

Effects of Viscous and Joule Dissipation on MHD Boundary Layer Flow of a Casson Fluid over an Exponentially Stretching Sheet with Thermal Radiation and Heat Source/Sink in the presence of Suction/Blowing

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Abstract- This paper presents the study of boundary layer flow due to an exponentially stretching surface in the presence of an applied magnetic field. Casson fluid model is used to characterize the non-Newtonian fluid behavior. The flow is subjected to suction/blowing at the surface. Analysis is carried out in the presence of thermal radiation and Chemical reaction. The governing partial differential equations are firstly converted into nonlinear ordinary differential equations by using appropriate transformations and then solved numerically by using Runge Kutta Shooting technique. The effect of Casson parameter, Magnetic field, Porous parameter, Radiation parameter, Heat generation parameter, Chemical reaction parameter, Prandtl number, Suction parameter and Eckert number on the flow variables is analyzed. Numerical results for local skin-friction, local Nusselt number, and local Sherwood number are tabulated for different physical parameters. The effect of increasing values of the Casson parameter is to suppress the velocity field. But the temperature is enhanced with increasing Casson parameter.

Keywords – Casson fluid, exponentially stretching surface, porous medium, Heat generation, Chemical reaction.

I. INTRODUCTION

The study of boundary layer flow over a stretching sheet is of considerable attention because of its ever increasing industrial applications and important bearing on several technological processes. Examples are materials manufactured by extrusion, cooling of metallic plates in cooling bath, glass fiber and paper production, metal spinning, drawing plastic films, glass blowing etc. Most of the available literature concentrated on the study of boundary layer flow over a stretching sheet where the velocity of the stretching surface is assumed to be linearly proportional to the distance from the fixed origin. Rajagopal *et al.* (1984) discussed the flow of second-order fluid over a stretched sheet. Anderson *et al.* (1992) considered the effect of magnetic field on the flow of a viscoelastic fluid past a stretching sheet. Char (1994) studied heat and mass transfer in a hydromagnetic flow of viscoelastic fluid over a stretching sheet. Abel *et al.* (2005) analyzed MHD boundary layer flow over continuously moving stretching surface embedded in a porous medium by considering the Buoyancy force and thermal radiation effects. Mukhopadaya *et al.* (2008) discussed the free convective boundary layer flow with variable viscosity over a stretching surface with thermal radiation. Dulal pal (2009) investigated the mixed convection flow of an incompressible fluid over a stretching sheet in the presence of radiation.

However, realistically stretching of plastic sheet may not necessarily be linear. Flow and heat transfer characteristics past an exponentially stretching sheet have many applications in technology such as annealing and thinning of copper wires, the final product depending on the rate of heat transfer at the stretching continuous surface with exponentially stretching velocity and temperature distribution. During such processes, both the kinematics of stretching and the simultaneous heating or cooling has a decisive influence on the quality of the final products. Magyari and Keller (1999) discussed the similarity solution of flow and thermal boundary layers on an exponentially stretching surface. These solutions involve an exponential dependence of the temperature distribution in the direction parallel to that of the stretching. Elbashaeshy (2001) investigated the flow caused by exponentially continuous stretching surface. The effects of viscous dissipation on mixed convection flow and heat transfer over an exponentially stretching surface was studied by Partha *et al.* (2005). Bidin and Nazar (2009) studied the boundary layer flow over an exponential stretching sheet with thermal radiation, using Keller-box method. El-Aziz (2009), Ishak (2011) described the flow and heat transfer past an exponentially stretching sheet. Bhattacharyya (2012) discussed the steady boundary layer flow and reactive mass transfer past an exponentially stretching surface in an exponentially moving free stream.

Convective heat transfer plays a vital role during the handling and processing of non-Newtonian fluid flows. Mechanics of non-Newtonian fluid flows presents a special challenge to Engineers, Physicists, and Mathematicians. Because of the complexity of these fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids. In literature, the vast majority of non-Newtonian fluid models is concerned with simple models like the power law and grade two or three. The power-law model has a vast usage in modeling fluids with shear-dependent viscosity but it cannot predict the effects of elasticity. The fluids of grade two or three can calculate the effects of elasticity but the viscosity in these models is not shear dependent, and also they cannot predict the effects of stress relaxation. Forced convection heat transfer on a flat plate embedded in porous media for power-law fluids was investigated by Hady and Ibrahim (1997). Free convection heat and mass transfer of non-Newtonian power-law fluids with yield stress from a vertical flat plate which is saturated in porous media was analyzed by Jumah and Mujumdar (2000). The combined effect of magnetic field and viscous dissipation on a power-law fluid over plate with variable surface heat flux embedded in a porous medium was studied by El-Amin (2003). Finite-element method for the effect of various injection parameters on heat transfer for a power-law non-Newtonian fluid over a continuously stretched surface with thermal radiation is presented by Seddeek (2006). MHD power-law fluid flow and heat transfer over a non-isothermal stretching sheet is investigated by Prasad *et al.* (2009).

There is another fluid model for non-Newtonian fluid known as Casson fluid. Casson fluid exhibits yield stress. It is well known that Casson fluid is a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear, i.e., if the applied shear stress is less than the yield stress, it behaves like a solid, whereas if the applied shear stress is greater than yield stress, it starts to move. The examples of Casson fluid are jelly, tomato sauce, honey, soup, concentrated fruit juices, etc. Moreover, human blood can also be treated as Casson fluid. Eldabe (1995) considered the heat transfer of Casson fluid flow between two rotating cylinders. The flow of Casson fluid in a tube was studied by Dash *et al.* (2000) and Nagarani *et al.* (2004). The unsteady boundary layer flow of a Casson fluid over a moving flat plate was studied by Mustafa *et al.* (2011). Shehzad (2013) discussed the effects of mass transfer on the MHD boundary layer flow of a Casson fluid with chemical reaction. Sarojamma *et al.* (2014) analyzed the effect of chemical reaction on MHD boundary layer flow of a Casson fluid over a stretching sheet.

Suction or injection (blowing) of a fluid through the bounding surface can significantly change the flow field. Injection or withdrawal of fluid through a porous bounding wall is of general interest in practical problems involving boundary layer control applications such as film cooling, polymer fiber coating, and coating of wires. The process of suction and blowing has also its importance in many engineering activities such as in the design of thrust bearing and radial diffusers and thermal oil recovery. Suction is applied to chemical processes to remove reactants. Blowing is used to add reactants, cool the surface, prevent corrosion or scaling and reduce the drag.

The heat and mass transfer of viscous fluids over an isothermal stretching sheet with suction or blowing have been studied by Gupta and Gupta (1997). Chen and Char (1998) investigated the heat transfer on continuous stretching surfaces with suction or blowing. Afify (2009) studied the effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection. Chamkha *et al.* (2010) studied the similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/ injection and chemical reaction effects. The influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time dependent suction was studied by Israel-Cooke *et al.* (2003). Abo-Eldahab *et al.* (2004) studied the blowing/suction effect on hydromagnetic heat transfer by mixed convection from an inclined stretching surface with internal heat generation/absorption.

In view of the above discussions, this paper presents the heat and mass transfer in radiative hydromagnetic chemical reactive Casson fluid over an exponentially stretching sheet with Joule effect and suction/ injection. The mathematical formulation of the problem with appropriate boundary conditions is given in section 2. The method of solution of the governing equations is presented in section 3. The Results and Discussion studying the influence of governing parameters on the flow is presented in section 4. The conclusions are given in section 5.

II. MATHEMATICAL FORMULATION

Consider laminar boundary-layer, two-dimensional steady flow of an incompressible, non-Newtonian Casson fluid over an exponentially stretching sheet embedded in a porous medium. Assume that the plate has a surface temperature (T_w) and concentration (C_w) and is placed in a quiescent fluid of uniform ambient temperature (T_∞) and concentration (C_∞). Choose the coordinate system such that x-axis is parallel to the surface

and y-axis normal to the surface as shown in Figure 1. A variable magnetic field $B(x) = B_0 e^{\frac{x}{2L}}$ is applied in the y direction. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field and Hall effects are negligible. We also considered the heat and mass transfer processes in the presence of chemical reaction and suction.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid can be written as (Mustafa *et al.* (2011))

$$\tau_{ij} = \begin{cases} 2 \left(\mu_B + \frac{P_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c \\ 2 \left(\mu_B + \frac{P_y}{\sqrt{2\pi_c}} \right) e_{ij}, & \pi < \pi_c \end{cases} \quad (1)$$

where μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, P_y is the yield stress of the fluid, e_{ij} denotes the (i, j) - th component of the deformation rate, $\pi = e_{ij} e_{ij}$ is the product of the component of deformation rate with itself, π_c is the critical value of π based on the non-Newtonian model.

Under the above assumptions and with the usual boundary layer approximations, the governing equations are:

Equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

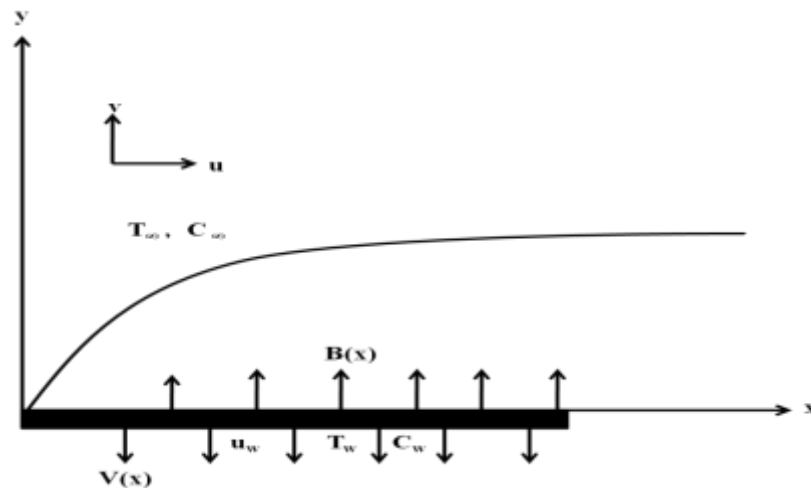


Figure 1: Physical Model and Coordinate system

Equation of momentum

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} u - \frac{v}{K'} u \quad (3)$$

Equation of energy

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{\sigma B^2}{\rho c_p} u^2 + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{Q(x)}{\rho c_p} (T - T_\infty) \quad (4)$$

Equation of mass diffusion

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_1 (C - C_\infty) \quad (5)$$

where u and v are the velocity components along the x and y directions, ν is the kinematic viscosity, σ is the electric conductivity of the fluid, ρ is the density, $\beta = \frac{\mu_B \sqrt{2\pi_c}}{P_y}$ is the Casson fluid parameter,

$K' = K_0 e^{-x/L}$ is the permeability of the porous medium, T is the temperature, C is the concentration, T_∞ is the temperature for away from the plate, C_∞ Species concentration of the ambient fluid, α is the thermal diffusivity, $Q(x) = Q_0 e^{x/L}$ is the heat generation parameter, D is the mass diffusivity, q_r is the radiative heat flux, $K_1 = \gamma_1 e^{x/L}$ is the chemical reaction rate constant. The second, third, fourth and fifth terms on the right hand side of the energy equation (4) represent the radiative heat flux, joule dissipation, viscous dissipation and heat generation effects respectively. The second term on the right hand side of the concentration Eq. (5) represents chemical reaction effects.

Assume that the exponentially stretching surface is maintained at the stretching velocity $u_w(x)$, exponential temperature distribution $T_w(x)$ and exponential concentration distribution $C_w(x)$, which are defined by

$$u_w = u_0 e^{x/L}, \quad T_w = T_\infty + T_0 e^{x/2L}, \quad C_w = C_\infty + C_0 e^{x/2L} \quad (6)$$

where the subscripts w, ∞ refers to the surface and ambient conditions, u_0 is the characteristic velocity, T_0 is the reference temperature, C_0 is the reference concentration, L is the reference length.

Hence, the boundary conditions of the flow are

$$u = u_w, \quad v = -V(x), \quad T = T_w, \quad C = C_w \text{ at } y = 0 \quad (7a)$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty \quad (7b)$$

where $V(x) = V_0 e^{\frac{x}{2L}}$, a special type of velocity at the wall, $V(x) > 0$ is the velocity of suction and $V(x) < 0$ is the velocity of blowing, V_0 is the initial strength of suction.

By using the Rosseland approximation (Brewster 1992), we can write the radiative heat flux q_r as

$$q_r = -\frac{4\sigma^*}{3s^*} \frac{\partial T^4}{\partial y} \quad (8)$$

where σ^* is the Stephen Boltzmann constant, s^* is the mean absorption coefficient. We assume that the temperature differences within the flow are sufficiently small so that T^4 can be expanded in a Taylor series about T_∞ and neglecting higher order terms result in

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \quad (9)$$

Substituting equations (8) and (9) into (4), we get

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3s^* \rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B^2}{\rho c_p} u^2 + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{Q(x)}{\rho c_p} (T - T_\infty) \quad (10)$$

III. METHOD OF SOLUTION

The continuity equation (2) is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

The momentum, energy and concentration equations were transformed to ordinary differential equations by using the following similarity variables:

$$u = u_0 e^{\frac{x}{2L}} f'(\eta), \quad v = -\left(\frac{v u_0}{2L} \right)^{\frac{1}{2}} e^{\frac{x}{2L}} [\eta f'(\eta) + f(\eta)], \quad T = T_\infty + T_0 e^{\frac{x}{2L}} \theta(\eta), \quad C = C_\infty + C_0 e^{\frac{x}{2L}} \phi(\eta) \quad (11)$$

where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature, $\phi(\eta)$ is the dimensionless concentration and prime denotes differentiation with respect to η

Using (11), the governing partial differential equations are reduced to

$$\left(1 + \frac{1}{\beta} \right) f''' + f f'' - 2(f')^2 - (M + K)f' = 0 \quad (12a)$$

$$\left(1 + \frac{4R}{3} \right) \theta'' + Pr[f\theta' - f'\theta + MEc(f')^2 + Ec(f'')^2 + Q_H\theta] = 0 \quad (12b)$$

$$\phi'' + Sc[f\phi' - f'\phi - \gamma\phi] = 0 \quad (12c)$$

The corresponding boundary conditions are

$$f' = 1, f = S, \theta = 1, \phi = 1 \quad \text{when } \eta = 0 \quad (13a)$$

$$f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (13b)$$

where

$$M = \frac{2L\sigma B_0^2}{\rho u_0} \quad (\text{Magnetic parameter}),$$

$$K = \frac{2vL}{K_0 u_0} \quad (\text{Porous parameter})$$

$$Pr = \frac{\rho c_p v}{k^*} \quad (\text{Prandtl number}),$$

$$R = \frac{4\sigma^* T_\infty^3}{s^* k^*} \quad (\text{Radiation parameter})$$

$$Ec = \frac{u_0^2}{c_p T_0} \quad (\text{Eckert number}),$$

$$Q_H = \frac{2LQ_0}{u_0 \rho c_p} \quad (\text{Heat generation parameter})$$

$$Sc = \frac{\nu}{D} \quad (\text{Schmidt number}),$$

$$\gamma = \frac{2\gamma_1 L}{u_0} \quad (\text{Chemical reaction parameter})$$

$$S = \frac{V_0}{\sqrt{\frac{u_0 \nu}{2L}}} \quad (\text{Suction } (V_0 > 0) \text{ and blowing } (V_0 < 0) \text{ parameter})$$

It should be noticed that the effects of viscous dissipation is characterized by the Eckert number Ec while Joule heating effects is represented by the product of Ec and M .

The physical quantities of interest are the skin friction coefficient C_f , the local Nusselt number Nu and local Sherwood number Sh which are given by

$$Re_x^{1/2} C_f = (1 + \frac{1}{\beta}) f''(0)$$

$$Re_x^{-1/2} Nu = -\theta'(0)$$

$$Re_x^{-1/2} Sh = -\phi'(0)$$

where $Re_x = x u_w(x)/\nu$ is the local Reynolds number.

IV. DISCUSSION OF THE RESULTS

The set non-linear differential equations 12 (a)-12(c) cannot be solved in closed-form, so it is required to solve this problem numerically in order to describe the physics of the problem well. The resulting nonlinear ordinary differential equations are solved by fourth-order Runge-Kutta method with shooting technique. In order to analyze the results, numerical computations have been carried out for various values of Casson parameter (β), Magnetic parameter (M), porosity parameter (K), Radiation parameter (R), Prandtl number (Pr), Eckert number (Ec), heat generation parameter (Q_H), Schmidt number (Sc), Chemical reaction parameter (γ), Suction parameter (S). For numerical results, we considered the non-dimensional parameter values as $K = 0.1$, $M = 0.1$, $\beta = 2$, $Q_H = 0.1$, $Pr = 1$, $R = 0.5$, $Ec = 0.1$, $\gamma = 0.1$, $Sc = 0.24$. All the graphs correspond to these values unless indicated on the appropriate graph. In order to assess the accuracy of the numerical scheme, the computed value of the Skin friction coefficient and rate of heat transfer were compared with those of Magyari Kellar (1999), Elbashbeshy (2001), El-Aziz (2009), Bidin and Nazur (2009) which is shown in Table 1-2

The skin-friction (C_f), Nusselt number (Nu) and Sherwood numbers (Sh) are evaluated for variations in the governing parameters and are presented in Tables 3-6. Table 3 shows the skin friction coefficient for the different values of β and S . By increasing the values of β , the value of the skin friction coefficient, Nusselt number, Sherwood number decreases, but it increases upon increasing suction parameter. Table 4 represents the variation of the skin-friction, Nusselt number and Sherwood numbers for various values of K and M . It is observed that the skin friction coefficient increases with the increase in magnetic parameter M and Porous parameter K while Nusselt number and Sherwood numbers decreases.

Table 1: A comparison of $f''(0)$ for Newtonian fluid in the absence of Magnetic field and in the case of Non permeable stretching sheet

Magyari Kellar (1999)	Elbashbeshy (2001)	Present study
-1.28180	-1.28181	-1.281813

Table 2: comparison of $-\theta'(0)$ for several values of Prandtl number, Magnetic Parameter, Radiation parameter when $Ec = 0$, $\beta \rightarrow \infty$, $S = 0$, $Q_H = 0$

M	K	Pr	Magyari Kellar (1999)	El-Aziz (2009)	Bidin and Nazur (2009)	Present study
0	0	1	0.954782	0.954785	0.9548	0.9548
		2			1.4714	1.4715

		3	1.869075	1.869074	1.8691	1.8691
		5	2.500135	2.500132		2.5001
		10	3.660379	3.660372		3.6604
1		1				0.8611
0	1				0.5315	0.5312
1						0.4505

Table 3: Numerical values of the skin-friction coefficient, Nusselt Number, Sherwood Number for different values of β and S when $Q_H = 0.1, K = 0.1, M = 0.1, \gamma = 0.1, Sc = 0.24, Ec = 0.1, R = 0.5, Pr = 1,$

β	$(1+1/\beta)f''(0)$			$-\theta'(0)$			$-\phi'(0)$		
	S = - 0.5	S = 0	S = 0.5	S = - 0.5	S = 0	S = 0.5	S = - 0.5	S = 0	S = 0.5
0.2	3.13563	3.36116	3.60522	0.623222	0.780894	0.965403	0.458087	0.518139	0.585083
0.4	2.32822	2.55006	2.79699	0.581921	0.73417	0.915748	0.435522	0.491804	0.555233
0.6	2.00275	2.22261	2.4715	0.557115	0.70503	0.883994	0.423047	0.476865	0.537949
∞	1.14815	1.35906	1.61772	0.448023	0.565924	0.724942	0.375475	0.417385	0.467429

Table 4: Numerical values of the skin-friction coefficient, Nusselt Number, Sherwood Number for different values of M and K when $Q_H = 0.1, \gamma = 0.1, Sc = 0.24, Ec = 0.1, R = 0.5, Pr = 1, \beta = 2$

M	K	$(1+1/\beta)f''(0)$		$-\theta'(0)$		$-\phi'(0)$	
		S = - 0.5	S = 0.5	S = - 0.5	S = 0.5	S = - 0.5	S = 0.5
0	0.1	1.40516	1.8724	0.509197	0.809898	0.399783	0.501271
0.1		1.44982	1.91896	0.496318	0.799391	0.395273	0.497576
0.4		1.57732	2.05099	0.460172	0.770289	0.383071	0.487683
0.1	0.1	1.44982	1.91896	0.496318	0.799391	0.395273	0.497576
	0.2	1.49335	1.96417	0.486324	0.791426	0.390994	0.494093
	0.3	1.53582	2.00815	0.476635	0.78377	0.386932	0.490801

Table 5: Numerical values of the Nusselt Number for different values of R, Pr, Ec, Q_H when $K = 0.1, M = 0.1, \gamma = 0.1, Sc = 0.24, \beta = 2$

R	Pr	Ec	Q_H	S = - 0.5	S = 0.5
0.5	1	0.1	0.1	0.496318	0.799391
1				0.417533	0.618821
1.5				0.36901	0.515503
0.5	0.5			0.351145	0.479171
	0.7			0.413313	0.609562
	1			0.496318	0.799391
		0.1		0.496318	0.799391
		0.2		0.473522	0.765502
		0.3		0.450726	0.782447
			0.1	0.496318	0.799391
			0.2	0.425144	0.736948
			0.3	0.334696	0.663003

Table 5 highlights the effect of the Radiation parameter, Prandtl number, viscous dissipation and heat generation on Nusselt number. It is evident that all the three parameters except Prandtl number reduce the values of the wall temperature gradient while the opposite trend is observed in case of Prandtl number. The values of Sherwood number are shown in table 6 for different values of the Schmidt number and chemical reaction parameter. This table concludes that values of Sherwood number increase with the increase in Schmidt number and Chemical reaction parameter.

Table 6: Numerical values of the Sherwood Number for different values of Sc and γ when $Q_H = 0.1, K = 0.1, M = 0.1, Ec = 0.1, R = 0.5, Pr = 1, \beta = 2$

Sc	γ	S = - 0.5	S = 0.5
0.24		0.395273	0.497576
0.48		0.561923	0.798245

0.62		0.640962	0.95828
0.24	0.1	0.395273	0.497576
	0.2	0.425208	0.528352
	0.3	0.453185	0.557077

We first concentrate on the effects of Casson parameter β on velocity and temperature profiles. Figure 2 illustrates the effect of Casson parameter β on velocity profile in the presence of suction/blowing. Velocity is found to decrease with increasing the values of Casson parameter β because of increase in β plastic dynamic viscosity increases and it causes resistance to fluid motion. It is also noticed that momentum boundary layer thickness decreases with increasing β . Fluid velocity is much more suppressed in case of suction ($S = 0.5$) than that of blowing ($S = -0.5$).

The influence of Casson parameter on temperature profiles in the presence of suction/blowing is shown in figure 3. Thermal boundary layer thickness increases with increasing β . The thickness of the boundary layer occurs due to the increase in elasticity stress parameter. It is clearly shown that with increasing Casson fluid parameter β , the temperature is found to decrease for both the cases of suction and blowing.

Figure 4 shows the variation of velocity profile against the magnetic parameter in the presence of suction/blowing. We notice that the effect of the magnetic parameter is to reduce the velocity of the fluid in the boundary layer region. This clearly reveals that the transverse magnetic field opposes the fluid transport due to increasing Lorentz force associated with increasing magnetic parameter. Figure 5 exhibits the effect of magnetic parameter on temperature profiles in the presence of suction/blowing. It is noticed that the temperature is found to be increased. The thermal boundary layer thickness increases, as magnetic parameter increases. Effects of porous parameter on velocity and temperature field in the presence of suction/blowing is exhibited in figure 6 and 7. It is observed that rise in the value of porous parameter declines the velocity profiles and enhances the temperature. It is due to the fact that increase in porosity widens the porous layer and increases the momentum boundary layer thickness.

Figure 8 represents the effect of heat generation parameter on temperature profiles in the presence of suction/blowing. Physically speaking, the presence of heat generation effects has the tendency to increase the fluid temperature. Figure 9 depicts the effect of radiation on temperature profiles in the presence of suction/blowing. It indicates that thermal radiation enhances the temperature in the boundary layer region. Thus radiation should keep at its minimum in order to facilitate better cooling environment.

The effect of Eckert number on heat transfer in the presence of suction/blowing is shown in Figure 10. It is found that temperature in the boundary layer region increases with increase in the viscous dissipation parameter. Figure 11 depicts the effects of Prandtl number Pr on temperature profiles. It shows that the temperature decreases with increasing Pr . Moreover, the thermal boundary layer thickness decreases with increasing Prandtl number.

The influences of Schmidt number and Chemical reaction parameter on concentration profiles are illustrated in figures 12 and 13. It can be observed that increase in both Schmidt number and Chemical reaction parameter reduces the concentration boundary layer. We also observed that the parameters affecting the energy equation namely Prandtl number, the radiation parameter, Eckert number and Heat generation parameter do not alter velocity and concentration profiles.

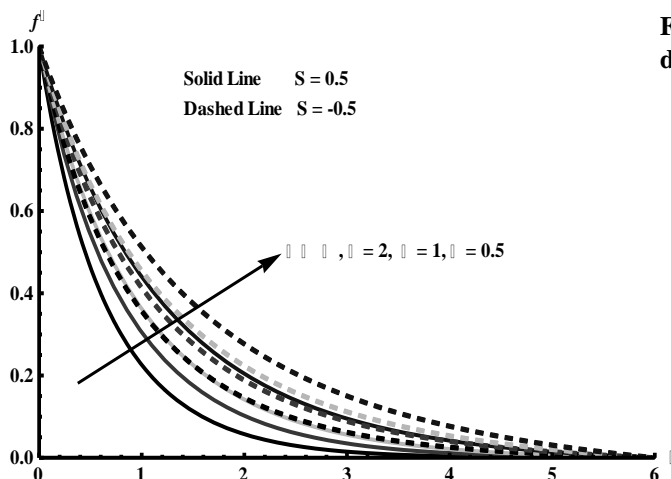


Figure 2: Variation of velocity profile for different values of β

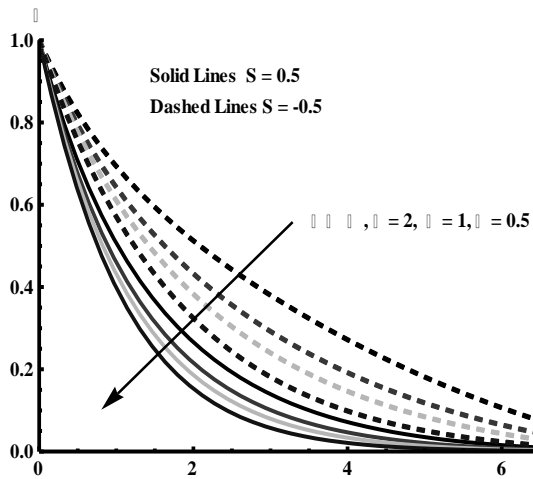


Figure 3: Variation of temperature profile for different values of β

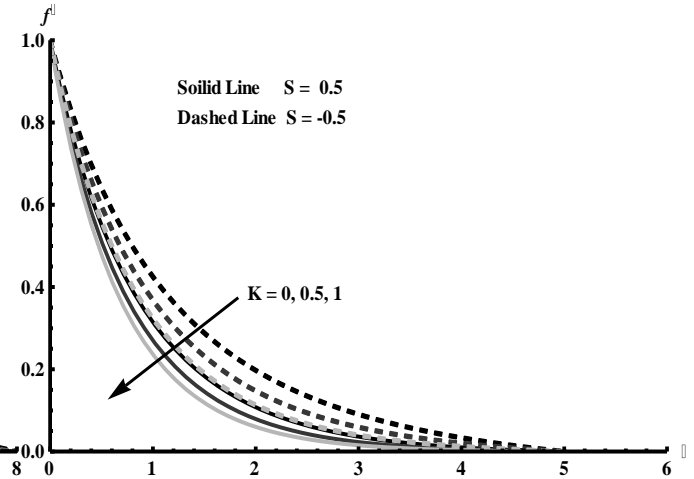


Figure 6: Variation of velocity for different values of K

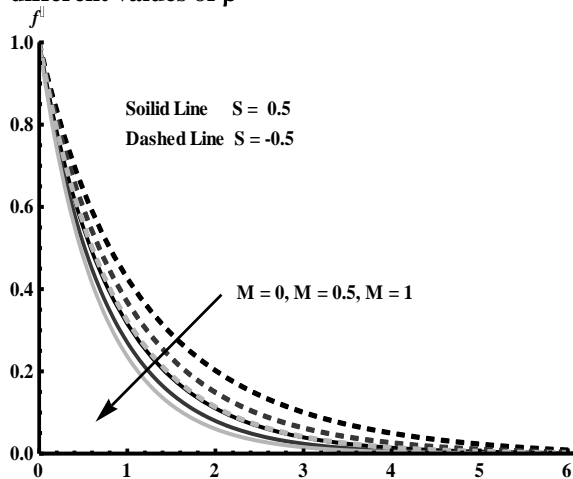


Figure 4: Variation of velocity for different values of M

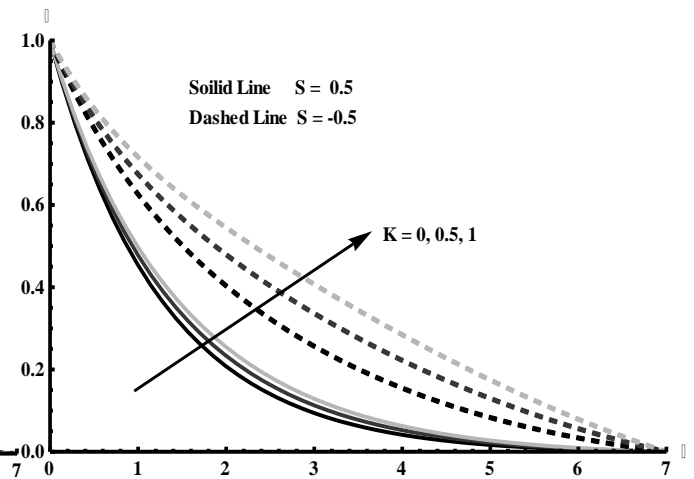


Figure 7: Variation of temperature profile for different values of K

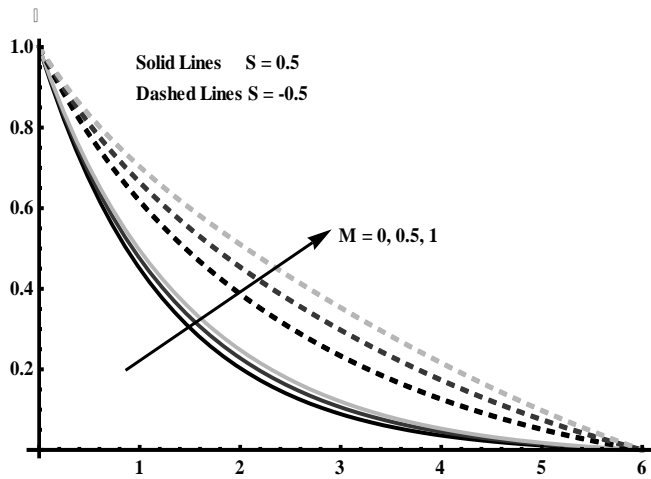


Figure 5: Variation of temperature profile for different values of M

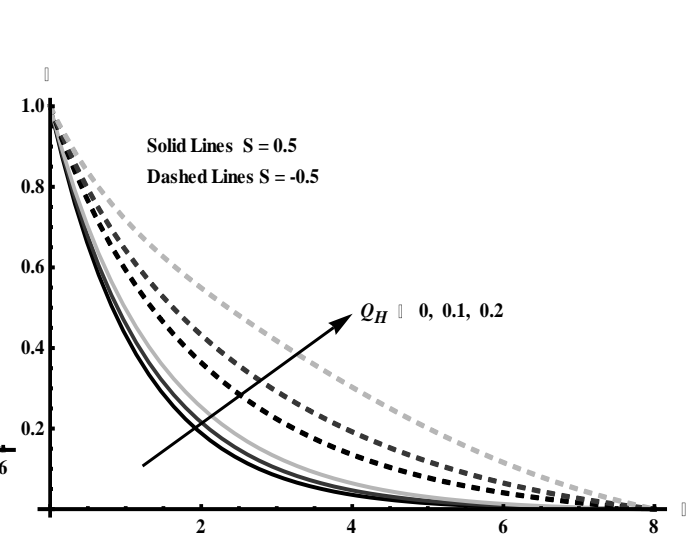


Figure 8: Variation of temperature profile for different values of Q_H

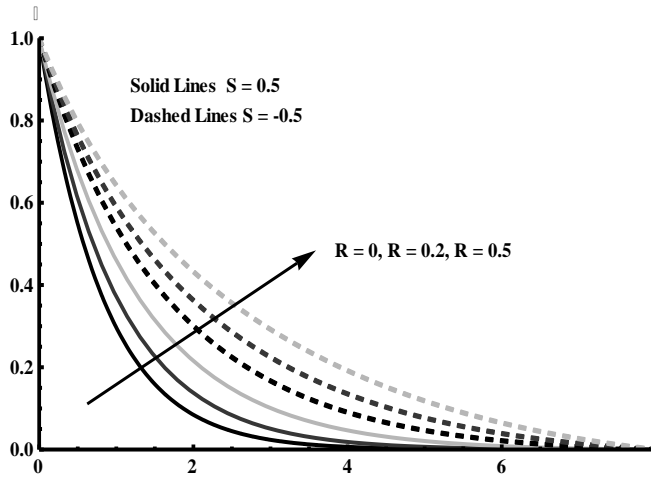


Figure 9: Variation of temperature profile for different values of R

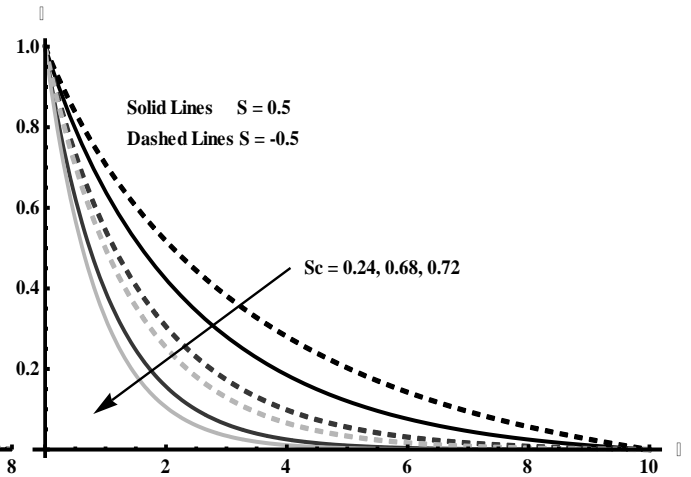


Figure 12: Variation of Concentration profile for different values of Sc

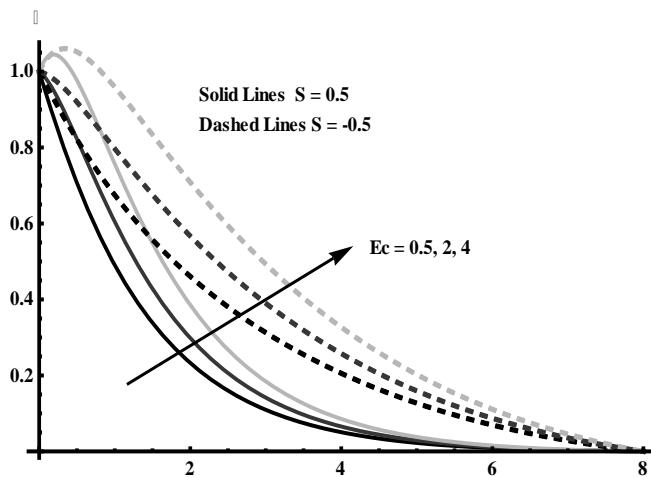


Figure 10: Variation of temperature profile for different values of Ec

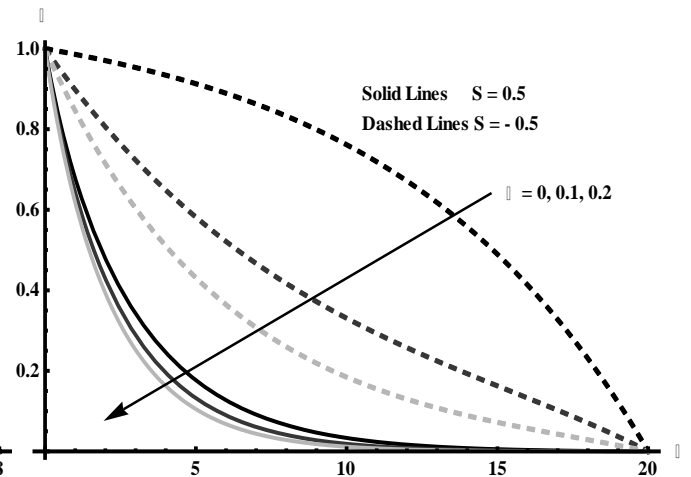


Figure 13: Variation of Concentration profile for different values of γ

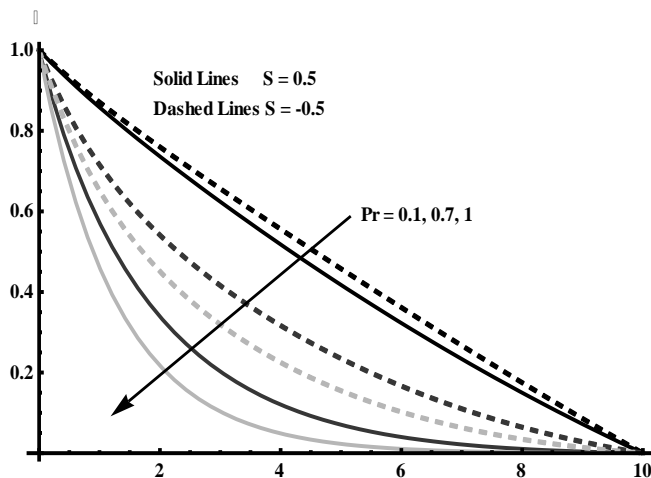


Figure 11: Variation of temperature profile for different values of Ec

IV.CONCLUSION

The present paper provides the numerical solution of Heat and mass transfer problem of Radiative, dissipative MHD flow in a Casson fluid over an exponentially stretching sheet embedded in a porous medium with heat generation, Joule dissipation and suction/blowing. By using similar transformation, the governing equations are reduced in to ordinary differential equations which are solved by Runge Kutta fourth Order Shooting technique. The main findings can be summarized as follows

- * Increase in Casson parameter causes reduction in momentum boundary layer thickness but the thermal boundary layer thickness increases in this case.
- * Magnetic parameter reduces the rate of transport.
- * Prandtl number reduces the thermal boundary layer thickness.
- * Due to chemical reaction the concentration of the fluid decreases.
- * Effect of suction parameter on a fluid is to suppress the velocity field which in turn causes the enhancement of skin friction coefficient.
- * Skin friction coefficient is higher for suction than for blowing.

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