolean algebra by Rajbongshi [12] , in this paper we wish to formulate the structure of fuzzy magnified Boolean algebra using the concept of fuzzy magnification and prove several properties of it .

In fact we may recall the following definitions:

1

Definition 1.1 *In a Boolean algebra B if $a+b=b$ (or $a.b=a$) then a is said to be a subelement of b where '$+$' and '$.$' are two binary operations.*

Definition 1.2 *In a Boolean algebra B an element $e \neq 0$ is said to be an atom of B if for every $x \in B$, $x \leq e \Rightarrow$ either $x=0$ or $x=e$.*

Definition 1.3 *The dual concept of an atom in a Boolean algebra is a co-atom.*

Definition 1.4 *Let μ be a non-empty fuzzy subset of a Boolean algebra B (i.e. $\mu(x) \neq 0$ for some $x \in B$) and $\beta \in [0,1]$. Then the fuzzy magnification μ_β^m of μ in B is defined as*

$$\mu_\beta^m = \beta.\mu(x) \text{ for all } x \in B.$$

μ_β^m is also a fuzzy subset of B.

Let $S=\{s_0, s_1, ..., s_{p-1}\}$ be a finite set with p elements. Then the fuzzy magnification of fuzzy subsets of S are the mappings from S to I where $I=[0,1]$. Let M be the set of membership values taken under fuzzy magnification of fuzzy subsets of S such that

$$\begin{aligned} M &= \left\{\beta.0, \beta.\frac{1}{n}, \beta.\frac{2}{n}, \beta.\frac{3}{n}, ..., \beta.\frac{n-1}{n}, \beta.\frac{n}{n}\right\} \\ &= \{0, \beta h, 2\beta h, 3\beta h, ..., (n-1)\beta h, n\beta h\}, \\ \text{where } h &= \frac{1}{n} \text{ and n is a positive integer.} \end{aligned}$$

Then the set M forms a poset under the operation "$\leq$" where the symbol has its usual meaning. Then the mapping from S to M contains $|M|^{|S|}$ functions which are fuzzy subsets. Let F be the set of all fuzzy subsets of S with M of membership values. If we consider a subset of M containing 0 and any one element from the rest say $m\beta h$ where $1 \leq m \leq n$ then the set of mappings from S to $\{0, m\beta h\}$ are the fuzzy subsets of F which are different from crisp subsets. We denote this set of fuzzy subsets of F by F_m and it is contained in F. The numbers of elements in F_m is 2^p which namely are:

$$I_0 = \phi = \{(s_0,0), (s_1,0), ..., (s_{p-2},0), (s_{p-1},0)\},$$

$$I_1 = \{(s_0,0), (s_1,0), ..., (s_{p-2},0), (s_{p-1},m\beta h)\},$$

$$I_2 = \{(s_0,0), (s_1,0), ..., (s_{p-2},m\beta h), (s_{p-1},0)\},$$

$$I_3 = \{(s_0,0), (s_1,0), ..., (s_{p-2},m\beta h), (s_{p-1},m\beta h)\},$$

....

....

$$I_{2^p-2} = \{(s_0,m\beta h), (s_1,m\beta h), ..., (s_{p-2},m\beta h), (s_{p-1},0)\} \text{ and}$$

2

$$I_{2^p-1} = \{(s_0, m\beta h), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\}.$$

Hence $F_m = \{I_0, I_1, I_2, ..., I_{2^p-1}\}$ is the family of fuzzy subsets. In F_m, I_0 is identical with the empty subset ϕ of F. On the otherhand, I_{2^p-1} is the universal fuzzy subset of F_m which may be different from the universal set of F.

The following definitions are relevant in developing the methodology of the paper:

Definition 1.5 *For any two fuzzy Boolean algebras B_p and B_q where $1 \le p < q \le n$, the scalar multiplication is defined as:*

$$\mu_{B_p}(x_i) = \frac{p}{q}\mu_{B_q}(x_i), \ \forall x_i \in S$$

where $\mu_{B_p}(x_i)$ and $\mu_{B_q}(x_i)$ are the membership values of the i-th element of the fuzzy subsets of B_p and B_q respectively.

Definition 1.6 *An atom of a fuzzy Boolean algebra F is a fuzzy subset which does not have any non-trivial proper fuzzy subset. Alternatively, an atom of a fuzzy Boolean algebra F is a fuzzy subset which cannot be expressed as the fuzzy union of other non-trivial proper fuzzy subsets of F.*

Definition 1.7 *A co-atom of a fuzzy Boolean algebra F is a fuzzy subset which does not have any proper fuzzy superset. Alternatively, a co-atom of a fuzzy Boolean algebra F is a fuzzy subset which cannot be expressed as the fuzzy intersection of other proper fuzzy subsets of F.*

Let two fuzzy subsets A and A' are said to be complementary to each other if

$$\mu_{A'}(x) = \{1 - \mu_A(x)\}.$$

Redefining this operation for the set F_k of fuzzy subsets of the set F we may formulate :

$$\mu_{A'}(x) = \{a - \mu_A(x)\}$$

where $a = m\beta h \in M$ with $1 \le m \le n$.

2 Theorems.

In this section we present the main results of the paper.

Theorem 2.1 *F_m forms a fuzzy magnified Boolean algebra.*

Proof. Let A, B and $C \in F_m$ be fuzzy subsets with

$$A = \{(x, \mu_A(x) : x \in S, \mu_A(x) \in \{0, a\})\},$$
$$B = \{(x, \mu_B(x) : x \in S, \mu_B(x) \in \{0, a\})\}$$

and

$$C = \{(x, \mu_C(x) : x \in S, \mu_C(x) \in \{0, a\})\}$$

where $a = m\beta h$ with $1 \le m \le n$.

3

Therefore $\forall x \in S$ we have

$$\min\left\{\mu_A(x), \mu_B(x)\right\} = \min\left\{\mu_B(x), \mu_A(x)\right\}$$
$$i.e.,\ A \cdot B = B \cdot A.$$

And

$$\max\left\{\mu_A(x), \mu_B(x)\right\} = \max\left\{\mu_B(x), \mu_A(x)\right\}$$
$$i.e., A + B = B + A.$$

So the binary operations $'+'$ and $'\cdot'$ are both commutative.
Now

$$A + (B \cdot C) = \max\left\{\mu_A(x), \min\left\{\mu_B(x), \mu_C(x)\right\}\right\}$$

and

$$(A + B) \cdot (A + C) = \min\left\{\max\left\{\mu_A(x), \mu_B(x)\right\}, \max\left\{\mu_A(x), \mu_C(x)\right\}\right\}.$$

Also without loss of generality let $\mu_A(x) \leq \mu_B(x) \leq \mu_C(x), \forall x \in S$
Then

$$\begin{aligned} A + (B \cdot C) &= \max\left\{\mu_A(x), \min\left\{\mu_B(x), \mu_C(x)\right\}\right\} \\ &= \max\left\{\mu_A(x), \mu_B(x)\right\} = \mu_B(x) \end{aligned}$$

and

$$\begin{aligned} (A + B) \cdot (A + C) &= \min\left\{\max\left\{\mu_A(x), \mu_B(x)\right\}, \max\left\{\mu_A(x), \mu_C(x)\right\}\right\} \\ &= \min\left\{\mu_B(x), \mu_C(x)\right\} = \mu_B(x) \end{aligned}$$

$$\therefore A + (B \cdot C) = (A + B) \cdot (A + C).$$

Similarly we get that

$$\therefore A \cdot (B + C) = (A \cdot B) + (A \cdot C).$$

Hence $'+'$ and $'\cdot'$ are both distributive over each other.
Further,

$$\min\left\{\mu_A(x), \mu_{I_{2^p-1}}(x)\right\} = \mu_A(x)$$
$$i.e., A \cdot I_{2^p-1} = A$$

and

$$\max\left\{\mu_A(x), \mu_\phi(x)\right\} = \max\left\{\mu_A(x), 0\right\} = \mu_A(x)$$
$$i.e., A + \phi = A.$$

Thus ϕ and I_{2^p-1} are identity elements for $'+'$ and $'\cdot'$ respectively.

Moreover, let A' be the complement of A in F_m. Therefore in view of Definition 1.6 and for $\forall x \in S$ we have

$$\min\left\{\mu_A(x), \mu_{A'}(x)\right\} = \min\left\{\mu_A(x), a - \mu_A(x)\right\}$$

i.e., for every $x \in S$ either $\mu_A(x) = 0$ or $a - \mu_A(x) = 0$.
Therefore $A \cdot A' = I_0$
And

$$\max\left\{\mu_A(x), \mu_{A'}(x)\right\} = \max\left\{\mu_A(x), a - \mu_A(x)\right\}.$$

Therefore complementary laws with respect to both $'+'$ and $'\cdot'$ holds in F_m. Hence F_m forms a magnified Boolean algebra. ■

4

Theorem 2.2 *The number of atoms of F_m is equal to the cardinality of I_{2^p-1}.*

Proof. Let S be a finite set of p elements and also let

$$\begin{aligned} M &= \left\{\beta.0, \beta.\frac{1}{n}, \beta.\frac{2}{n}, \beta.\frac{3}{n}, ..., \beta.\frac{n-1}{n}, \beta.\frac{n}{n}\right\} \\ &= \{0, \beta h, 2\beta h, 3\beta h, ..., (n-1)\beta h, n\beta h\} \text{ where } h = \frac{1}{n}, \text{ n being a positive integer.} \end{aligned}$$

of membership values under fuzzy magnification of fuzzy subsets of S. So for the type of mappings $S \to \{0, m\beta h\}$ where $1 \leq m \leq n$ as considered we get exactly p-fuzzy subsets containing exactly one non-zero membership which cannot be expressed as the join of non-trivial proper fuzzy subsets. Therefore, only these fuzzy subsets are the atoms for a fuzzy Boolean algebra under fuzzy magnification. Hence the number of atoms of a fuzzy Boolean algebra under fuzzy magnification is equal to the cardinality of I_{2^p-1}.

Again for the mappings $S \to \{0, \beta h\}, S \to \{0, 2\beta h\}, ..., S \to \{0, n\beta h\}$, n-number of F_m can be obtained. Therefore the total number of atoms that can be obtained from all fuzzy magnified Boolean algebras formed by fuzzy subsets of S is always equal to $p \times n$.

This proves the theorem. ■

Remark 2.1 *Theorem 2.2 improves Theorem 5 of [12].*

Remark 2.2 *In view of Theorem 2.2 the number of elements of F_m can be written as $2^{|atom|}$.*

Theorem 2.3 *The number of co-atoms of F_m is equal to the cardinality of I_{2^p-1}.*

Proof. Let S be a finite set of p elements and also let

$$\begin{aligned} M &= \left\{\beta.0, \beta.\frac{1}{n}, \beta.\frac{2}{n}, \beta.\frac{3}{n}, ..., \beta.\frac{n-1}{n}, \beta.\frac{n}{n}\right\} \\ &= \{0, \beta h, 2\beta h, 3\beta h, ..., (n-1)\beta h, n\beta h\} \text{ where } h = \frac{1}{n}, \text{ n being a positive integer.} \end{aligned}$$

of membership values under fuzzy magnification of fuzzy subsets of S.So considering the mapping $S \to \{0, m\beta h\}$ where $1 \leq m \leq n$ there are exactly p-fuzzy subsets containing exactly one zero membership which cannot be expressed as the fuzzy intersection of proper fuzzy subsets. Therefore only those fuzzy subsets are the co-atoms of a fuzzy Boolean algebra under fuzzy magnification. Hence the number of co-atoms of F_m is equal to the cardinality of I_{2^p-1}. Thus the theorem is established. ■

Remark 2.3 *It is evident from Theorem 2.3 that the number of elements of F_m is $2^{|co-atom|}$.*

Remark 2.4 *The number of atoms and co-atoms of F_m is equal.*

Theorem 2.4 *All the fuzzy Boolean algebra of F_m have same number of atoms.*

Proof. Since the number of atoms of F_m is equal to the total number of elements of I_{2^p-1} , then in view of Theorem 2.2 the result follows. ■

5

Theorem 2.5 *All the fuzzy Boolean algebra of F_m have same number of co-atoms.*

Proof. Since the number of co-atoms of F_m is equal to the cardinality of I_{2^p-1}, therefore from Theorem 2.3 the result can be easily derived. ■

Theorem 2.6 *In F_m each fuzzy subset contains exactly one non-zero membership element.*

Proof. From the definition of fuzzy union of two fuzzy subsets of F_m it follows that only those fuzzy subsets which contain exactly one non-zero membership element cannot be expressed as the fuzzy union of non trivial proper fuzzy subsets and all other non trivial proper fuzzy subsets can be expressed as the join of remaining non trivial proper fuzzy subsets. Hence these fuzzy subsets are the only atoms of F_m. Hence the result follows. ■

Theorem 2.7 *In F_m the co-atoms are the fuzzy subsets which contain exactly one zero membership element.*

Proof. In view of definition of fuzzy intersection of fuzzy sets of F_m , we see that fuzzy subsets containing exactly one zero membership element cannot be expressed as the fuzzy intersection of proper fuzzy subsets and any proper fuzzy subset can be expressed as the intersection of remaining proper fuzzy subsets. Hence, these fuzzy subsets are the only co-atoms of F_m. Thus the theorem is established. ■

Theorem 2.8 *The co-atom and atoms of F_m are complementary to each other.*

Proof. Let S be a finite set of p elements and also let

$$\begin{aligned} M &= \left\{\beta.0, \beta.\frac{1}{n}, \beta.\frac{2}{n}, \beta.\frac{3}{n}, ..., \beta.\frac{n-1}{n}, \beta.\frac{n}{n}\right\} \\ &= \{0, \beta h, 2\beta h, 3\beta h, ..., (n-1)\beta h, n\beta h\} \text{ where } h = \frac{1}{n}, \text{ n being a positive integer.} \end{aligned}$$

of membership values under fuzzy magnification of fuzzy subsets of S. Since any atom of F_m are the fuzzy subsets containing exactly one non-zero membership element, so the set of all atoms A(say) of F_m is of the form :

$$\begin{aligned} A &= A_0 = \{(s_0,0), (s_1,0), ..., (s_{p-2},0), (s_{p-1}, m\beta h)\}, \\ A_1 &= \{(s_0,0), (s_1,0), ..., (s_{p-2}, m\beta h), (s_{p-1},0)\}, \\ & \quad .... \\ & \quad .... \\ A_{p-2} &= \{(s_0,0), (s_1, m\beta h), ..., (s_{p-2},0), (s_{p-1},0)\} \text{ and} \\ A_{p-1} &= \{(s_0, m\beta h), (s_1,0), ..., (s_{p-2},0), (s_{p-1},0)\}. \end{aligned}$$

Again since the co-atoms of F_m are the fuzzy subsets containing exactly one zero membership element, so the set of all co-atoms C (say) of F_m is of the

6

form:

$$\begin{aligned} C &= C_0 = \{(s_0, m\beta h), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1}, 0)\}, \\ C_1 &= \{(s_0, m\beta h), (s_1, m\beta h), ..., (s_{p-2}, 0), (s_{p-1}, m\beta h)\}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_{p-2} &= \{(s_0, m\beta h), (s_1, 0), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\} \text{ and} \\ C_{p-1} &= \{(s_0, 0), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\}. \end{aligned}$$

In view of the definition of complementation in F_m, we have

$$(A_0)' = C_0, (A_1)' = C_1, \text{...and } (A_{p-1})' = C_{p-1},$$

i.e., atoms and co-atoms of F_m are complementary to each other.

This proves the theorem. ■

The following example ensures the notion of fuzzy magnification in fuzzy boolean algebra.

Example 2.9 *Let $S = \{s_0, s_1, s_2, s_3\}$ be universal set. Again let $M = \{0, \beta h, 2\beta h, 3\beta h = 1\}$ where $h = \frac{1}{3}, \beta = 0.8$. Further considering a mapping from E to $\{0, \beta h\}$, we gat a fuzzy Boolean algebra C under fuzzy magnification B_1 which is written as follows*

$$\begin{aligned} C &= C_0 = 0 = \{(s_0, 0), (s_1, 0), (s_2, 0)\}, \\ C_1 &= 1 = \{(s_0, 0), (s_1, 0), (s_2, 0.8h)\}, \\ C_2 &= \{(s_0, 0), (s_1, 0.8h), (s_2, 0)\}, \\ C_3 &= \{(s_0, 0.8h), (s_1, 0), (s_2, 0)\}, \\ C_4 &= \{(s_0, 0), (s_1, 0.8h), (s_2, 0.8h)\}, \\ C_5 &= \{(s_0, 0.8h), (s_1, 0), (s_2, 0.8h)\}, \\ C_6 &= \{(s_0, 0.8h), (s_1, 0.8h), (s_2, 0)\} \text{ and} \\ C_7 &= \{(s_0, 0.8h), (s_1, 0.8h), (s_2, 0.8h)\}. \end{aligned}$$

Where 0 and C_7 are respectively the empty and universal fuzzy subset of C under B_1.

Theorem 2.10 *I_0 and I_{2^p-1} are respectively the infimum and supremum of the set of all atoms of F_m.*

Proof. Let S be a finite set of p elements and also let

$$\begin{aligned} M &= \left\{\beta.0, \beta.\frac{1}{n}, \beta.\frac{2}{n}, \beta.\frac{3}{n}, ..., \beta.\frac{n-1}{n}, \beta.\frac{n}{n}\right\} \\ &= \{0, \beta h, 2\beta h, 3\beta h, ..., (n-1)\beta h, n\beta h\}, \quad h = \frac{1}{n}, \text{ n being a positive integer.} \end{aligned}$$

of membership values under fuzzy magnification of fuzzy subsets of S. Since any atom of F_m are the fuzzy subsets containing exactly one non-zero membership

7

element, so the A(say) of F_m is of the form:

$$\begin{aligned} A &= A_0 = \{(s_0,0), (s_1,0), ..., (s_{p-2},0), (s_{p-1}, m\beta h)\}, \\ A_1 &= \{(s_0,0), (s_1,0), ..., (s_{p-2}, m\beta h), (s_{p-1},0)\}, \\ &\quad .... \\ &\quad .... \\ A_{p-2} &= \{(s_0,0), (s_1, m\beta h), ..., (s_{p-2},0), (s_{p-1},0)\} \text{ and} \\ A_{p-1} &= \{(s_0,m\beta h), (s_1,0), ..., (s_{p-2},0), (s_{p-1},0)\}. \end{aligned}$$

As we know that the supremum of all atoms of F_m is the fuzzy union of them, then it can be written as

$$\begin{aligned} & A_0 \vee A_1 \vee A_2 \vee ... \vee A_{p-1} \\ = & \{(s_0,m\beta h), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\} \\ = & I_{2^p-1}. \end{aligned}$$

Again as the infimum of all the atoms of F_m is the fuzzy intersection of them, then it can be expressed as

$$\begin{aligned} & A_0 \wedge A_1 \wedge A_2 \wedge ... \wedge A_{p-1} \\ = & \{(s_0,0), (s_1,0), ..., (s_{p-2},0), (s_{p-1},0)\} \\ = & I_0. \end{aligned}$$

Thus the theorem is established. ■

Theorem 2.11 *I_0 and I_{2^p-1} are respectively infimum and supremum of the set of all co-atoms of F_m .*

Proof. Let S be a finite set of p elements and also let

$$\begin{aligned} M &= \left\{\beta.0, \beta.\frac{1}{n}, \beta.\frac{2}{n}, \beta.\frac{3}{n}, ..., \beta.\frac{n-1}{n}, \beta.\frac{n}{n}\right\} \\ &= \{0, \beta h, 2\beta h, 3\beta h, ..., (n-1)\beta h, n\beta h\} \text{ where } h = \frac{1}{n}, \text{n being a positive integer.} \end{aligned}$$

of membership values under fuzzy magnification of fuzzy subsets of S. As the co-atoms of F_m are the fuzzy subsets containing exactly one zero membership element, so the set of all co-atoms C (say) of F_m is of the form:

$$\begin{aligned} C &= C_0 = \{(s_0,m\beta h), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1},0)\}, \\ C_1 &= \{(s_0,m\beta h), (s_1, m\beta h), ..., (s_{p-2},0), (s_{p-1}, m\beta h)\}, \\ &\quad .... \\ &\quad .... \\ C_{p-2} &= \{(s_0,m\beta h), (s_1,0), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\} \text{ and} \\ C_{p-1} &= \{(s_0,0), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\}. \end{aligned}$$

As we know that the supremum of all the co-atoms of F_m is fuzzy union of them, then it follows that

$$\begin{aligned} & C_0 \vee C_1 \vee C_2 \vee ... \vee C_{p-1} \\ = & \{(s_0,m\beta h), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\} \\ = & I_{2^p-1}. \end{aligned}$$

Further, as the infimum of all the co-atoms of F_m is fuzzy intersection of them, then it can be written as

$$\begin{aligned} & C_0 \wedge C_1 \wedge C_2 \wedge ... \wedge C_{p-1} \\ = & \{(s_0, 0), (s_1, 0), ..., (s_{p-2}, 0), (s_{p-1}, 0)\} \\ = & I_0. \end{aligned}$$

Hence the proof of the theorem.. ■

In view of Theorem 2.7, Theorem 2.8, Theorem 2.9 and Theorem 2.10 the following remark is immediate:

Remark 2.5 *I_0 and I_{2^p-1} are respectively the infimum and supremum of the set of all atoms and co-atoms of F_m .*

Theorem 2.12 *The set of atoms of F_m has one to one correspondence to the set of atoms of another fuzzy magnified Boolean algebra under the same universal set.*

Proof. Let $S = \{s_0, s_1, s_2, ..., s_{p-1}\}$ with p elements. In the light of Theorem 2.8, the set of all atoms A of any fuzzy Boolean algebra under fuzzy magnification B_1 is of the following form

$$\begin{aligned} A &= A_0 = \{(s_0, 0), (s_1, 0), ..., (s_{p-2}, 0), (s_{p-1}, m\beta h)\}, \\ A_1 &= \{(s_0, 0), (s_1, 0), ..., (s_{p-2}, m\beta h), (s_{p-1}, 0)\}, \\ & \ldots\ldots\ldots\ldots\ldots\ldots \\ & \ldots\ldots\ldots\ldots\ldots\ldots \\ A_{p-2} &= \{(s_0, 0), (s_1, m\beta h), ..., (s_{p-2}, 0), (s_{p-1}, 0)\} \text{ and} \\ A_{p-1} &= \{(s_0, m\beta h), (s_1, 0), ..., (s_{p-2}, 0), (s_{p-1}, 0)\}. \end{aligned}$$

Similarly in I_{2^p-1} we can get another set of atoms C (say) of another fuzzy Boolean algebra under fuzzy magnification B_2 which is of the following form:

$$\begin{aligned} C &= C_0 = \{(s_0, 0), (s_1, 0), ..., (s_{p-2}, 0), (s_{p-1}, m_1\beta h)\}, \\ C_1 &= \{(s_0, 0), (s_1, 0), ..., (s_{p-2}, m_1\beta h), (s_{p-1}, 0)\}, \\ & \ldots\ldots\ldots\ldots\ldots\ldots \\ & \ldots\ldots\ldots\ldots\ldots\ldots \\ C_{p-2} &= \{(s_0, 0), (s_1, m_1\beta h), ..., (s_{p-2}, 0), (s_{p-1}, 0)\} \text{ and} \\ C_{p-1} &= \{(s_0, m_1\beta h), (s_1, 0), ..., (s_{p-2}, 0), (s_{p-1}, 0)\}. \end{aligned}$$

Now we can define a function f from A to C in such a manner that

$$f(A_0) = C_0, f(A_1) = C_1, \text{...and } f(A_{p-1}) = C_{p-1}.$$

Hence f is one to one and onto.

Thus the theorem follows. ■

Theorem 2.13 *The set of co-atoms of F_m has one to one correspondence to the set of co-atoms of another fuzzy magnified Boolean algebra with the same I_{2^p-1}.*

Proof. Let $S = \{s_0, s_1, s_2, ..., s_{p-1}\}$ with p elements. Since, the co-atoms of F_m are the fuzzy subsets containing exactly one zero membership element, so set of all co-atoms C(say) of a fuzzy Boolean algebra under fuzzy magnification B_1 is of the following form:

$$\begin{aligned} C &= C_0 = \{(s_0, m\beta h), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1}, 0)\}, \\ C_1 &= \{(s_0, m\beta h), (s_1, m\beta h), ..., (s_{p-2}, 0), (s_{p-1}, m\beta h)\}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_{p-2} &= \{(s_0, m\beta h), (s_1, 0), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\} \text{ and} \\ C_{p-1} &= \{(s_0, 0), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\}. \end{aligned}$$

Similarly in I_{2^p-1} we can get another set of co-atoms D(say) of another fuzzy Boolean algebra under fuzzy magnification B_2 which is of the form :

$$\begin{aligned} D &= D_0 = \{(s_0, m\beta h), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1}, 0)\}, \\ D_1 &= \{(s_0, m\beta h), (s_1, m\beta h), ..., (s_{p-2}, 0), (s_{p-1}, m\beta h)\}, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ D_{p-2} &= \{(s_0, m\beta h), (s_1, 0), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\} \text{ and} \\ D_{p-1} &= \{(s_0, 0), (s_1, m\beta h), ..., (s_{p-2}, m\beta h), (s_{p-1}, m\beta h)\}. \end{aligned}$$

Now we may define a function f from C to D such that

$$f(C_0) = D_0, f(C_1) = D_1, ... \text{ and } f(C_{p-1}) = D_{p-1}.$$

Hence f is one to one and onto.
Thus the proof of the theorem is established. ■

References

[1] C. Byliński: Functions from a set to a set, Formalized Mathematics, Vol. 1, No.1(1990), pp. 153-164 .

[2] S. K. Datta: Fuzzy magnified translation in rings, International Journal of Pure and Applied Mathematics, Vol. 50, No.4(2009), pp. 523-528.

[3] M. Dhar: Fuzzy sets towards forming boolean algebra, International Journal of energy, Information and Communication, Vol. 2, No.4 (2011).

[4] Dubois and J. Didier: Fuzzy sets and systems: theory and applications, Vol. 144(1980), Access Online via Elsevier.

[5] O. Frink : Ideals in partially ordered sets, The American Mathematical Monthly,Vol. 61,No.4 (1954), pp. 223-234.

[6] S.Givant and P. Halmos: Introduction to Boolean algebras, Undergrad, Texts Math. Springer, New York (2009).

[7] Kandasamy and W.B. Vasantha: Samarandache fuzzy algebra, American Research Press,Rehoboth (2003), pp.151-154.

[8] S.K. Majumder and S.K. Sarder: On fuzzy magnified translation, Communicated.

[9] J. D.Monk and R.Bonnet: Handbook of Boolean algebras Vol. 3 (1989), North Holland.

[10] T. J. Neog and D. K. Sut: Complement of an extended fuzzy Set, IJCA , Vol. 29, No.3 (2011).

[11] S. K. Rajbongshi and D. Talukdar: Some aspects of fuzzy Boolean algebras formed by fuzzy subsets, IJARCSSE, Vol.3, No.7(2013), pp.1-8.

[12] S.K.Rajbongshi: Some aspects of isomorphic fuzzy Boolean rings in the set of fuzzy subsets of a finite set (2017).

[13] K. H. Rosen and K.Krithivasan: Discrete mathematics and its applications ,Vol. 6, New York: McGraw-Hill (1999).

[14] R. J. Squires: An introduction to Boolean algebra, Journal of the Institute of Actuaries Students Society Vol.17(1964), pp. 314.

[15] L.A. Zadeh: Fuzzy sets, Information and Control, Vol.8(1965), pp. 338-353.