

# Effect of double dispersion on hydromagnetic convective heat and mass transfer flow past a vertical wavy wall with variable viscosity thermal conductivity, thermal radiation, Chemical reaction in the presence of non-uniform heat sources

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**Abstract :** The aim of this paper is to investigate effect of magnetic field and heat sources on double dispersion effects on mixed convective boundary layer flow from a vertical wavy surface embedded in a fluid saturated porous medium with variable properties.

**Keywords:** Non-Uniform heat sources, heat and mass transfer, vertical wavy wall, thermal conductivity, chemical reaction.

## 1. INTRODUCTION:

The flow, heat and mass transport in a fluid saturated porous medium are often occurred in many engineering processes, natural environments and geophysical applications such as seepage of water through river bed, migration of pollutants into the soil and aquifers and flow of moisture through porous industrial materials etc. A good review of convective heat and mass transfer flows in a fluid saturated Darcy porous medium is given by Nield and Bejan [21]. In recent years, a great deal of interest has been generated in the study of convective boundary layer flow over a vertical wavy surface because of its enormous applications in engineering and industry like grain storage container where walls are buckled, condensation process, heat transfer devices such as flat plate condensers and flat plate solar collectors in refrigerators and so on. Several researchers have reported the characteristics of heat and mass transfer over a wavy surface in a fluid saturated porous medium. Maria [17] studied natural convective flow over a vertical wavy surface in non-Darcy porous medium with heat and mass flux conditions. Narayana et.al [20] studied cross-diffusion effects on heat and mass transfer flow past a horizontal wavy surface in a porous medium. Ahmed and Aziz [1] investigated steady and unsteady effects on Darcy free convection of a nanofluid over a vertical wavy surface. Most of the studies reported in the literature have assumed the fluid properties are constant. The variable viscosity and thermal conductivity effects on flow characteristics through porous media have been more important in industrial and engineering fields such as ground water pollution, crude oil extraction and geothermal systems, etc. Rashad [25] studied variable viscosity effect on unsteady MHD flow of a rotating fluid over a stretching surface in a porous medium with thermal radiation. Recently, Srinivasacharya et.al [28], [30] studied variable properties and cross diffusion effects on mixed convective flow over a vertical wavy surface in a fluid saturated porous medium and also discussed with Dispersion Effects on Mixed Convection over a Vertical Wavy Surface in a Porous Medium with Variable Properties.

The hydrodynamic mixing is called dispersion, which is a secondary effect of a porous medium on the fluid flow takes place in the result of mixing and recirculation of local fluid particles through tortuous paths formed by the porous medium solid particles. There has been renewed interest in studying double diffusive convection due to the effect of thermal and solutal dispersions; these are additional energy and concentration mass transport process. In certain thermal and solutal dispersion applications such as those involving oil reservoir and geothermal engineering applications such as ceramic processing, sensible heat storage beds and petroleum recovery etc.

A brief review on double dispersion (thermal and solutal dispersion) effects can be found in Bear [3] and Nield and Bejan [21]. Kvernold and Tyvand [11] investigated thermal dispersion effects and they compared and found better agreement between the experimental and theoretical results. The analysis on thermal and solutal dispersion is dealt with at length in works by Cheng [6], Telles and Trevisan [31], Ramasubba Reddy et. al [23], Murthy and Sing [19], Hassenien et.al. [8], Murthy [18], El-Hakiem [7], Chamkha et.al [5], El-Emin [2], Khaled and Vafai [10]. Recently Ibrahim et.al [9], Srinivasacharya et.al [29], and Ramreddy [24], Mallikarjuna [16] investigated thermal and solutal dispersion effects on Darcy and non-Darcy convective heat and mass transfer flow over a different geometries in a porous medium. Recently, Srinivasacharya et al [30] studied dispersion effects on mixed convection over a vertical wavy surface in a porous medium with variable properties .

In all the above mentioned work, double dispersion effects on convective heat and mass transfer flow along a vertical wavy surface is not analyzed. Hence, the aim of this paper is to investigate effect of magnetic field and heat sources on double dispersion effects on mixed convective boundary layer flow from a vertical wavy surface embedded in a fluid saturated porous medium with variable properties.

**2. FORMULATION OF THE PROBLEM:**

We consider a steady incompressible two-dimensional laminar natural convective heat and mass transfer flow over a vertical wavy surface embedded in a saturated porous medium. The porous medium is uniform and local thermal equilibrium with the fluid. The Darcy law is used to describe the fluid saturated porous medium. The fluid is assumed to be gray, absorbing-emitting radiation but non-scattering medium.

The wavy surface profile is given by

$$y = \bar{\sigma}(\bar{x}) = \bar{a} \sin\left(\frac{\pi \bar{x}}{l}\right) \tag{1}$$

where  $l$  is the characteristic length of wavy surface and  $\bar{a}$  is the amplitude of the wavy surface. The wavy surface is maintained at constant temperature  $T_w$  which are higher than the ambient fluid temperature  $T_\infty$ .

We consider the hydromagnetic natural convection-radiation flow in the presence of non-uniform heat sources to be governed by the following equations under Boussinesq approximations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\frac{\partial}{\partial y} \left( \frac{\mu}{k} \bar{u} \right) - (\sigma \mu_e^2 H_o^2) \frac{\partial u}{\partial y} = \frac{\partial}{\partial x} \left( \frac{\mu}{k} \bar{v} \right) + \rho g (\beta_T \frac{\partial T}{\partial y} + \beta_c \frac{\partial C}{\partial y}) \tag{3}$$

$$\bar{u} \frac{\partial T}{\partial x} + \bar{v} \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} (\alpha_x \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (\alpha_y \frac{\partial T}{\partial y}) + \frac{1}{C_o} q''' - \frac{1}{C_p} \nabla \cdot q_r \tag{4}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{\partial}{\partial x} (D_x \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (D_y \frac{\partial T}{\partial y}) \tag{5}$$

The relevant boundary conditions are

$$\bar{u} = 0, \bar{v} = 0, T = T_w, C = C_w \text{ at } \bar{y} = \bar{\sigma}(\bar{x}) = \bar{a} \sin\left(\frac{\pi \bar{x}}{l}\right) \tag{6}$$

$$\bar{u} \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } \bar{y} \rightarrow \infty$$

where  $\bar{u}$  and  $\bar{v}$  are the velocity components in the directions of  $x$  and  $y$  respectively  $T, C$  are temperature, Concentration respectively  $\rho$  is the density of the fluid  $\mu$  is the dynamic viscosity of the fluid is the permeability of the porous medium  $\sigma$  is the electrical conductivity  $\mu_e$  is the magnetic permeability,  $H_o$  is the strength of the magnetic field,  $D_B$  is the molecular diffusivity  $k_c$  is the coefficient of chemical reaction  $\beta_T$  are the coefficients of thermal expansion  $\beta_c$  is the volumetric coefficient of mass fraction,  $q_r$  is the radiative heat flux  $g$  is the acceleration due to gravity and  $q'''$  is the non-uniform heat source,  $K_T$  is the thermal diffusion ratio,  $T_m$  is the mean fluid temperature and  $k_c$  is the chemical reaction coefficient.  $\alpha_x, D_x$  and  $\alpha_y, D_y$  are the effective thermal and solutal diffusivities respectively, have the contributions of both molecular diffusion and hydrodynamic dispersion, these can be described as (Telles and Trevisan [31])

$$\alpha_x = \alpha + \gamma \bar{v}, \alpha_y = \alpha + \gamma \bar{u} \tag{7}$$

$$D_x = D + \zeta \bar{v}, D_y = D + \zeta \bar{u}$$

Where  $\alpha$  is the thermal conductivity,  $D$  is the molecular diffusivities,  $\gamma$  is the coefficient of thermal dispersion and  $\zeta$  is the solutal dispersion.

By applying Rosseland approximation (Brewster [4]) the radiative heat flux  $q_r$  is given by

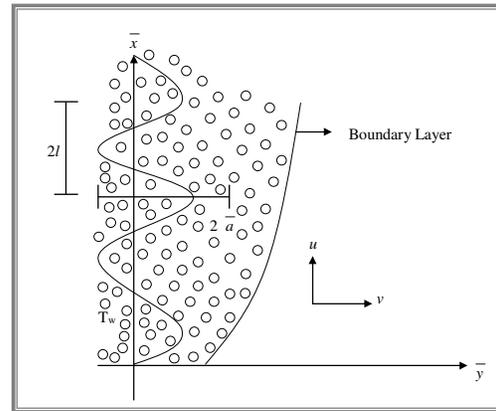


Fig. - 1 : Physical Configuration and Co-ordinate System

$$q_r = -\left(\frac{4\sigma^*}{3\beta_R}\right)\frac{\partial}{\partial y}[T'^4] \quad (8)$$

Where  $\sigma^*$  is the Stephan – Boltzmann constant and mean absorption coefficient.

Assuming that the difference in temperature within the flow are such that  $T'^4$  can be expressed as a linear combination of the temperature. We expand  $T'^4$  in Taylor's series about  $T_e$  as follows

$$T'^4 = T_\infty^4 + 4T_0^3(T - T_0) + 6T_0^2(T - T_0)^2 + \dots \quad (9)$$

Neglecting higher order terms beyond the first degree in  $(T - T_\infty)$ , we have

$$T'^4 \cong -3T_0^4 + 4T_0^3T \quad (10)$$

Differentiating equation (6) with respect to  $y$  and using (7) we get

$$\frac{\partial(q_R)}{\partial y} = -\frac{16\sigma^*T_0^3}{3\beta_R}\frac{\partial^2 T}{\partial y^2} \quad (11)$$

On using equations(11) in the last term of equation(4) we get

$$\begin{aligned} \bar{u}\frac{\partial T}{\partial \bar{y}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{\partial}{\partial \bar{x}}((\alpha + \gamma tv)\frac{\partial T}{\partial \bar{x}}) + \frac{\partial}{\partial \bar{y}}((\alpha + \gamma tu)\frac{\partial T}{\partial \bar{y}}) + \frac{1}{C_p}q''' \\ + \frac{16\sigma^*T_0^3}{C_p\beta_R}\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right) \end{aligned} \quad (12)$$

The coefficient  $q'''$  is the rate of internal heat generation (>0) or absorption (<0). The internal heat generation /absorption  $q'''$  is modelled as

$$q''' = \left(\frac{k_f u_s}{x\nu}\right)(T_w - T_\infty)(A_1 u + B_1\left(\frac{T - T_\infty}{T_w - T_\infty}\right))$$

where  $A_1$  and  $B_1$  are coefficients of space dependent and temperature dependent internal heat generation or absorption respectively. It is noted that the case  $A_1 > 0$  and  $B_1 > 0$ , corresponds to internal heat generation. The fluid properties are assumed to be constant except fluid viscosity and thermal conductivity. Therefore we assume that the viscosity of the fluid is to be an inverse function of the temperature and it can be expressed as [Lai and Kulacki [12]]

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}(1 + \delta(T - T_\infty)) \text{ or } \frac{1}{\mu} = b((T - T_\infty)) \quad (13)$$

where  $b = \frac{\delta}{\mu_\infty}$  and  $T = T_\infty - \frac{1}{\delta}$ . Both  $b$  and  $T_r$  are constants and their values depend on the reference state

and the thermal property of the fluid i.e.  $\delta$ . In general  $b > 0$  for liquids and  $b < 0$  for gases.  $\theta_r$  which is defined by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\delta(T_w - T_\infty)} \quad (14)$$

is constant. The parameter  $\theta_r$  was first introduced by Ling and Dybbs. It is important to note that for  $\delta \rightarrow 0$  (i.e.  $\mu = \mu_\infty = \text{constant}$ ) then  $\theta_r \rightarrow \infty$  the effect of viscosity is negligible. The value of  $\theta_r$  is determined by the temperature difference  $(T_w - T_\infty)$  and viscosity  $\delta$  of the fluid in consideration. A smaller values of  $\theta_r$  implies either the fluid viscosity changes considerably or the temperature difference is high. On the otherhand for a larger values of  $\theta_r$  implies either  $(T_w - T_\infty)$  or  $\delta$  is small and therefore the effects of variable viscosity can be neglected. In either case the influence of variable viscosity plays very important role and the liquid viscosity varies differently with temperature than that of gases. Therefore  $\theta_r$  is positive for gases and negative for liquids respectively.

Also we assume that the fluid thermal conductivity  $\alpha$  is to be varying as a linear function of temperature in the form [Seddeek and Salem[26]]

$$\alpha = \alpha_o(1 + E(T - T_\infty))$$

where  $\alpha_o$  is the thermal diffusivity at the wavy surface temperature  $T_w$  and  $E$  is a constant depending on the nature of the fluid. It is worth mentioning here that  $E$  is positive for fluids such as air and  $E$  is negative for fluids such as lubrication oils.

This can be written in the non-dimensional form [Slattery[27]] as

$$\alpha = \alpha_o (1 + \beta\theta) \quad (15)$$

Where  $\beta = E(T - T_\infty)$  is the thermal conductivity parameter. The variation of  $\beta$  can be taken in the range  $-0.1 \leq \beta \leq 0$  for lubrication oils  $0 \leq \beta \leq 0.12$  for water and  $0 \leq \beta \leq 6$  for air.

In view of the continuity equation(2) we define the stream function  $\psi$  as

$$\bar{u} = \frac{\partial \bar{\psi}}{\partial y}, \quad v = -\frac{\partial \bar{\psi}}{\partial x} \quad (16)$$

In order to write the governing equations in the dimensionless form we introduce the following non-dimensional variables as

$$x = \frac{\bar{x}}{l}, y = \frac{\bar{y}}{l}, a = \frac{\bar{a}}{l}, \sigma = \frac{\bar{\sigma}}{l}, \psi^* = \frac{\bar{\psi}}{l}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad (17)$$

The equations (12)-(15).equations(2).(3) and (12) reduce to

$$\left(\frac{1}{\theta - \theta_r}\right) \left(\frac{\partial \theta}{\partial y} \frac{\partial \psi^*}{\partial y} - \frac{\partial \theta}{\partial x} \frac{\partial \psi^*}{\partial x}\right) + \left(\frac{\partial^2 \psi^*}{\partial x^2} + \frac{\partial^2 \psi^*}{\partial y^2}\right) + \quad (18)$$

$$+ Ra \left(1 - \frac{\theta}{\theta_r}\right) \left(\frac{\partial \theta}{\partial y} + N_r \frac{\partial \phi}{\partial y}\right) - M^2 \frac{\partial^2 \psi^*}{\partial y^2}$$

$$\left(\frac{\partial \theta}{\partial x} \frac{\partial \psi^*}{\partial y} - \frac{\partial \theta}{\partial y} \frac{\partial \psi^*}{\partial x}\right) = \left(1 + \beta\phi + \frac{4Rd}{3}\right) \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + \beta \left(\left(\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2\right) \quad (19)$$

$$+ \left(A_1 \frac{\partial \psi^*}{\partial y} + B_1 \theta\right) + \frac{\gamma d}{l} \left(\frac{\partial \theta}{\partial x} \frac{\partial^2 \psi^*}{\partial y^2} - \frac{\partial \theta}{\partial y} \frac{\partial^2 \psi^*}{\partial x^2} - \frac{\partial^2 \theta}{\partial x^2} \frac{\partial \psi^*}{\partial y} + \frac{\partial^2 \theta}{\partial y^2} \frac{\partial \psi^*}{\partial x}\right)$$

$$Le \left(\frac{\partial \phi}{\partial x} \frac{\partial \psi^*}{\partial y} - \frac{\partial \phi}{\partial y} \frac{\partial \psi^*}{\partial x}\right) = \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) + \frac{\zeta d}{l} \left(\frac{\partial \phi}{\partial x} \frac{\partial^2 \psi^*}{\partial y^2} - \frac{\partial \phi}{\partial y} \frac{\partial^2 \psi^*}{\partial x^2} - \frac{\partial^2 \phi}{\partial x^2} \frac{\partial \psi^*}{\partial y} + \frac{\partial^2 \phi}{\partial y^2} \frac{\partial \psi^*}{\partial x}\right) \quad (20)$$

Where  $Ra_d = \frac{gK\beta_r(T_w - T_\infty)d}{\alpha_o\nu}$  is the pore diameter dependent Rayleigh number which describes the

relative intensity of the buoyancy force, such that  $d$  is the pore diameter.  $\nu = \frac{\mu_\infty}{\rho}$  is the kinematic viscosity of

the fluid,  $Rd = \frac{4\sigma^*T_\infty^3}{k_f\beta_R}$  is the Radiation parameter  $Q = \frac{Q_H l^2}{\alpha_o C_p}$  is heat source parameter  $Q_0 = Q_H x^{-1}$ ,

$M^2 = \frac{\sigma\mu_e^2 H_0^2 l^2}{\mu}$  is the magnetic parameter.  $Le = \frac{\nu}{D_B}$  is the Lewis number.  $N = \frac{\beta_c (C_w - C_\infty)}{\beta_0 (T_w - T_\infty)}$

is the buoyancy ratio

The transformed boundary conditions are

$$\psi^* = 0, \theta = 1, \phi = 1 \quad \text{at } y = a \sin(x)$$

$$\frac{\partial \psi^*}{\partial y} \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow \infty \quad \text{as } y \rightarrow \infty \quad (21)$$

### 3.METHOD OF SOLUTION:

We can transform the effect of wavy surface from the boundary conditions into the governing equations by using suitable coordinate transformation with boundary layer scaling for the case of free convection. The Cartesian coordinates(x,y) are transformed into the new variables( $\xi, \eta$ ).

We incorporate the effect of effect of wavy surface and the usual boundary layer scaling into the governing equations(16)-(19) for free convection using the transformations and  $Ra \rightarrow \infty$  (i.e boundary layer approximation)

$$x = \xi, \hat{\eta} = \frac{y - a \sin(x)}{\xi^{1/2} Ra^{-1/2}}, \psi^* = Ra^{1/2} \psi \quad (22)$$

These transformations are similar to those presented in for instance Rees and Pop[17]. We obtain the following boundary layer equations :

$$\left(\frac{1}{\theta - \theta_r}\right)(1 + a^2 \cos^2 \xi) \frac{\partial \theta}{\partial \eta} \frac{\partial \psi}{\partial \eta} + (1 + a^2 \cos^2 \xi) \frac{\partial^2 \psi}{\partial \eta^2} = Ra \xi^{1/2} \quad (23)$$

$$\left(1 - \frac{\theta}{\theta_r}\right) \left(\frac{\partial \theta}{\partial \eta}\right) + N \frac{\partial \phi}{\partial \eta} - M^2 \frac{\partial^2 \psi}{\partial \eta^2}$$

$$\xi^{1/2} \left(\frac{\partial \theta}{\partial \xi} \frac{\partial \psi^*}{\partial \eta} - \frac{\partial \theta}{\partial \eta} \frac{\partial \psi^*}{\partial \xi}\right) = (1 + a^2 \cos^2(\xi)) \left(1 + \beta \theta + \frac{4Rd}{3}\right) \left(\frac{\partial^2 \theta}{\partial \eta^2}\right) + \beta \left(\frac{\partial \theta}{\partial \eta}\right)^2 \quad (24)$$

$$+ (A_1 \frac{\partial \psi^*}{\partial y} + B_1 \theta + \frac{\gamma d}{l} \left(\frac{\partial \theta}{\partial \xi} \frac{\partial^2 \psi^*}{\partial \eta^2} - \frac{\partial \theta}{\partial \eta} \frac{\partial^2 \psi^*}{\partial \xi^2} - \frac{\partial^2 \theta}{\partial \xi^2} \frac{\partial \psi^*}{\partial \eta} + \frac{\partial^2 \theta}{\partial \eta^2} \frac{\partial \psi^*}{\partial \xi}\right))$$

$$\xi^{1/2} Le \left(\frac{\partial \phi}{\partial \xi} \frac{\partial \psi}{\partial \eta} - \frac{\partial \phi}{\partial \eta} \frac{\partial \psi}{\partial \xi}\right) = (1 + a^2 \cos^2(\xi)) \left(\frac{\partial^2 \phi}{\partial \eta^2}\right) + \frac{\zeta d}{l} \quad (25)$$

$$\left(\frac{\partial \phi}{\partial \xi} \frac{\partial^2 \psi^*}{\partial \eta^2} - \frac{\partial \phi}{\partial \eta} \frac{\partial^2 \psi^*}{\partial \xi^2} - \frac{\partial^2 \phi}{\partial \xi^2} \frac{\partial \psi^*}{\partial \eta} + \frac{\partial^2 \phi}{\partial \eta^2} \frac{\partial \psi^*}{\partial \xi}\right)$$

We now introduce the following similarity variables as

$$\eta = \frac{\bar{\eta}}{(1 + a^2 \cos^2(\xi))}, \psi = \xi^{1/2} f(\eta), \theta = \theta(\eta), \phi = \phi(\eta)$$

In equations(23)-(25) we obtain a system of ordinary differential equations as follows:

$$f'' + \left(\frac{1}{\theta - \theta_r}\right) \theta' f' - \frac{M^2}{(1 + a^2 \cos^2 \xi)} f'' = Ra \left(1 - \frac{\theta}{\theta_r}\right) (\theta' + N_r \phi') \quad (26)$$

$$\beta (\theta')^2 + \left(1 + \beta \theta + \frac{4Rd}{3}\right) \theta'' + D_s \frac{(1 + a^3 \cos^3(\xi))}{(1 + a^2 \cos^2(\xi))} (f'' \theta' + f' \theta'') + \quad (27)$$

$$+ \frac{1}{(1 + a^2 \cos^2(\xi))} (A_1 f' + B_1 \theta) = 0$$

$$\phi'' + \frac{Le}{2} f \phi' + D_c \frac{(1 + a^3 \cos^3(\xi))}{(1 + a^2 \cos^2(\xi))} (f'' \phi \theta + f' \phi'') = 0 \quad (28)$$

where prime denotes differentiation with respect to  $\eta$ .  $D_s = \gamma Ra_d$  is the thermal dispersion parameter,

$D_c = \zeta Ra_d$  is the solutal dispersion parameter.

The corresponding boundary conditions are

$$f = 0, \theta = 1, \phi = 1 \text{ at } \eta = 0, \quad f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (29)$$

In equation(3.6) the radiation parameter  $Rd = \frac{4\sigma^* T_\infty^3}{k_f \beta_R}$  means that the rate of thermal radiation contribution

relative to the thermal conditions. As  $Rd \rightarrow \infty$  influence of thermal radiation is high in the boundary layer

regime. For  $Rd \rightarrow 0$  the term  $4Rd/3$  tends to zero. For  $Rd=1$  thermal radiation and thermal conduction will give equal contribution.

From equations(27) and(28),we say that in natural convection due to vertical wavy surface in a fluid saturated porous medium,the field variable,flow,heat and mass transfer characteristics are not similar because the  $\xi$ -coordinate can not be eliminated. However,we found the local non-similarity solutions for some convective boundary layer flows dealing with Darcy porous medium,the technique is more complex to extend in this case.Hence ,for case of analysis,it is decided to proceed with evaluating local similarity solutions for the equations(26)-(28).For that purpose we take  $\xi=x/l$  and then vary  $\xi$ -location to study the influence of various parameters.

The associated boundary conditions are

$$\begin{aligned} f = 0, \theta = 1 \text{ and } \phi = 1 \text{ at } \eta = 0 \\ f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \tag{30}$$

**4. SKIN FRICTION,NUSELT and SHERWOOD NUMBER**

The main results of practical interest in many applications are Skin friction coefficient , heat transfer coefficient ,mass transfer coefficient at the surface.

The Skin friction coefficient (Cf) is given by

$$C_f = \frac{f''(0)(1+a^2 \cos^2(\xi))Ra^{1/2}}{(\Gamma+M^2+a^2 \cos^2(\xi))} \tag{31}$$

The heat and mass transfer coefficients are expressed in terms of Nusselt and Sherwood numbers  $Nux$   $Shx$ .

Nusselt number  $Nu_\xi$  and Sherwood number  $Sh_\xi$  are given by

$$Nu_\xi = -(1+Ds) \frac{f'(0)}{(1+a^2 \cos^2(\xi))} - \frac{\theta'(0)Ra_\xi^{1/2}}{(1+a^2 \cos^2(\xi))}, \quad Sh_\xi = -(1+Dc) \frac{f'(0)}{(1+a^2 \cos^2(\xi))} - \frac{\phi'(0)Ra_\xi^{1/2}}{(1+a^2 \cos^2(\xi))} \tag{32}$$

**5. COMPARISON**

**Table. 1** - In the absence of non-uniform heat sources( $A1=0$  &  $B1=0$ ),and magnetic field( $M=0$ ) the results are in good agreement with *Mallikarjuna* [16]

Parameters				$NuxRa^{-1/2}$	$ShxRa^{-1/2}$	$NuxRa^{-1/2}$	$ShxRa^{-1/2}$
$\theta_r$	$\beta$	$Ds$	$Dc$	<i>Mallikarjun I</i> [16]		Present Results	
-1.5	0.5	0.3	0.3	0.52456	0.65234	0.52452	0.65233
-2	0.5	0.3	0.3	0.48766	0.60454	0.48760	0.60456
-3	0.5	0.3	0.3	0.46578	0.58769	0.46572	0.58767
-1.5	0.1	0.3	0.3	0.61234	0.67890	0.61232	0.67889
-1.5	0.3	0.3	0.3	0.46784	0.60567	0.46780	0.60569
-1.5	0.5	0.1	0.3	0.50766	0.57555	0.50767	0.57558
-1.5	0.5	0.5	0.5	0.65456	0.67896	0.65458	0.67899
-1.5	0.5	0.3	0.1	0.47865	0.60674	0.47866	0.60676
-1.5	0.5	0.3	0.5	0.58976	0.71234	0.58979	0.71232

**6. RESULTS AND DISCUSSION:**

The aim of this analysis is to investigate the combined influence of double dispersion effects along with variable properties on the flow characteristics .

The variation of the non-dimensional velocity,temperature and concentration profiles with  $\eta$  for different values of thermal dispersion( $Ds$ ), solutal dispersion( $Dc$ ), temperature dependent viscosity parameter( $\theta_r$ ), thermal conductivity parameter( $\beta$ ), radiation parameter( $Rd$ ), heat source parameters( $A1,B1$ ), Buoyancy parameter( $Nr$ ) magnetic parameter( $M$ ), Rayleigh number( $Ra$ )Lewis parameter( $Le$ ) ,surface amplitude of the wavy surface( $a$ ) and stream wise coordinate ( $\xi$ ) are presented in figs.14a-14c.

Figs.2a-2c illustrate the variation of velocity temperature and concentration with Rayleigh number( $Ra$ ).It can observed from the profiles that the axial velocity enhances in the flow region. An increase in  $Ra$  reduces the temperature and concentration .This may be attributed to the fact that the thickness of the thermal and solutal boundary layers reduce with increasing values of  $Ra$ .

Fig.3a-3c depict the variation of velocity temperature and concentration with magnetic parameter( $M$ ).From the profiles we find that the velocity reduce ,The temperature and concentration enhances with increasing values of magnetic parameter.This is due to the fact that in increase in  $M$  grows the thickness of the thermal and solutal boundary layer.

Figs.4a-4c depict the variation of velocity temperature and concentration with buoyancy ratio( $Nr$ ).It can be seen from the profiles that when the molecular buoyancy force dominates over the thermal buoyancy force the velocity enhances in the region when the buoyancy forces are in the same direction and for the forces acting in opposite directions it reduces in the region(fig.4a). The temperature and concentration reduce with increase in  $Nr>0$  and for  $Nr<0$ ,the temperature and the concentration enhance in the flow region(figs.4b&c) .

Figs.5a-5c show the variation of velocity temperature and concentration with the influence of radiation parameter( $Rd$ ).From fig.5a we find that the velocity reduces in the flow region (0,1.0) and enhances in the region(1.0,4.0). This means that the thickness of the momentum boundary layer reduces in the narrow region adjacent to the boundary and enhances far away from the wall with increasing values of  $Rd$ . Fig.5b &5c represent the temperature and concentration with  $Rd$ . It can be seen from the profiles that an increase in  $Rd$  leads to a growth in thickness of the thermal boundary layer and the solutal boundary layer which results in an enhancement of the temperature and concentration in the flow region.

From 6a-6c represent the effect of thermal conductivity parameter  $\beta$  on the non-dimensional velocity temperature and concentration.Fig.6a shows the variation of velocity with  $\beta$ .In this case the velocity is found to reduce in the flow region (0,0.5) and enhances in the region (0.5,4.0).From fig.6b &6c we find that as the thermal conductivity parameter  $\beta$  increases the temperature and the concentration decreases. This is due to the thickening of the thinning of the thermal and solutal boundary layers as a result of increasing values of thermal conductivity.

The variation of non-dimensional velocity temperature and concentration profiles with  $\eta$  for different values of temperature dependent viscosity parameter ( $\theta_r$ ) is illustrated in figs.7a-7c.It is found that from fig.7a that the velocity of the fluid reduces in the flow region. This can be explained physically as the parameter  $\theta_r$  increases there is decay in the boundary layer thickness. From fig.7b&7c we notice that the temperature and the concentration profiles increase with increasing values of  $\theta_r$ . This can be attributed to the fact the an increase in  $\theta_r$  increases the thickness of the thermal and the thickness of the solutal boundary layer which results in an enhancement of the temperature and enhancement in the concentration in the flow region.

Figs.8a-8c represent  $u$ ,  $\theta$  and  $\phi$  with space dependent heat source parameter( $A1$ ).From fig.8a we find that the an increase in the strength of the space dependent heat source reduces the velocity in the region abutting the wall,enhances in the region(0.5,4.0) and the Concentration reduce in the flow region(fig. 8a&8c).The temperature enhances with increase in the strength of the space dependent heat source(fig.8b). In the case of space dependent heat sink,the velocity enhances in the region(0,0.5) and reduces in the flow region(0.5,4.0),the temperature reduces, the concentration enhances with increase in  $A1<0$ (figs.8a-8c)

Figs.9a-9c represent the variation of  $u$ , $\theta$  and  $\phi$  with temperature dependent heat source parameter( $B1$ ).In the presence of heat generating source, the velocity enhances  $0n$  (0,0.5) and reduces in the flow region(0.5,4.0),the temperature and concentration reduce in the flow region. In the presence of heat absorbing source( $B1<0$ ) the velocity, temperature and concentration enhance in the entire flow region(figs.9a-9c).

Figs.10a-10c shows the variation of  $u$ , $\theta$  and  $\phi$  with thermal dispersion( $Ds$ ).It can be seen from the profiles that the velocity( $u$ ) reduces in  $u$  in the narrow region(0,0.5) and enhances in the region (0.5,4.0) far away from the wall .The temperature enhances and the concentration reduces with increasing  $g$  values of  $Ds$ (figs.10a-10c).

Figs.11a-11c exhibit the effect of solutal dispersion( $Dc$ ) on the flow variables . An increase in  $Dc$  reduces the velocity in a narrow region(0,0.5)and enhances in the remaining region(0.5,4.0).The temperature and concentration experience an enhancement with increasing values of  $Dc$ (figs.11a-11c).

An increase in Lewis number ( $Le$ ) increases the velocity in the region(0,1.0)and reduces in the flow region(1.1,4.0).The temperature enhances and the concentration reduces in the flow region .(figs.12a-12c)

Figs.13a-13c represent  $u$ ,  $\theta$  and  $\phi$  with surface amplitude of the wavy ' $a$ '. From fig.13a we find that an increase in amplitude reduces the velocity in the entire flow region .The temperature and concentration reduces with increase in ' $a$ ' in the entire flow region(figs.13b&13c).

Figs.13a-13c depict the variation of  $u$ ,  $\theta$  and  $\phi$  with stream wise coordinate ( $\xi$ ). It can be seen from the profiles that an increase in stream wise coordinate decreases the velocity while increases the temperature and concentration in the flow region.

The skin friction( $C_f$ )at the wall is represented in table.2 for different variations. From the tabular values we find that the magnitude of the skin friction enhances with increase in  $Ra$  . $|C_f|$  increases at the wall with increase in buoyancy ratio( $Nr>0$ ) and reduces with  $Nr<0$ . Higher the viscosity parameter( $\theta_r$ )or Lorentz force or higher thermal radiation( $Rd$ ) or thermal conductivity ( $\beta$ ) smaller the skin friction at the wall.Higher the strength of the temperature gradient heat source larger the magnitude of  $C_f$  at the wall while it reduces with increase in space dependent heat source( $A1>0$ ),enhances with that of heat sink( $A1<0$ ) at  $\eta=0$ . Increasing Thermal dispersion( $Ds$ )/solutal dispersion( $Dc$ ) leads to a reduction in  $C_f$  at the wall. Higher the Lewis number( $Le$ )or

amplitude of the wavy surface 'a', larger the skin friction at the wall. Also an increase in stream wise coordinate ( $\xi$ ), leads to a depreciation in the magnitude of  $C_f$  at the wall.

The rate of heat transfer (Nu) at the wall is displayed in table.2. From the tabular values we find that rate of heat transfer at the wall decreases with increase in Rayleigh number (Ra) and increases with magnetic parameter (M). When the molecular buoyancy force dominates over the thermal buoyancy force Nu reduces when the buoyancy forces are in the same direction while for the forces acting in opposite directions it enhances on the wall. The Nusselt number enhances with increase in viscosity parameter ( $\theta_r$ ). The variation of Nu with heat source parameter (Q) shows that Nu increases with increase in temperature gradient heat source. The Nusselt number reduces with increase in Rd or thermal conductivity parameter ( $\beta$ ). Increasing thermal dispersion (Ds) or solutal dispersion (Dc) leads to a reduction in Nu at the wall. The rate of heat transfer reduces with increase in  $A_1 > 0$  and enhances with  $B_1 > 0$  while it enhances with increase in  $A_1 < 0 / B_1 < 0$  at  $\eta = 0$ . An increase in Lewis number (Le) leads to a depreciation in the rate of heat transfer at the wall. The variation of Nu with amplitude of the wavy surface (a) and stream wise coordinate ( $\xi$ ) shows that Nu increases with increase in 'a' and decreases with ' $\xi$ '.

The rate of mass transfer (Sh) at the wall is displayed in table.2. From the tabular values we find that rate of mass transfer at the wall increases with increase in Rayleigh number (Ra) and reduces with magnetic parameter (M). The Sherwood number reduces with increase in viscosity parameter ( $\theta_r$ ) or radiation parameter (Rd) or thermal conductivity parameter ( $\beta$ ). The variation of Sh with heat source parameter ( $A_1, B_1$ ) shows that the rate of mass transfer at the wall enhances with increase in the strength of the temperature gradient heat source and reduces with that of space dependent heat source ( $A_1 > 0$ ). Nu reduces with absorbing source ( $B_1 < 0$ ) / space dependent sink ( $A_1 < 0$ ). Increasing thermal dispersion (Ds) leads to an enhancement in Nu while it reduces with that of solutal dispersion (Dc). An increase in Le enhances the rate of mass transfer at  $\eta = 0$ . The variation of Sh with amplitude of the wavy surface (a) and stream wise coordinate ( $\xi$ ) shows that Sh increases with increase in 'a' and decreases with ' $\xi$ '.

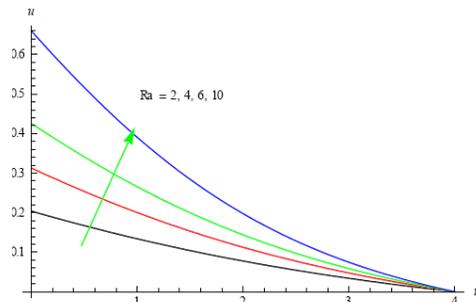


Fig.2a Variation of velocity (u) with Ra  
 $Ra=2.0, \theta_r=-2.0, \beta=0.5, A_1=0.2, B_1=0.2,$   
 $Nr=0.5, Rd=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

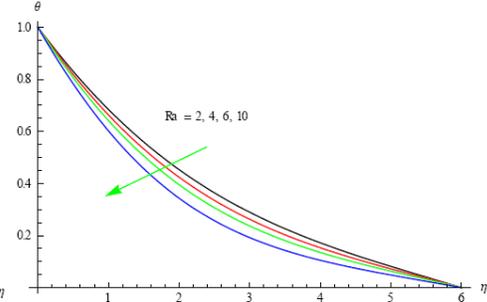


Fig.2b Variation of Temperature ( $\theta$ ) with Ra  
 $Ra=2.0, \theta_r=-2.0, \beta=0.5, A_1=0.2, B_1=0.2, Nr=0.5,$   
 $Rd=0.5, M=1.0, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

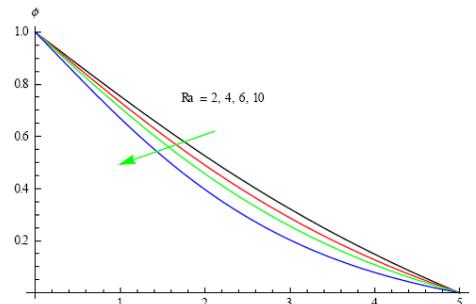


Fig.2c Variation of Concentration ( $\phi$ ) with Ra  
 $Ra=2.0, \theta_r=-2.0, \beta=0.5, A_1=0.2, B_1=0.2, Nr=0.5,$   
 $Rd=0.5, M=1.0, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

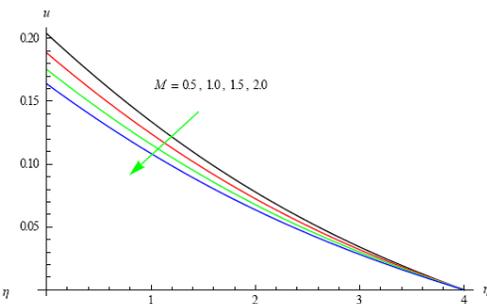


Fig.3a Variation of velocity (u) with M  
 $Ra=2.0, \theta_r=-2.0, \beta=0.5, A_1=0.2, B_1=0.2, Nr=0.5,$   
 $Rd=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

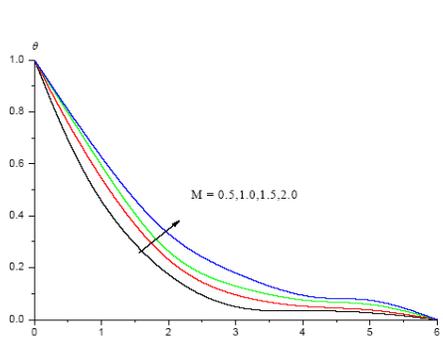


Fig.3b Variation of Temperature ( $\theta$ ) with M  
 $Ra=2.0, \theta_r=-2.0, \beta=0.5, A1=0.2, B1=0.2,$   
 $Nr=0.5, Rd=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

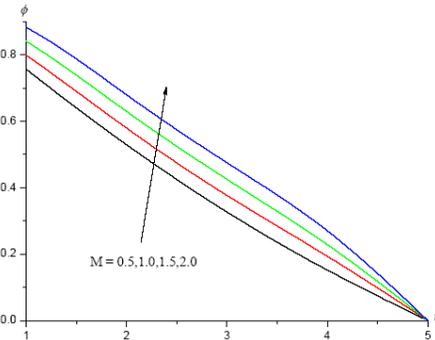


Fig.3c Variation of Concentration ( $\phi$ ) with M  
 $Ra=2.0, \theta_r=-2.0, \beta=0.5, A1=0.2, B1=0.2,$   
 $Nr=0.5, Rd=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

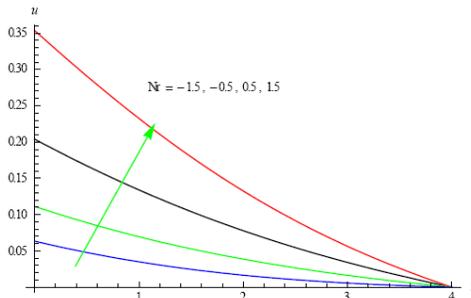


Fig.4a Variation of velocity ( $u$ ) with Nr  
 $Ra=2.0, \theta_r=-2.0, \beta=0.5, A1=0.2, B1=0.2,$   
 $Rd=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

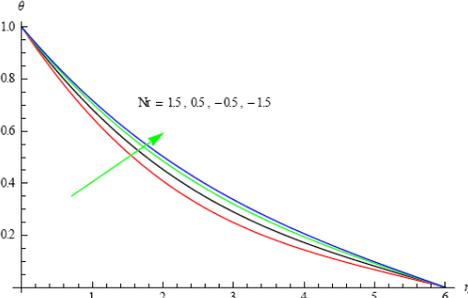


Fig.4b Variation of Temperature ( $\theta$ ) with Nr  
 $Ra=2.0, \theta_r=-2.0, \beta=0.5, A1=0.2, B1=0.2,$   
 $Rd=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

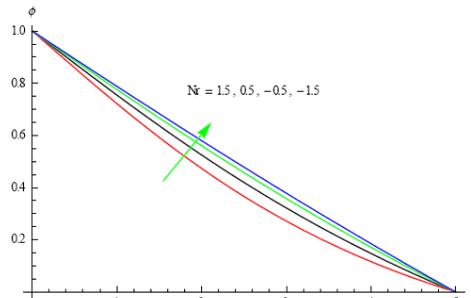


Fig.4c Variation of Concentration ( $\phi$ ) with Nr  
 $Ra=2.0, \theta_r=-2.0, \beta=0.5, A1=0.2, B1=0.2,$   
 $Rd=0.5, Nr=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

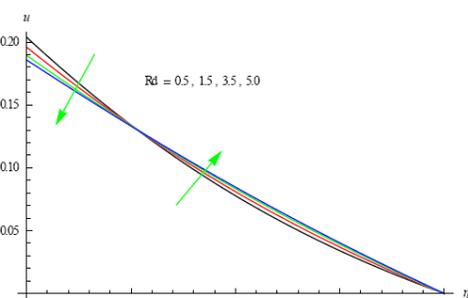


Fig.5a Variation of velocity ( $u$ ) with Rd  
 $Ra=2.0, M=1.0, Nr=1.0, \beta=0.5, A1=0.2, B1=0.2,$   
 $Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

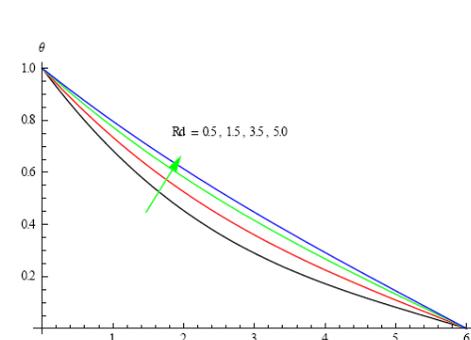


Fig.5b Variation of Temperature ( $\theta$ ) with Rd  
 $Ra=2.0, Nr=1.0, \beta=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

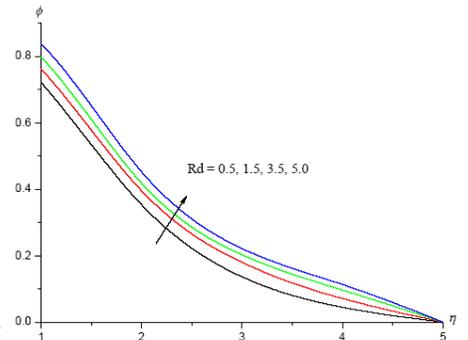


Fig.5c Variation of Concentration ( $\phi$ ) with Rd  
 $Ra=2.0, Nr=1.0, \beta=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

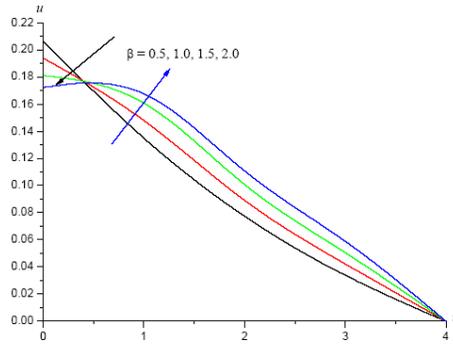


Fig. 6a Variation of velocity (u) with  $\beta$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2, M=1.0,$   
 $Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

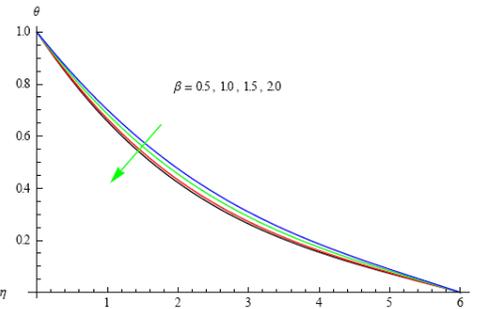


Fig. 6b Variation of Temperature ( $\theta$ ) with  $\beta$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

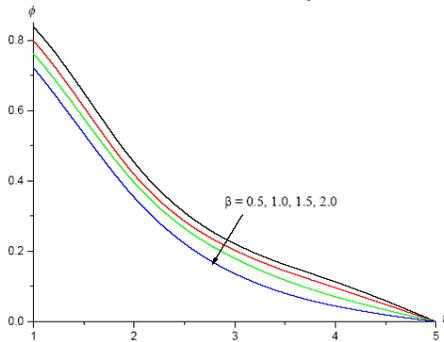


Fig. 6c Variation of Concentration ( $\phi$ ) with  $\beta$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

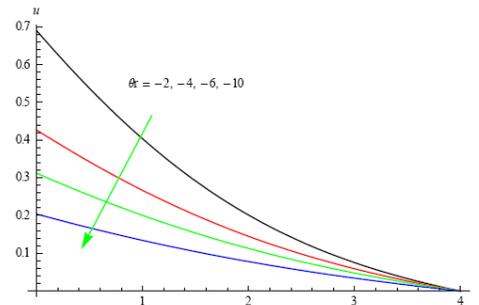


Fig. 7a Variation of velocity (u) with  $\theta_r$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

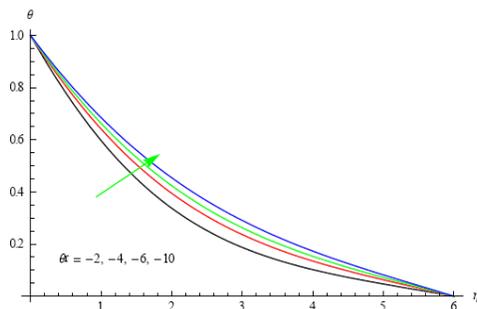


Fig. 7b Variation of Temperature ( $\theta$ ) with  $\theta_r$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

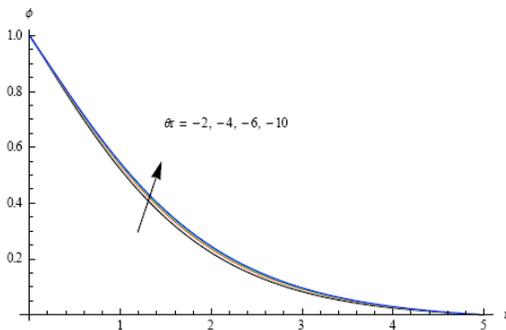


Fig. 7c variation of Concentration( $\phi$ )with  $\theta_r$   
 $M=0.5, Ra=2.0, Rd=0.5, \beta=0.5, Sc=0.24,$   
 $Ra=2, a=0.2, \xi=\pi/4$

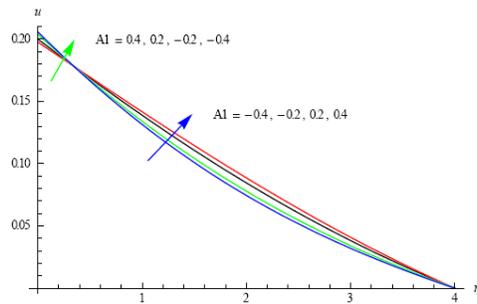


Fig. 8a Variation of velocity (u) with  $A1$   
 $Ra=2.0, Nr=1.0, Rd=0.5, B1=0.2,$   
 $M=1.0, \beta=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

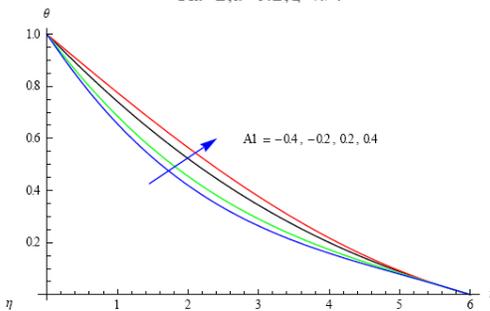


Fig. 8b Variation of Temperature ( $\theta$ ) with  $A1$   
 $Ra=2.0, Nr=1.0, Rd=0.5, B1=0.2,$   
 $M=1.0, \beta=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

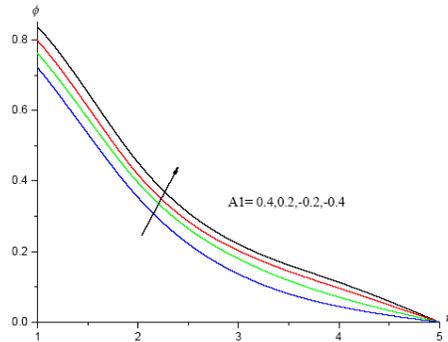


Fig.8c Variation of Concentration ( $\phi$ ) with A1  
 $Ra=2.0, Nr=1.0, Rd=0.5, B1=0.2, \beta=0.5,$   
 $M=1.0, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

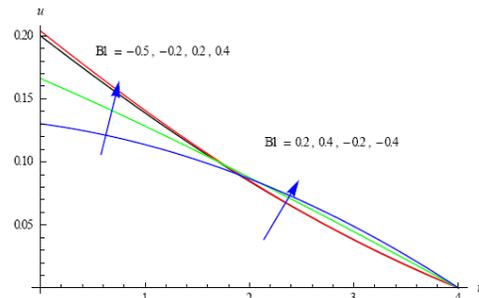


Fig.9a Variation of velocity ( $u$ ) with B1  
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, M=1.0,$   
 $\beta=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

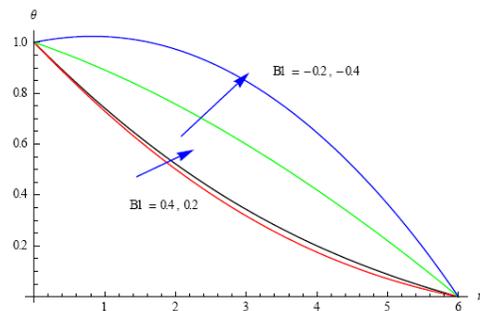


Fig.9b Variation of Temperature ( $\theta$ ) with B1  
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, M=1.0,$   
 $B=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

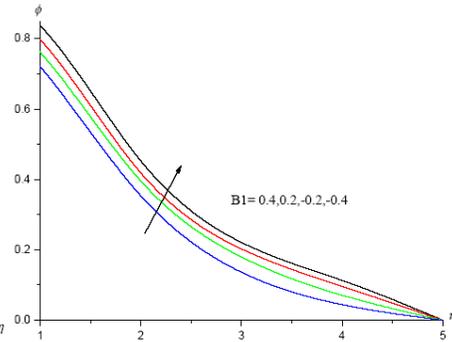


Fig.9c Variation of Concentration ( $\phi$ ) with B1  
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, M=1.0,$   
 $\theta r=-2, \beta=0.5, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

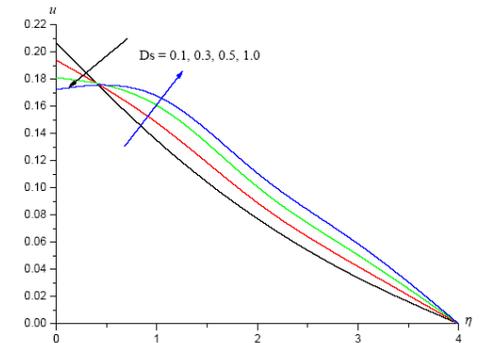


Fig.10a Variation of velocity ( $u$ ) with Ds  
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $\theta r=-2, M=1.0, \beta=0.5, Dc=0.2, a=0.2, \xi=\pi/4$

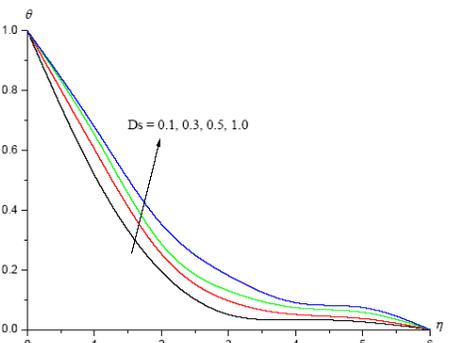


Fig.10b Variation of Temperature ( $\theta$ ) with Ds  
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta r=-2, Dc=0.2, a=0.2, \xi=\pi/4$

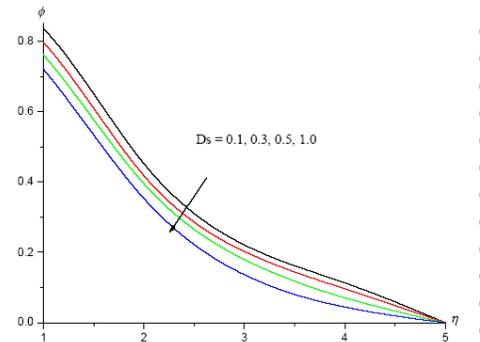


Fig.10c Variation of Concentration ( $\phi$ ) with Ds  
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta r=-2, Dc=0.2, a=0.2, \xi=\pi/4$

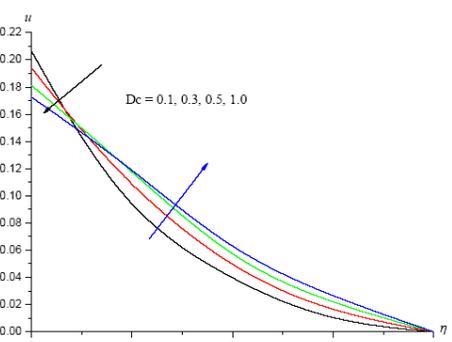


Fig.11a Variation of velocity ( $u$ ) with Dc  
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta r=-2, Ds=0.2, a=0.2, \xi=\pi/4$

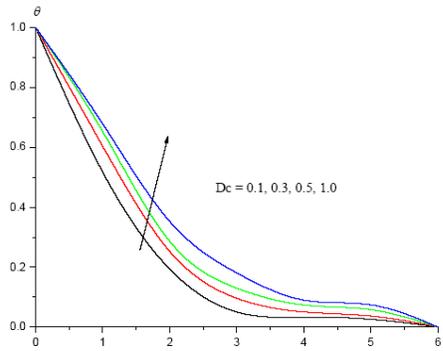


Fig. 11b Variation of Temperature ( $\theta$ ) with  $D_c$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, a=0.2, \xi=\pi/4$

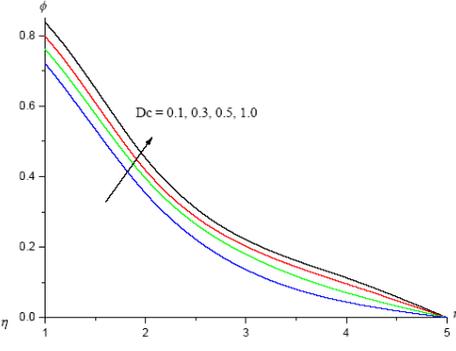


Fig. 11c Variation of Concentration ( $\phi$ ) with  $D_c$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, a=0.2, \xi=\pi/4$

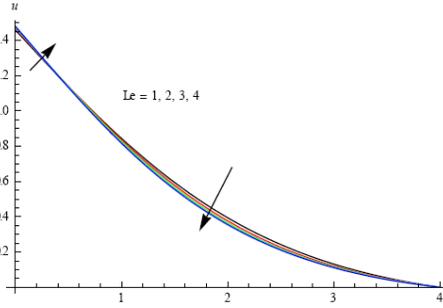


Fig. 12a Variation of axial velocity ( $u$ ) with  $Le$   
 $M=0.5, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

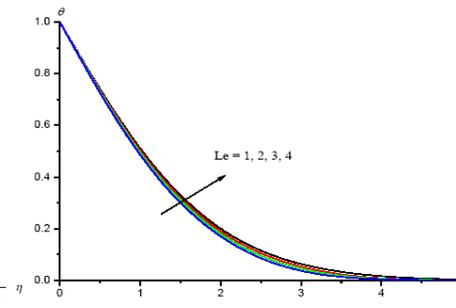


Fig. 12b Variation of temperature ( $\theta$ ) with  $Le$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

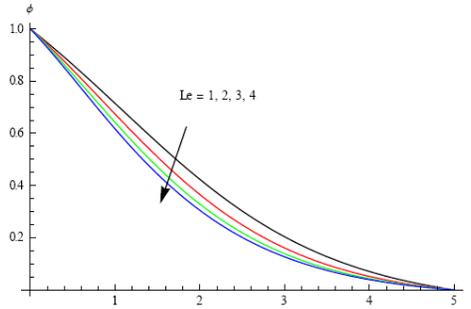


Fig. 12c Variation of Concentration ( $\phi$ ) with  $Le$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, Dc=0.2, a=0.2, \xi=\pi/4$

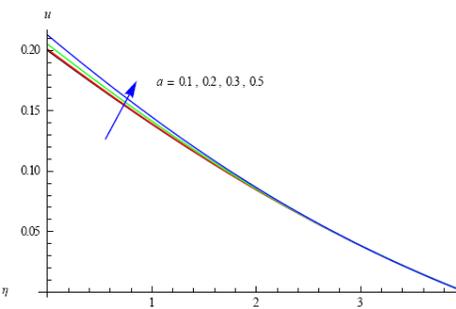


Fig. 13a Variation of velocity ( $u$ ) with  $a$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, Dc=0.2, \xi=\pi/4$

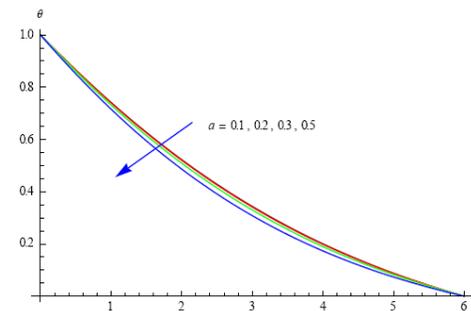


Fig. 13b Variation of Temperature ( $\theta$ ) with  $a$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, Dc=0.2, \xi=\pi/4$

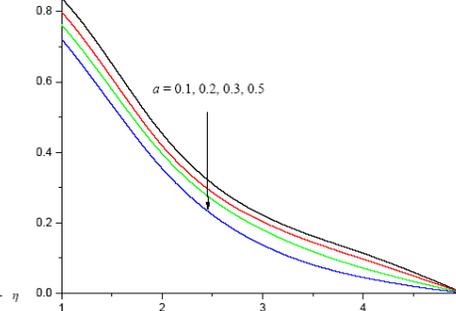


Fig. 13c Variation of Concentration ( $\phi$ ) with  $a$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, Dc=0.2, \xi=\pi/4$

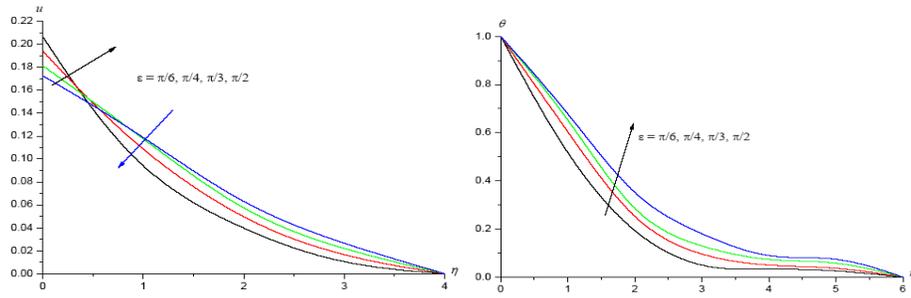


Fig. 14a Variation of velocity (u) with  $\xi$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, Dc=0.2, a=0.2$

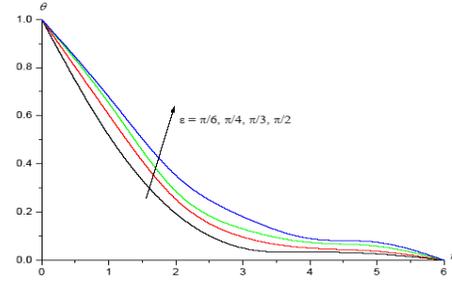


Fig. 14b Variation of Temperature ( $\theta$ ) with  $\xi$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, Dc=0.2, a=0.2$

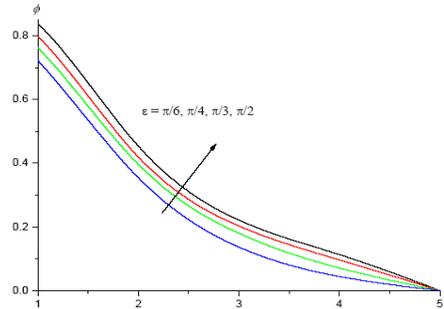


Fig. 14c Variation of Concentration ( $\phi$ ) with  $\xi$   
 $Ra=2.0, Nr=1.0, Rd=0.5, A1=0.2, B1=0.2,$   
 $M=1.0, \beta=0.5, \theta_r=-2, Ds=0.2, Dc=0.2, a=0.2$

**Table – 5.2 : Shear stress Skin friction(Cf),Nusselt Number (Nu) and Sherwood Number (Sh) at  $\eta = 0$**

Parameter		Cf (0)	Nu(0)	Sh(0)	Parameter	$\tau(0)$	Nu(0)	Sh(0)	
Ra	2	0.199938	0.277499	0.248876	B1	0.2	0.199938	0.277499	0.248876
	4	0.413496	0.251022	0.300434		0.4	0.210783	0.399211	0.248111
	6	0.638986	0.226932	0.351553		-0.2	0.133878	-0.130833	0.244299
	10	0.873794	0.204696	0.399876		-0.4	0.448626	0.801791	0.271652
M	0.5	0.199938	0.277499	0.248876	Ds	0.1	0.199938	0.277499	0.248876
	1.0	0.185079	0.278453	0.245192		0.3	0.199469	0.273084	0.248888
	1.5	0.172356	0.280092	0.24202		0.5	0.199143	0.270027	0.248897
	2.0	0.161269	0.281525	0.23926		1.0	0.198818	0.267065	0.248904
Nr	0.5	0.199938	0.277499	0.248876	Dc	0.1	0.199938	0.277499	0.248876
	1.5	0.349202	0.258571	0.284849		0.3	0.199535	0.276543	0.24201
	-0.5	0.0621004	0.295621	0.212544		0.5	0.199272	0.276528	0.236182
	-1.5	-0.0607707	0.315078	0.177849		1.0	0.19901	0.276516	0.230707
Rd	0.5	0.199938	0.277499	0.248876	Le	1	0.199938	0.277499	0.248876
	1.5	0.193002	0.240922	0.248166		2	0.203624	0.276615	0.305304
	3.5	0.187594	0.211633	0.247626		3	0.20643	0.27681	0.361527
	5.0	0.18456	0.194905	0.24733		4	0.208431	0.277085	0.41576
$\beta$	0.5	0.199938	0.277499	0.248876	a	0.1	0.199938	0.277499	0.248876
	1.0	0.197874	0.259882	0.248874		0.2	0.201092	0.27969	0.249277
	1.5	0.195569	0.243538	0.248774		0.3	0.205218	0.290175	0.25058
	2.0	0.198743	0.301683	0.20017		0.5	0.212718	0.311716	0.252851
$\theta_r$	-2	0.682492	0.223618	0.359623	$\xi$	$\pi/6$	0.199938	0.277499	0.248876
	-4	0.416681	0.250622	0.301279		$\pi/4$	0.199823	0.276551	0.248874
	-6	0.304771	0.263629	0.274576		$\pi/3$	0.199816	0.276532	0.248873
	-10	0.199827	0.276561	0.248875		$\pi/2$	0.199807	0.276507	0.24887
A1	0.2	0.199938	0.277499	0.248876					
	0.4	0.197462	0.227493	0.249827					
	-0.2	0.203741	0.361599	0.24724					
	-0.4	0.205699	0.40625	0.24639					

### 7. CONCLUSIONS:

- 1] An increase in  $Ra$  enhances the axial velocity, and reduces the temperature and concentration. This may be attributed to the fact that the thickness of the thermal and solutal boundary layers reduce with increasing values of  $Ra$ . The skin friction, Sherwood number increase, Nusselt number reduces at the wall  $\eta=0$  with increasing values of  $Ra$ .
- 2] Higher the Lorentz force smaller the velocity, larger the temperature and concentration in the flow region. This is due to the fact that in increase in  $M$  grows the thickness of the thermal and solutal boundary layer. The skin friction, Sherwood number decrease, Nusselt number enhances at the wall  $\eta=0$  with increasing values of  $M$ .
- 3] When the molecular buoyancy force dominates over the thermal buoyancy force the velocity enhances in the region when the buoyancy forces are in the same direction and for the forces acting in opposite directions it reduces in the region. The temperature and concentration reduce with increase in  $Nr>0$  and for  $Nr<0$ , the temperature and the concentration enhance in the flow region. The skin friction, Sherwood number increase, Nusselt number reduces at the wall  $\eta=0$  with increasing values of  $Nr>0$  while a reversed effect is seen with  $Nr<0$ .
- 4] Higher the thermal radiation smaller the velocity reduces in the flow region (0,1.0) and larger in the region (1.0,4.0). An increase in  $Rd$  leads to a growth in thickness of the thermal boundary layer and the solutal boundary layer which results in an enhancement of the temperature and concentration in the flow region. The skin friction, Nusselt and Sherwood numbers decrease at the wall  $\eta=0$  with increasing values of  $Rd$ .
- 5] An increase in thermal conductivity ( $\beta$ ) reduces the velocity in the flow region (0,0.5) and enhances in the region (0.5,4.0). As  $\beta$  increases the temperature and the concentration decreases. This is due to the thickening of the thinning of the thermal and solutal boundary layers as a result of increasing values of thermal conductivity. The skin friction, Sherwood number increase, Nusselt number reduce at the wall  $\eta=0$  with increasing values of  $\beta$ .
- 6] An increase in variable viscosity ( $\theta_r$ ) reduces the velocity of the fluid in the flow region. This can be explained physically as the parameter  $\theta_r$  increases there is decay in the boundary layer thickness. The temperature and the concentration profiles increase with increasing values of  $\theta_r$ . This can be attributed to the fact that an increase in  $\theta_r$  increases the thickness of the thermal and the thickness of the solutal boundary layer which results in an enhancement of the temperature and enhancement in the concentration in the flow region. The skin friction and Sherwood number decrease, Nusselt number increases at the wall  $\eta=0$  with increasing values of  $\theta_r$ .
- 7] An increase in the strength of the space dependent heat source reduces the velocity in the region abutting the wall, enhances in the region (0.5,4.0) and the Concentration reduce in the flow region. The temperature enhances with increase in the strength of the space dependent heat source. In the case of space dependent heat sink, the velocity enhances in the region (0,0.5) and reduces in the flow region (0.5,4.0), the temperature reduces, the concentration enhances with increase in  $A1<0$ . The skin friction, Nusselt number decrease, Sherwood number enhances at the wall  $\eta=0$  with increasing values of  $A1>0$  while a reversed effect is seen with  $A1<0$ .
- 8] In the presence of heat generating source, the velocity enhances in (0,0.5) and reduces in the flow region (0.5,4.0), the temperature and concentration reduce in the flow region. In the presence of heat absorbing source ( $B1<0$ ) the velocity, temperature and concentration enhance in the entire flow region. The skin friction, Nusselt number increase, Sherwood number reduces at the wall  $\eta=0$  with increasing values of  $B1>0$  while a reversed effect is seen with  $B1<0$ .
- 9] An increase in  $Ds$  reduces the velocity ( $u$ ) in the narrow region (0,0.5) and enhances in the region (0.5,4.0) far away from the wall. The temperature enhances and the concentration reduces with increasing  $g$  values of  $Ds$ . The skin friction, nusselt number decrease, Sherwood number enhances at the wall  $\eta=0$  with increasing values of  $Ds$ .
- 10] An increase in  $Dc$  reduces the velocity in a narrow region (0,0.5) and enhances in the remaining region (0.5,4.0). The temperature and concentration experience an enhancement with increasing values of  $Dc$ . The skin friction, Nusselt and Sherwood numbers decrease at the wall  $\eta=0$  with increasing values of  $Dc$ .
- 11] An increase in Lewis number ( $Le$ ) increases the velocity in the region (0,1.0) and reduces in the flow region (1.1,4.0). The temperature enhances and the concentration reduces in the flow region. The skin friction, Sherwood number increase, Nusselt number reduces at the wall  $\eta=0$  with increasing values of  $Le$ .
- 12] An increase in amplitude ( $a$ ) reduces the velocity in the entire flow region. The temperature and concentration reduces with increase in ' $a$ ' in the entire flow region. The skin friction, Nusselt and Sherwood number increase at the wall  $\eta=0$  with increasing values of ' $a$ '.

- 13] An increase in stream wise coordinate ( $\xi$ ) decreases the velocity while increase<sup>49</sup> the temperature and concentration in the flow region. The skin friction, Nusselt and Sherwood number decrease at the wall  $\eta=0$  with increasing values of ' $\xi$ '.

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