On Intuitionistic fuzzy contra β^{**} generalized continuous functions

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Abstract

In this paper, we introduce the notion of contra β^{**} generalized continuous functions and almost contra β^{**} generalized continuous functions in intuitionistic fuzzy topological spaces. Furthermore we provide some properties of the same function and discuss some fascinating theorems.

Keywords : Intuitionistic fuzzy set, intuitionistic fuzzy topology, intuitionistic fuzzy β^{**} generalized closed set, intuitionistic fuzzy β^{**} generalized open set, intuitionistic fuzzy contra β^{**} generalized continuous functions, intuitionistic fuzzy almost contra β^{**} generalized continuous functions.

1. Introduction

In 1965, Zadeh [12] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of this notion. The notion of intuitionistic fuzzy set is introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. Later this was followed by the introduction of intuitionistic fuzzy β generalized closed sets by Saranya, M and Jayanthi, D [6] in 2016 which was simultaneously followed by the introduction of intuitionistic fuzzy β generalized continuous functions [7] by the same authors. The concept of intuitionistic fuzzy β^{**} generalized closed sets is introduced by Sudha, S. M and Jayanthi, D [8] in 2020. In this paper, we introduce the notion of intuitionistic fuzzy contra β^{**} generalized continuous functions and almost contra β^{**} generalized continuous functions. Furthermore we investigate some of their properties.

2. Preliminaries

Definition 2.1: [1] An ***intuitionistic fuzzy set*** (IFS) A is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$$

where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X. An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A \rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$.

Definition 2.2: [1] Let A and B be two IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X\}$,

(d) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X\}$,

(e) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1 \rangle$ and $1_\sim = \langle x, 1, 0 \rangle$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An ***intuitionistic fuzzy topology*** (IFT) on X is a family τ of IFSs in X satisfying the following axioms :

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{ G_i : i \in J \} \subseteq \tau$

In this case the pair (X,τ) is called the ***intuitionistic fuzzy topological space*** (IFTS) and any IFS in τ is known as an ***intuitionistic fuzzy open set*** (IFOS) in X. The complement A^c of an IFOS A in an IFTS (X,τ) is called an ***intuitionistic fuzzy closed set*** (IFCS) in X.

Definition 2.4: [4] A function $f : (X, \tau) \to (Y, \sigma)$ is called an

1. ***intuitionistic fuzzy contra continuous*** if $f^{-1}(B)$ is an IFCS in X for every IFOS B in Y.
2. ***intuitionistic fuzzy contra α continuous*** if $f^{-1}(B)$ is an IFαCS in X for every IFOS B in Y.
3. ***intuitionistic fuzzy contra pre continuous*** if $f^{-1}(B)$ is an IFPCS in X for every IFOS B in Y.

Definition 2.5: [8] An IFS A of an IFTS (X, τ) is said to be an ***intuitionistic fuzzy β** generalized closed set*** (IFβ^{**}GCS) if $cl(int(cl(A))) \cap int(cl(int(A))) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Definition 2.6: [8] The complement A^c of an IFβ^{**}GCS A in an IFTS (X, τ) is called an ***intuitionistic fuzzy β** generalized open set*** (IFβ^{**}GOS) in X. The family of all IFβ^{**}GOSs of an IFTS (X, τ) is denoted by IFβ^{**}GO(X).

Definition 2.7: [11] An IFTS (X, τ) is an ***intuitionistic fuzzy*** **β*****p$T_{1/2}$***(IFβ******p$T_{1/2}$) ***space*** if every IFβ**GCS is an IFPCS in X.

Definition 2.8: [11] An IFTS (X, τ) is an ***intuitionistic fuzzy*** **β*****g$T_{1/2}$*** (IFβ******g$T_{1/2}$) ***space*** if every IFβ**GCS is an IFGCS in X.

3. Intuitionistic fuzzy contra β** generalized continuous functions

In this section we have introduced intuitionistic fuzzy contra β^{**} generalized continuous functions and investigate some of their properties.

Definition 3.1: A function $f : (X, \tau) \to (Y, \sigma)$ is said to be an ***intuitionistic fuzzy contra β** generalized*** (IF contra β^{**}G) ***continuous function*** if $f^{-1}(A)$ is an IFβ^{**}GCS in X for every IFOS A in Y.

Example 3.2: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b) \rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here the IFS $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$ is an IFOS in Y and $f^{-1}(G_3) = \langle x, (0.3_a, 0.4_b), (0.5_a, 0.6_b) \rangle$ is an IFβ^{**}GCS in (X, τ). Therefore f is an IF contra β^{**}G continuous function in (X, τ).

Proposition 3.3: Every IF contra continuous function is an IF contra β^{**}G continuous function but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra continuous function. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFCS in X, by hypothesis. Since every IFCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous function.

Example 3.4: In example 3.2, f is an IF contra β^{**}G continuous function, but not an IF contra continuous function, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFCS in X, as $cl(f^{-1}(G_3)) = G_1^c \neq f^{-1}(G_2)$.

Proposition 3.5: Every IF contra semi continuous function is an IF contra β^{**}G continuous function but not conversely in general.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra semi continuous function. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFSCS in X, by hypothesis. Since every IFSCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous function.

Example 3.6: In example 3.2, f is an IF contra β^{**}G continuous function but not an IF contra semi continuous function, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_2))) = G_1 \not\subseteq f^{-1}(G_3)$.

Proposition 3.7: Every IF contra pre continuous function is an IF contra β^{**}G continuous function but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra pre continuous function. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFPCS in X, by hypothesis. Since every IFPCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous function.

Example 3.8: Let X = {a, b} and Y = {u, v}. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b) \rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v) \rangle$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is an IF contra β^{**}G continuous function but not an IF contra pre continuous function, since $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v) \rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3))) = G_2^c \not\subseteq f^{-1}(G_2)$.

Proposition 3.9: Every IF contra α continuous function is an IF contra β^{**}G continuous function but not conversely in general.

Proof: Let f : (X, τ) $\rightarrow$ (Y, σ) be an IF contra α continuous function. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFαCS in X, by hypothesis. Since every IFαCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous function.

Example 3.10: In example 3.2, f is an IF contra β^{**}G continuous function but not an IF contra α continuous function, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFαCS in (X, τ), as $cl(int(cl(f^{-1}(G_3)))) = G_1^c \not\subseteq f^{-1}(G_3)$.

Proposition 3.11: Every IF contra β continuous function is an IF contra β^{**}G continuous function but not conversely in general.

Proof: Let f : (X, τ) $\rightarrow$ (Y, σ) be an IF contra β continuous function. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFβCS in X, by hypothesis. Since every IFβCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous function.

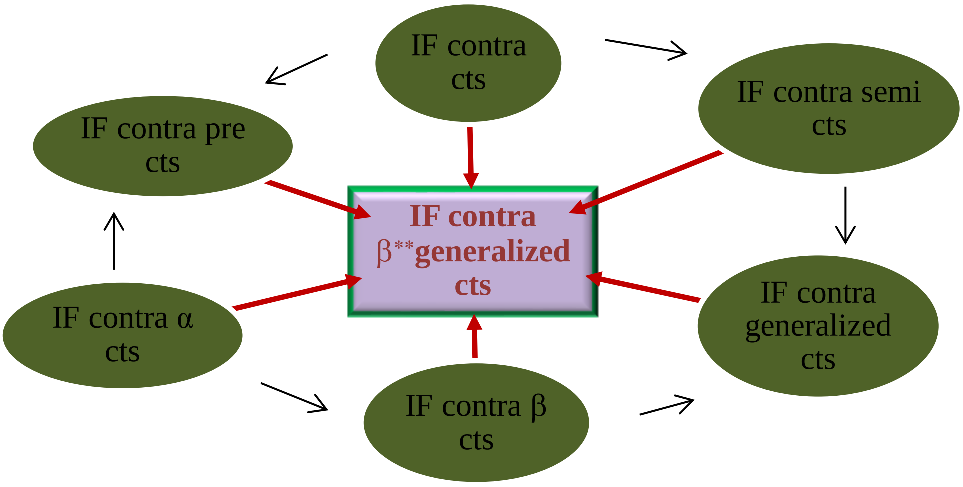
Example 3.12: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function f : (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Here f is an IF contra β^{**}G continuous function but not an IF contra β continuous function, since $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v) \rangle$ is an IFOS in Y, but $f^{-1}(G_2)$ is not an IFβCS in (X, τ), as $int(cl(int(f^{-1}(G_2)))) = 1_\sim \not\subseteq f^{-1}(G_2)$.

Proposition 3.13: Every IF contra generalized continuous function is an IF contra β^{**}G continuous function but not conversely in general.

Proof: Let f : (X, τ) $\rightarrow$ (Y, σ) be an IF contra generalized continuous function. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFGCS in X, by hypothesis. Since every IFGCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous function.

Example 3.14: In example 3.2, f is an IF contra β^{**}G continuous function but not an IF contra generalized continuous function, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFGCS in X, as $cl(f^{-1}(G_3)) = G_1^c \not\subseteq G_1$, whereas $f^{-1}(G_3) \subseteq G_1$.

The relation between various types of intuitionistic fuzzy contra continuity is given in the following diagram. In this diagram 'cts' means continuous.



The reverse implications are not true ingeneral in the above diagram.

Proposition 3.15: A function f : (X, τ) → (Y, σ) is an IF contra β^{**}G continuous function if and only if the inverse image of each IFCS in Y is an IFβ^{**}GOS in X.

Proof :Necessity : Let A be an IFCS in Y. This implies A^c is an IFOS in Y. Then $f^{-1}(A^c)$ is an IFβ^{**}GCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβ^{**}GOS in X.

Sufficiency: Let A be an IFOS in Y. Then A^c is an IFCS in Y. By hypothesis $f^{-1}(A^c)$ is IFβ^{**}GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous function.

Proposition 3.16: Let f : (X, τ) → (Y, σ) be a bijective function, suppose that one of the following properties hold :

(i) $f^{-1}(cl(B)) \subseteq int(\beta cl(f^{-1}(B)))$ for each IFS B in Y.

(ii) $cl(\beta int\,(f^{-1}(B))) \subseteq f^{-1}\,(int\,(B))$ for each IFS B in Y.

(iii) $f\,(cl(\beta int(A))) \subseteq int(f(A))$ for each IFS A in X.

(iv) $f\,(cl(A)) \subseteq int(f(A))$ for each IFβOS A in X.

Then f is an IF contra β^{**}G continuous function.

Proof : (i) ⇒ (ii) is obvious by taking complement in (i).

(ii) $\Rightarrow$ (iii) : Let A $\subseteq$ X, Then B = f(A) $\subseteq$ Y. This implies A = f^{-1}(f(A)) = f^{-1}(B) in X. Now cl(βint(A)) = cl(βint(f^{-1}(B))) $\subseteq$ f^{-1}(int(B)) by (ii). Therefore f(cl(βint(A))) $\subseteq$ f(f^{-1}(int(B))) = int(B) = int(f(A)).

(iii) $\Rightarrow$(iv): Let A $\subseteq$ X be an IFβOS. Then βint(A) = A. By hypothesis, f(cl(βint(A))) $\subseteq$ int(f(A)). Therefore f(cl(A)) = f(cl(βint(A))) $\subseteq$ int(f(A)).

Suppose (iv) holds. Let A be an IFOS in Y. Then f^{-1}(A) is an IFS in X and βint(f^{-1}(A)) is an IFβOS in X. Hence by hypothesis, f(cl(βint(f^{-1}(A)) $\subseteq$ int(f(βint(f^{-1}(A)))) $\subseteq$ int(f(f^{-1}(A))) = int(A) = A. Therefore cl(βint(f^{-1}(A))) = f^{-1}(f(cl(βint(f^{-1}(A)))) $\subseteq$ f^{-1}(A). Now cl(int(f^{-1}(A))) $\subseteq$ cl(βint(f^{-1}(A))) $\subseteq$ f^{-1}(A). This implies f^{-1}(A) is an IFPCS in X and hence an IFβ^{**}GCS in X. Thus f is an IF contra β^{**}G continuous function.

Proposition 3.17: Let f : (X, τ) $\to$ (Y, σ) be a function where X is an IFβ^{**}p$T_{1/2}$ space. Suppose that one of the following properties hold :

(i) f(βcl(A)) $\subseteq$ int(f(A)) for each IFS A in X,

(ii) βcl(f^{-1}(B)) $\subseteq$ f^{-1} (int (B) for each IFS B in Y,

(iii) f^{-1} (cl(B)) $\subseteq$ βint(f^{-1}(B)) for each IFS B in Y.

Then f is an IF contra β^{**}G continuous function.

Proof : (i) $\Rightarrow$ (ii): Let B $\subseteq$ Y, then f^{-1}(B) is an IFS in X. By hypothesis, f(βcl(f^{-1}(B))) $\subseteq$ int(f(f^{-1}(B))) $\subseteq$ int(B). Now βcl(f^{-1}(B)) $\subseteq$ f^{-1}(f(βcl(f^{-1}(B)))) $\subseteq$ f^{-1}(int(B)).

(ii) $\Rightarrow$ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds. Let A be an IFCS in Y. Then cl(A) = A and f^{-1}(A) is an IFS in X. Now f^{-1}(A) = f^{-1}(cl(A)) $\subseteq$ βint(f^{-1}(A)) = pint(A) $\subseteq$ f^{-1}(A), since X is an IFβ^{**}p$T_{1/2}$ space. This implies f^{-1}(A) is an IFPOS in X and hence an IFβ^{**}GOS in X. Therefore f is an IF contra β^{**}G continuous function.

Proposition 3.18: Let f : (X, τ) $\to$ (Y, σ) be a bijective function. Then f is an IF contra β^{**}G continuous function if cl(f(A)) $\subseteq$ f(βint(A)) for every IFS A in X.

Proof: Let A be an IFCS in Y. Then cl(A) = A and f^{-1}(A) is an IFS in X. By hypothesis cl(f(f^{-1}(A))) $\subseteq$ f(βint(f^{-1}(A))). Since f is bijective, f(f^{-1}(A)) = A. Therefore A= cl(A) =

$cl(f(f^{-1}(A))) \subseteq f(\beta int(f^{-1}(A)))$. Now $f^{-1}(A) \subseteq f^{-1}(f(\beta int(f^{-1}(A)))) = \beta int\,(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFβOS in X and hence an IFβ**GOS in X. Thus f is an IF contra β**G continuous function.

Proposition 3.19: If f : (X, τ) → (Y, σ) is an IF contra β**G continuous function and g : (Y, σ) → (Z, δ) is an IF continuous function [5] then g o f : (X, τ) → (Z, δ) is an IF contra β**G continuous function.

Proof: Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFOS in Y, since g is an IF continuous function. Since f is an IF contra β**G continuous function, $f^{-1}(g^{-1}(V))$ is an IFβ**GCS in X. Therefore g o f is an IF contra β**G continuous function.

Proposition 3.20: If f : (X, τ) → (Y, σ) is an IF contra β**G continuous function and g : (Y, σ) → (Z, δ) is an IF contra continuous function, then g o f : (X, τ) → (Z, δ) is an IFβ**G continuous function [9].

Proof: Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFCS in Y, since g is an IF contra continuous function. Since f is an IF contra β**G continuous function, $f^{-1}(g^{-1}(V))$ is an IFβ**GOS in X. Therefore g o f is an IFβ**G continuous function.

Proposition 3.21: A function f : (X, τ) → (Y, σ) is an IF contra β**G continuous function if $f^{-1}(\beta cl(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y.

Proof: Let $B \subseteq Y$ be an IFCS. Since every IFCS is an IFβCS, βcl(B) = B. Then by hypothesis, $f^{-1}(B) = f^{-1}(\beta cl(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = int(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an IFOS in X. Hence f is an IF contra continuous function. Then by Proposition 3.3, f is an IF contra β**G continuous function.

Proposition 3.22: If f : (X, τ) → (Y, σ) is an IFβ**G irresolute function and g : (Y, σ) → (Z, δ) is an IF contra continuous function, then g o f : (X, τ) → (Z, δ) is an IF contra β**G continuous function.

Proof: Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFCS in Y, since g is an IF contra continuous function. As every IFCS is an IFβ**GCS, $g^{-1}(V)$ is an IFβ**GCS in Y. Since f is an IFβ**G irresolute function, $f^{-1}(g^{-1}(V))$ is an IFβ**GCS in X. Therefore g o f is an IF contra β**G continuous function.

4. Intuitionistic fuzzy almost contra β^{**} generalized continuous functions

In this section we have introduced intuitionistic fuzzy almost contra β^{**} generalized continuous functions and studied some of its properties.

Definition 4.1: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be an ***intuitionistic fuzzy almost contra β^{**} generalized*** (IF almost contra β^{**}G) ***continuous function*** if $f^{-1}(A)$ is an IFβ^{**}GCS in X for every IFROS A in Y.

Example 4.2: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b) \rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF almost contra β^{**}G continuous function in (X, τ).

Proposition 4.3: Every IF contra continuous function is an IF almost contra β^{**}G continuous function but not conversely in general.

Proof : Let $A \subseteq Y$ be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous function.

Example 4.4: In example 4.2, f is an IF almost contra β^{**}G continuous function, but not an IF contra continuous function.

Proposition 4.5: Every IF contra semi continuous function is an IF almost contra β^{**}G continuous function but not conversely in general.

Proof: Let $A \subseteq Y$ be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFSCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous function.

Example 4.6: In example 4.2, f is an IF almost contra β^{**}G continuous function but not an IF contra semi continuous function.

Proposition 4.7: Every IF contra pre continuous function is an IF almost contra β^{**}G continuous function but not conversely in general.

Proof : Let $A \subseteq Y$ be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFPCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous function.

Example 4.8: Let X = {a, b} and Y = {u, v}. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b) \rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v) \rangle$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is an IF almost contra β^{**}G continuous function but not an IF contra pre continuous function.

Proposition 4.9: Every IF contra α continuous function is an IF almost contra β^{**}G continuous function but not conversely in general.

Proof: Let $A \subseteq Y$ be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFαCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous function.

Example 4.10: In example 4.2, f is an IF almost contra β^{**}G continuous function but not an IF contra α continuous function.

Proposition 4.11: Every IF contra β continuous function is an IF almost contra β^{**}G continuous function but not conversely in general.

Proof: Let $A \subseteq Y$ be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFβCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous function.

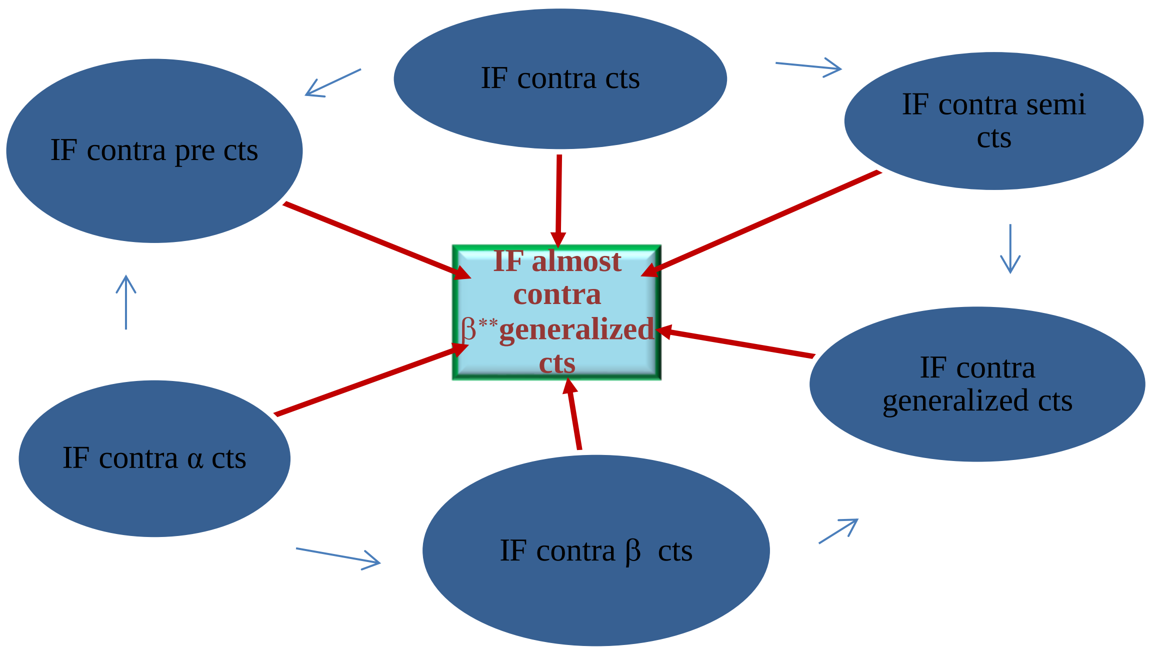
Example 4.12: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is an IF almost contra β^{**}G continuous function but not an IF contra β continuous function.

Proposition 4.13: Every IF contra generalized continuous function is an IF almost contra β^{**}G continuous function but not conversely in general.

Proof: Let $A \subseteq Y$ be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFGCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous function.

Example 4.14: In example 4.2, f is an IF almost contra β^{**}G continuous function but not an IF contra generalized continuous function.

The relation between various types of intuitionistic fuzzy contra continuity is given in the following diagram. In this diagram 'cts' means continuous.



The reverse implications are not true in general in the above diagram.

Proposition 4.15 : If $f : (X, \tau) \to (Y, \sigma)$ is a function, then the following are equivalent:

(i) f is an IF almost contra β^{**}G continuous function

(ii) $f^{-1}(A) \in$ IFβ^{**}GO(X) for every $A \in$ IFRC(Y).

Proof : (i) $\Rightarrow$ (ii): Let A be an IFRCS in Y. Then A^c is an IFROS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFβ^{**}GCS in X. Therefore $f^{-1}(A)$ is an IFβ^{**}GOS in X as $f^{-1}(A^c) = f^{-1}(A)^c$.

(iii) $\Rightarrow$ (i): Let A be an IFROS in Y. Then A^c is an IFRCS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFβ^{**}GOS in X. Therefore $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IF almost contra β^{**}G continuous function.

Proposition 4.16 : If $f : (X, \tau) \to (Y, \sigma)$ be an IFβ^{**}G continuous function and g : $(Y, \sigma) \to (Z, \delta)$ is an IF almost continuous function [9], then $g \circ f : (X, \tau) \to (Z, \delta)$ is an IF almost contra β^{**}G continuous function.

Proof : Let A be an IFROS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since f is an IFβ^{**}G continuous function, $f^{-1}(g^{-1}(A))$ is an IFβ^{**}GCS in X. Hence $g \circ f$ is an IF almost contra β^{**}G continuous function.

Proposition 4.17 : If $f : (X, \tau) \to (Y, \sigma)$ is a function, then the following are equivalent:

(i) f is an IF almost contra β^{**}G continuous function

(ii) $f^{-1}(A) \in$ IFβ^{**}GO(X) for every $A \in$ IFRC(Y)

(iii) $f^{-1}(int(cl(G))) \in$ IFβ^{**}GC(X) for every IFOS $G \subseteq Y$.

Proof : (i) $\Leftrightarrow$(ii) is obvious from the Proposition 4.15.

(ii) $\Rightarrow$ (iii) Let G be any IFOS in Y. Then int(cl(G)) is an IFROS in Y. By hypothesis, $f^{-1}(int(cl(G)))$ is an IFβ^{**}GCS in X. Hence $f^{-1}(int(cl(G))) \in$ IFβ^{**}GC(X).

(iii) $\Rightarrow$ (i) Let A be any IFROS in Y. Then Ais an IFOS in Y. By hypothesis, we have $f^{-1}(int(cl(A))) \in$ IFβ^{**}GC(X). That is $f^{-1}(A) \in$ IFβ^{**}GC(X), since int(cl(A)) = A. Hence f is an IF almost contra β^{**}G continuous function.

Proposition 4.18 : Let $f : (X, \tau) \to (Y, \sigma)$ be a function and let $f^{-1}(A)$ be an IFRCS in X for every IFROS A in Y. Then f is an IF almost contra β^{**}G continuous function.

Proof : Let A be an IFROS in Y. By hypothesis, $f^{-1}(A)$ is an IFRCS in X. Since every IFRCS is an IFβ^{**}GCS, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IF almost contra β^{**}G continuous function.

Proposition 4.19: Let f : (X, τ) → (Y, σ) be an IF almost contra β^{**}G continuous function and X an IFβ**p$T_{1/2}$ space. Then f is an IF contra pre continuous function.

Proof : Let B be an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an IFβ^{**}GCS in X. Since X is an IFβ**p$T_{1/2}$ space, $f^{-1}(B)$ is an IFPCS in X. Hence f is an IF contra pre continuous function.

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