Behavioral Analysis of Single Unit System Using RPGT

Dr. Reena Garg
Assistant Professor
Faculty of Sciences (Department of Mathematics)
J.C.Bose University of Science and Technology,YMCA-121006
Faridabad(Haryana)-India
Correspondence id : reenagargymca@gmail.com

Abstract : In this paper , reliability and behavioral analysis of single unit system with scheduled maintenance using Regenerative Point Graphical Technique (RPGT) is discussed for system parameters .These parameters are used to measure system performance.

1. Mean Time To System Failure(MTSF).

2.Total fraction of time for which the system is available (Availability).

3. The busy period of the Repairman doing any given job.

4. The number of the Repairman's visits or replacement.

Tables and graphs are prepared to represent to study the behavior.

Keywords : Availability , Steady State , Primary Circuit , Secondary Circuit , Tertiary Circuit , Base - State ,Regenerative Point Graph Technique ,Mean Time to the System Failure, Busy Period of the Server, Expected number of Server's visits.

Assumptions :

The following assumptions have been associated with this model :

1. Initially, the whole system is good and operable.

2. All failures follow exponential time distribution .

3. All repairs follow general time distribution and are perfect.

4. Pre-emptive resume policy has been adopted for repair purpose.

5. There are two bank computers working in parallel redundancy.

6. On failure of any one bank computer, the whole system works in degraded state.

Notations :

λ : failure rate of operative unit

γ_1/γ_2 : rate of decrease/ increase of demand so as to become < / ≥ production

γ_3 : rate of going from upstate to downstate

γ_4 : rate of change of state from down to up when there is no production with system and demand is there

β : rate of requirement of schedualed maintenance

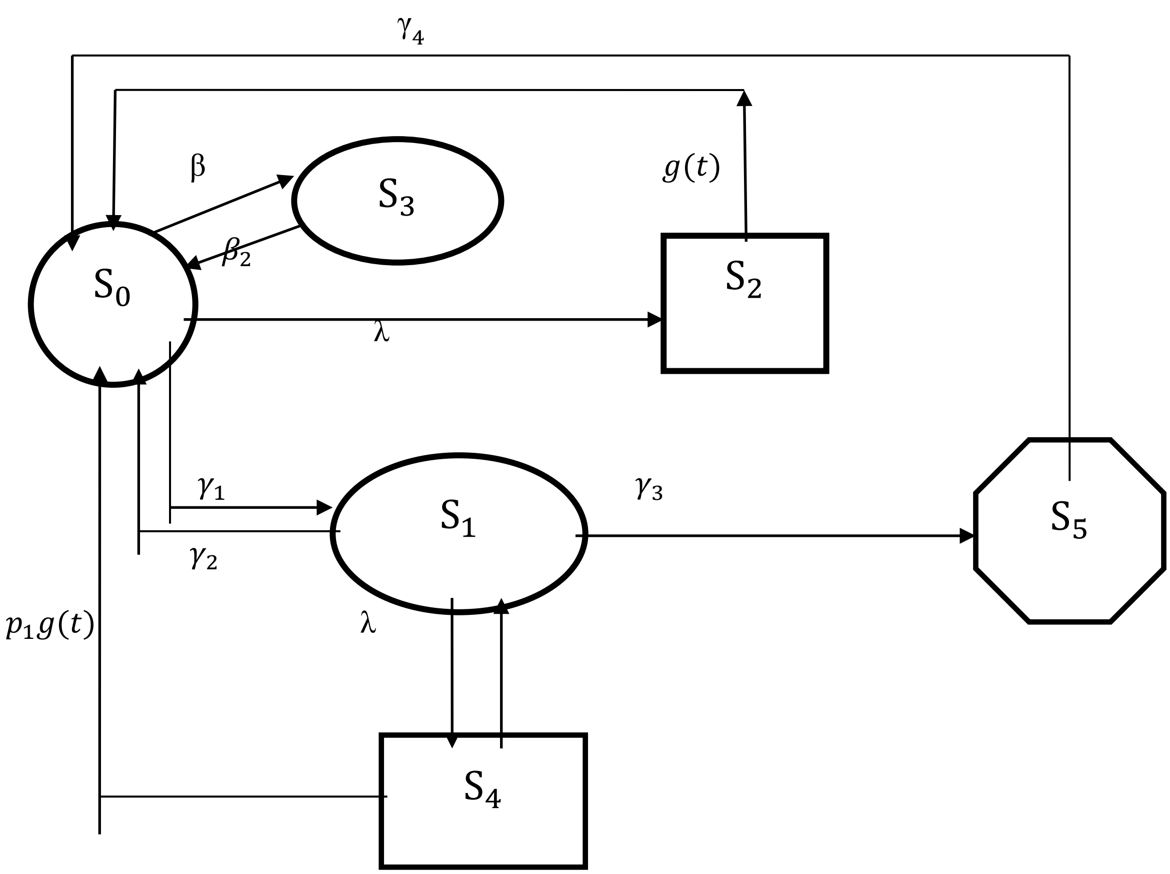
β_2 : rate of doing schedualed maintenance

p_1 : probability that during repair time period demand ≥ production

p_2 : probability that during repair time period demand < production

g(t) : p.d.f of repair time of unit.

Transition Diagram of the System :



Evaluation of Parameters of the System :

Analysis of System*:* They key parameters (under steady state conditions) are evaluated by determining a base-state and applying RPGT. The MTSF is evaluated w.r.t. initial state '0' and other parameters are obtained by using base – state.

Determination of base state*:* From the transition diagram the various paths from state i to reachable state j at all vertices, are shown in Table 1:

Table 1: Paths from state 'i' to the reachable state 'j':

Vertex	0	1	2	3	4	5
0	(0,1,0) (0,2,0) (0,3,0) (0,4,0) (0,5,0)	(0,1)	(0,2)	(0,3)	(0,1,4)	(0,5)
1	(1,0) (1,5,0) (1,4,0)	(1,0,2) (1,4,0,2) (1,5,0,2)	(0,1,0) (0,1,0) (0,1,0)	(1,0,3) (1,5,0,2) (1,4,0,2)	(1,4)	(1,5)
2	(2,0)	(2,0,1)	(2,0,2)	(2,0,3)	(2,0,1,4)	(2,0,1,5)
3	(3,0)	(3,0,1)	(3,0,2)	(3,0,3)	(3,0,1,4)	(3,0,1,5)
4	(4,0) (4,1,0) (4,1,5,0)	(4,1) (4,0,1)	(4,0,2) (4,1,0,2)	(4,1,5,0,3) (4,0,3)	(4,1,4) (4,0,4,1)	(4,1,5) (4,0,1,5)
5	(5,0)	(5,0,1)	(5,0,2)	(5,0,3)	(5,0,4)	(5,0,1,5)

Table 2: Primary, Secondary and Tertiary circuits at the vertices

	Primary	Secondary	Tertiary
0	(0,1,0) (0,2,0) (0,3,0)	(1,4,1) NIL NIL	NIL
1	(1,0,1) (1,4,1)	(0,3,0) (0,2,0)	NIL
2	(2,0,2)	(0,3,0) (0,1,0) (0,1,4,0)	NIL
3	(3,0,3)	(0,1,0) (0,2,0) (0,1,5,0) (0,1,4,0)	(1,4,1) NIL
4	(4,1,4)	(1,0,1) (1,5,0,1)	(0,2,0) (0,3,0)
5	(5,0,5,1)	(0,1,0) (0,2,0) (0,3,0) (1,0,1) (1,4,1)	(1,4,1) NIL

Transition Probability and the Mean sojourn times :

$\boldsymbol{q_{i,j}(t)}$: Probability density function (p.d.f.) of the first passage time from a regenerative state 'i' to a regenerative state 'j' or to a failed state 'j' without visiting any other regenerative state in (0,t].

$\boldsymbol{p_{i,j}}$: Steady state transition probability from a regenerative state 'i' to a regenerative state 'j' without visiting any other regenerative state.

$p_{i,j} = \boldsymbol{q^*_{i,j}(0)}$; where * denotes Laplace transformation.

Table 3: Transition probabilities

$q_{i,j}$	$p_{i,j}$
$q_{01} = \gamma_1 e^{[-(\beta+\lambda+\gamma_1)]t}$	$p_{01} = \frac{\gamma_1}{\beta+\lambda+\gamma_1}$
$q_{02} = \lambda e^{[-(\beta+\lambda+\gamma_1)]t}$	$p_{02} = \frac{\lambda}{\beta+\lambda+\gamma_1}$
$q_{03} = \beta e^{[-(\beta+\lambda+\gamma_1)]t}$	$p_{03} = \frac{\beta}{\beta+\lambda+\gamma_1}$
$q_{14} = \lambda e^{[-(\gamma_3+\gamma_2+\lambda)]t}$	$p_{14} = \frac{\lambda}{\gamma_2+\gamma_3+\lambda}$
$q_{15} = \gamma_3 e^{[-(\gamma_3+\gamma_2+\lambda)]t}$	$p_{15} = \frac{\gamma_3}{\gamma_2+\gamma_3+\lambda}$
$q_{10} = \lambda_2 e^{[-(\lambda_3+\lambda_2+\lambda)]t}$	$p_{10} = \frac{\gamma_2}{\gamma_2+\gamma_3+\lambda}$
$q_{20} = g(t)e^{[-g(t)]t}$	$p_{20} = 1$
$q_{30} = \beta_2 e^{[-(\beta_2)}$	$p_{30} = 1$
$q_{41} = p_2 g(t)e^{[-(p_2 g(t)+p_1 g(t))]t}$	$p_{41} = \frac{p_2 g(t)}{p_2 g(t)+p_1 g(t)}$
$q_{40} = p_1 g(t)e^{[-(p_1 g(t)+p_2 g(t))]t}$	$p_{40} = \frac{p_1 g(t)}{p_1 g(t)+p_2 g(t)}$
$q_{50} = \gamma_4 e^{[-(\lambda_4)]t}$	$p_{50} = 1$

Table 4 : Mean Sojourn Times

$\mu_0 = \frac{1}{\beta + \lambda + \gamma_1}$
$\mu_1 = \frac{1}{\gamma_2 + \gamma_3 + \lambda}$
$\mu_2 = \frac{1}{g(t)}$
$\mu_3 = \frac{1}{\beta_2}$
$\mu_4 = \frac{1}{p_2 g(t) + p_1 g(t)}$
$\mu_5 = \frac{1}{\gamma_4}$

Transition Probability Factors :

The mean time to system failure and all the key parameters of the system under steady state condition are evaluated, applying RPGT and using '0'as the base state of the system as under :

The transition probability factor of all the reachable states from the base state 'ξ'='0' are:

V_{00}= (0,1,0) + (0,2,0) + (0,3,0) = $p_{01}\, p_{10} + p_{02}\, p_{20} + p_{03} p_{30}$

=$[\frac{\gamma_1}{\beta+\lambda+\gamma_1} * \frac{\gamma_2}{\gamma_3+\gamma_2+\lambda}] + \frac{\lambda}{\beta+\lambda+\gamma_1} + \frac{\beta}{\beta+\lambda+\gamma_1}$

V_{01}= (0,1) = p_{01} = $[\frac{\gamma_1}{\beta+\lambda+\gamma_1}]$

V_{02}= (0,2) = p_{02} = $[\frac{\lambda}{\beta+\lambda+\gamma_1}]$

V_{03} = (0,3) = p_{03} = $[\frac{\beta}{\beta+\lambda+\gamma_1}]$

V_{04}= (0,1,4) = $p_{01}\, p_{14}$ = $[\frac{\gamma_1}{\beta+\lambda+\lambda_1} * \frac{\lambda}{\gamma_3+\gamma_2+\lambda}]$

V_{05} = (0,1,5) = $p_{01}\, p_{15}$ = $[\frac{\gamma_1}{\beta+\lambda+\gamma_1} * \frac{\gamma_3}{\gamma_3+\gamma_2+\lambda}]$

(a) MTSF(T_0) : The regenerative un-failed states to which the system can transit(initial state '0'), before entering any failed state are: 'i' = 0,1,2,3 taking 'ξ' = '0'.

MTSF (T_0) =

$$\left[\Sigma_{i,sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}i\right)\right\}\mu i}{\Pi_{m_{1\neq\xi}}\left\{1-V_{\overline{m_1 m_1}}\right\}}\right\}\right] \div \left[1-\Sigma_{sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}\xi\right)\right\}}{\Pi_{m_{2\neq\xi}}\left\{1-V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$T_0 = (V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,3}\mu_3)/[1-(0,1,0)+(0,3,0)]$

=

$$\frac{[(\gamma_1 * \gamma_2/\,(\beta+\lambda+\gamma_1) * \ (\gamma_2+\gamma_3+\lambda\,)] + [(\,\beta+\lambda\,/\,(\beta+\lambda+\gamma_1) * (\,\beta+\lambda+\gamma_1)] + [(\gamma_1/\,(\beta+\lambda+\gamma_1) * \ (}{1-[(\gamma_1 * \gamma_2/\,(\beta+\lambda+\gamma_1) * \ (\gamma_2+\gamma_3+\lambda\,)] - \ \beta\,/(\beta+\lambda+\gamma_1)}$$

(b) Availability of the System(A_0): The regenerative states at which the system is available are 'j' = 0,1,2,3 and the regenerative states are 'i' = 0 to 4 taking 'ξ' = '0' the total fraction of time for which the system is available is given by

$$A_0 = \left[\Sigma_{j,s_r}\left\{\frac{\left\{pr\left(\xi\xrightarrow{s_r}j\right)\right\}f_j\mu_j}{\Pi_{m_{1\neq\xi}}\left\{1\text{-}V_{m_1,m_1}\right\}}\right\}\right] \div \left[\Sigma_{i,s_r}\left\{\frac{\left\{pr\left(\xi\xrightarrow{s_r}i\right)\right\}}{\Pi_{m_{2\neq\xi}}\left(1\text{-}V_{m_2,m_2}\right)}\right\}\right]$$

$$A_0 = \left[\Sigma_j V_{\xi,j}.\, f_j.\, \mu_j\right] \div \left[\Sigma_i V_{\xi,i}\, .\, \mu_i^1\right]$$

$$A_0 = \frac{V_{00} \ * \ f_0 * \ \mu_0 + V_{01} \ * \ \ f_1 * \mu_1 + \ V_{03} * \ f_3 * \mu_3}{V_{00} \ * \ \mu_0' + \ V_{01} \ * \ \mu_1' + V_{02} \ * \ \mu_2' \ + V_{03} \ * \ \mu_3' + \ V_{04} \ * \ \mu_4' + V_{05} \ * \ \mu_5'}$$

Put , $f_j = 1 \qquad \forall\, j = 0,1,3$

$\mu_j' = \ \mu_j \quad \forall\, j = 0,1,2,3,4,5$

$$\therefore \ A_0 \ = \frac{K_1 \ + K_2 + \ K_3}{K_1 \ + K_2 + \ K_3 + K_4 \ + K_5}$$

Put , $K = \ K_1 \ + K_2 + \ K_3 + K_4 \ + K_5$

$$\therefore \ A_0 = \frac{K_1 \ + K_2 + \ K_3}{K}$$

(c) Busy Period of the Server(B_0): The regenerative states where server 'j' = 1,2,3,4,5 and regenerative states are 'i' = 0 to 5, taking ξ = '0', the total fraction of time for which the server remains busy is

$$B_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}n_j}{\prod_{m_1 \neq \xi}\{1-V_{m_1,m_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}\mu_i^1}{\prod_{m_2 \neq \xi}(1-V_{m_2,m_2})}\right\}\right]$$

$$B_0 = \sum_j V_{\xi,j}.n_j \div \sum_i V_{\xi,i}.\mu_i^1$$

$$B_0 = \frac{V_{01} * n_1 + V_{02} * n_2 + V_{03} * n_3 + V_{04} * n_4 + V_{05} * n_5}{V_{00} * \mu_0 + V_{01} * \mu_1 + V_{02} * \mu_2 + V_{03} * \mu_3 + V_{04} * \mu_4 + V_{05} * \mu_5}$$

and taking $n_j = \mu_j \ \forall \ j = 0,1,2,3,4,5$

$$B_0 = 1 - \frac{V_{00} * \mu_0}{K} = 1 - \frac{K_1}{K}$$

(d) Expected number of inspection by repair man(V_0): The regenerative states where the repair man do this job j = 1,2,3 Taking 'ξ' = '0', the number of visit by the repair man is given by

$$V_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}}{\prod_{k_1 \neq \xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}\mu_i^1}{\prod_{k_2 \neq \xi}(1-V_{k_2,k_2})}\right\}\right]$$

$$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}.\mu_i^1\right]$$

$$V_0 = \frac{V_{01} + V_{03}}{V_{00} * \mu_0 + V_{01} * \mu_1 + V_{02} * \mu_2 + V_{03} * \mu_3 + V_{04} * \mu_4}$$

$$= \frac{V_{01} + V_{03}}{K} = \frac{P_{01} + P_{03}}{K} = \frac{\frac{\gamma_1}{\beta + \lambda + \gamma_1} + \frac{\beta}{\beta + \lambda + \gamma_1}}{K} = \frac{\gamma_1 + \beta}{K(\beta + \lambda + \gamma_1)}$$

Particular Cases:

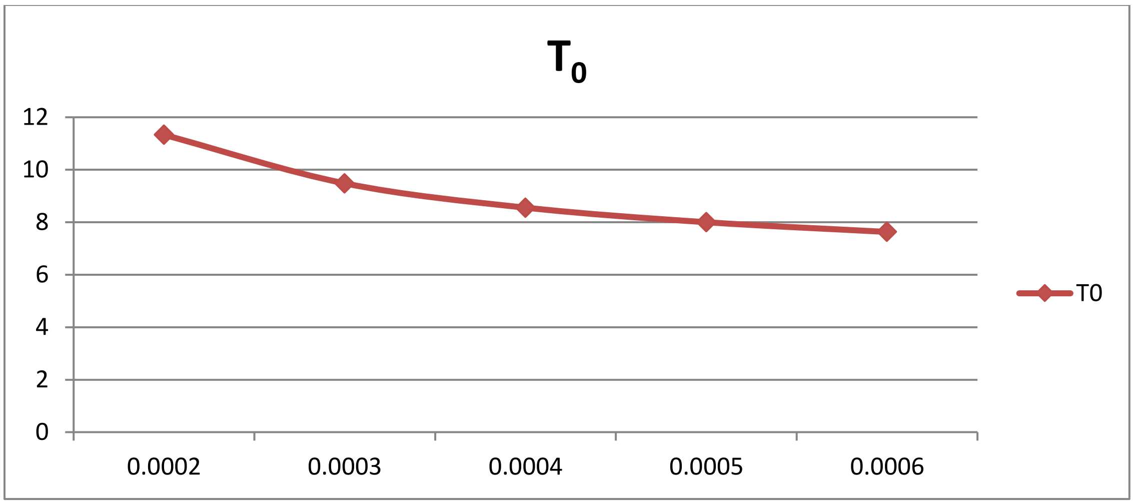
Take the value of β=.0001,β_2=0.0002 to .0010,λ=.02,γ_1=.07,$\gamma_2 = .235$,

$\gamma_3 = .353$,$\gamma_4 = .4213$, $p_1 = .665$,$p_2 = .335$

MTSF : The MTSF of the system is calculated for different values of failure rates by taking failure rates β_2 = 0.005, 0.006, .007, .009 and .010.

Table 6 : MTSF

β_2	T_0
0.0002	11.329544
0.0003	9.4797805
0.0004	8.5548495
0.0005	7.9999105
0.0006	7.629951
0.0007	7.365694



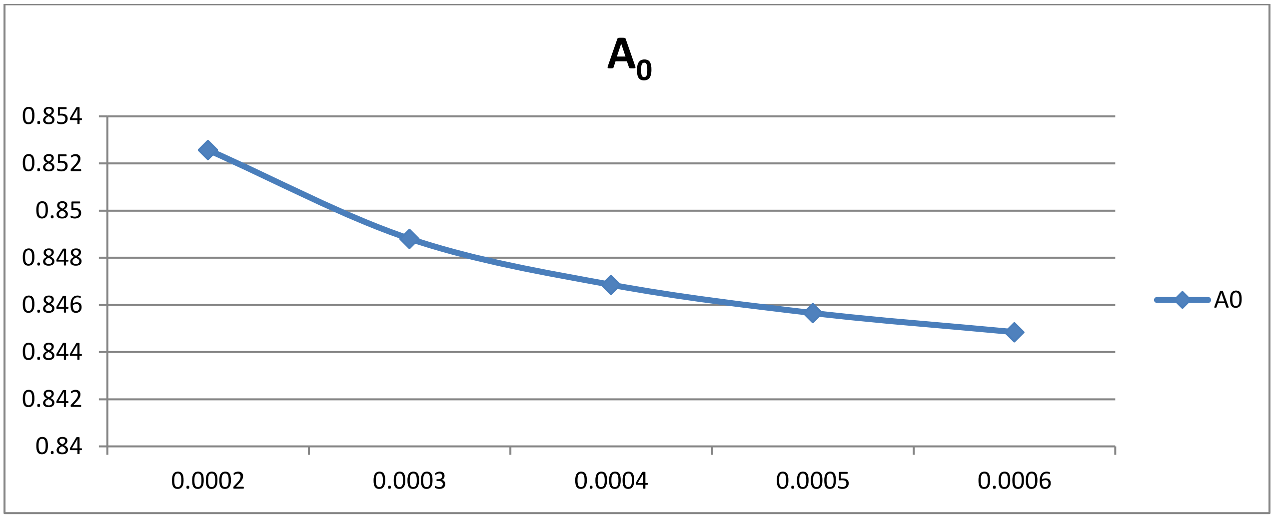
Conclusion : It can be concluded that values of MTSF shows expected trend for different values of failure rate. MTSF decreases with increase in the failure rate.

Availability: The Availability of the system is calculated for different values of failure rates by taking failure rates β_2 = 0.002, 0.003, .004, .005,.006 and .007 and $g(t) = e^t$, t = 0.03.

Table 7 : Availability

β_2	A_0
0.002	0.8525735
0.003	0.8488095
0.004	0.8468545
0.005	0.8456571
0.006	0.8448483

0.007	0.8442654

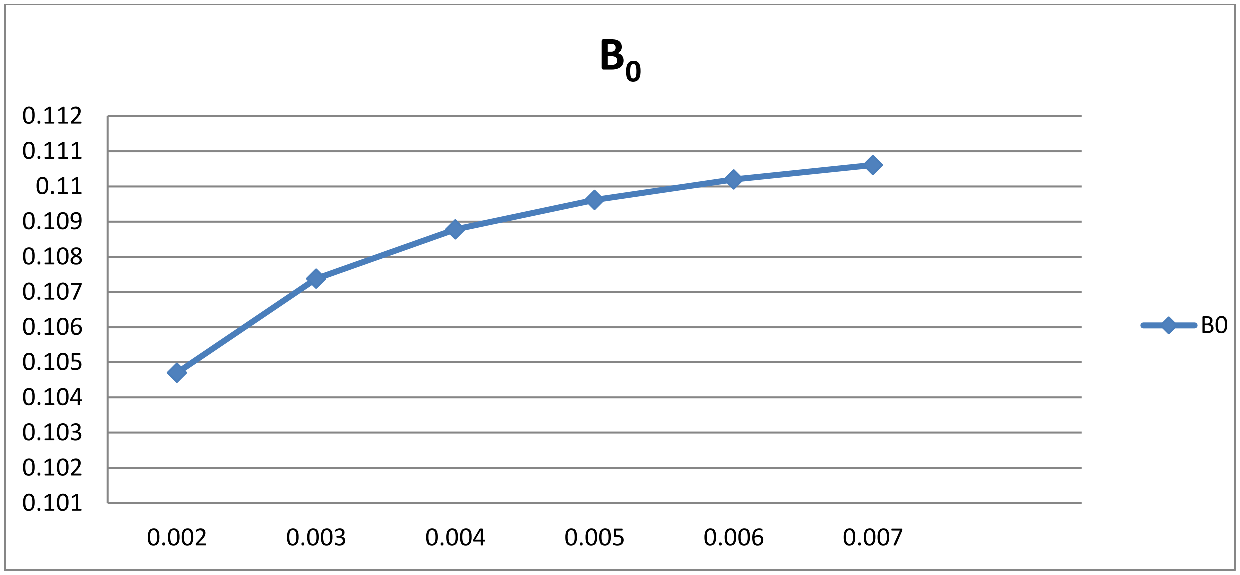


Conclusion : It can be concluded that values of Availability shows expected trend for different values of failure rate. Availability decreases with increase in the failure rate.

Busy period: The Busy Period of the system is calculated for different values of failure rates by taking failure rates β_2 = 0.002, 0.003, .004, .005,.006 and .007.

Table 7 : Busy Period

β_2	β_0
0.002	**0.1047061096**
0.003	**0.107379239**
0.004	**0.108776648**
0.005	**0.109618063**
0.006	**0.110192433**
0.007	**0.11060639**

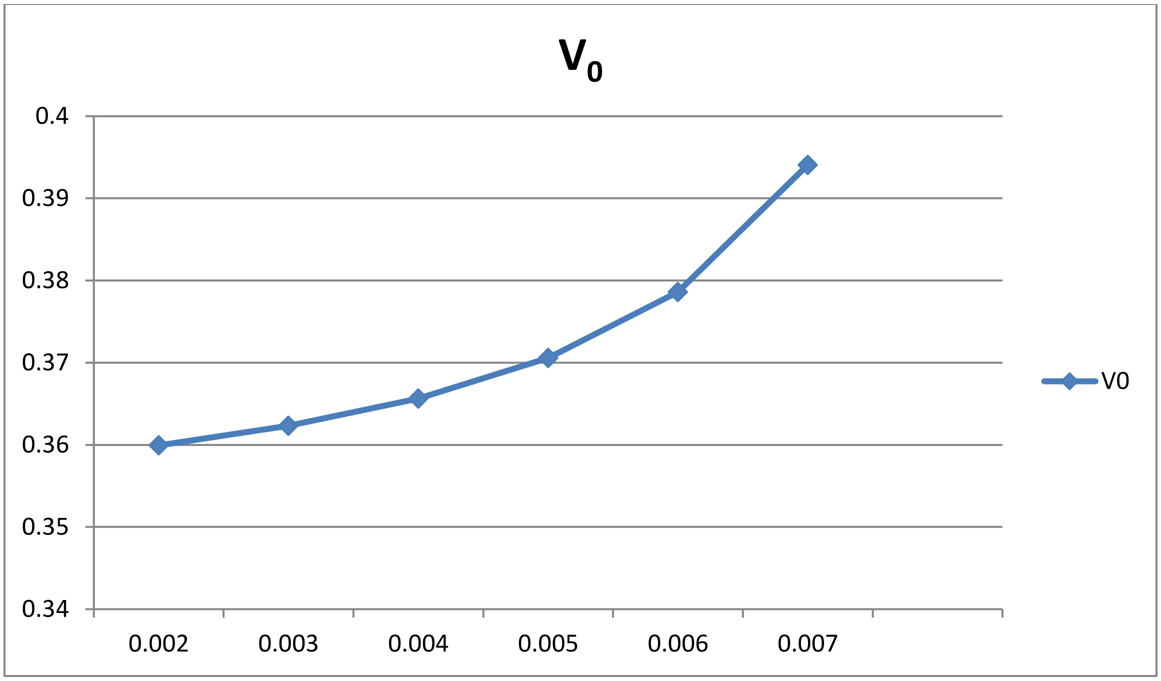


Conclusion : It can be concluded that values of Busy Period shows expected trend for different values of failure rate. Busy Period increases with increase in the failure rate.

Expected number of server's visits : The Expected number of Server's visits of the system is calculated for different values of failure rates by taking failure rates β_2 = 0.002, 0.003, .004, .005,.006 and .007.

Table 8: Expected number of Server's visits

β_2	V_0
0.002	0.3599339348
0.003	0.3623294899
0.004	0.3656532991
0.005	0.3705745479
0.006	0.3786091109
0.007	0.3940781928



Conclusion : It can be concluded that values of Expected number of server's visits shows expected trend for different values of failure rate. Expected number of server's visits increase with increase in the failure rate.

References :

1. Gulshan Taneja, Reetu Malhotra and Ashok Chitkara ," Comparative Profit Analysis of two reliability models with varying demand ", Aryabhatta J. of Maths & Info. Vol. 8, No. 2,July-Dec 2015.
2. P. Goel, "Availability Analysis of a Dairy Plant With Priority in Repair Using RPGT," 2014.
3. S. Malik, "Reliability modeling of a computer system with preventive maintenance and priority subject to maximum operation and repair times," International Journal of System Assurance Engineering and Management, vol. 4, pp. 94-100, 2013
4. P. G. Navneet Malik, "Behavioral Analysis of Single Module under Software and Hardware Failure using Regenerative Point Graphical Technique," International Journal Of Informative And Futuristic Research (Ijifr), vol. 1, 2013.
5. S. Malik and J. Anand, "Reliability and economic analysis of a computer system with independent hardware and software failures," Bulletin of Pure & Applied Sciences-Mathematics and Statistics, vol. 29, pp. 141-154, 2010.
6. Gupta,P., Lal,A.K., Sharma,R.K., and Singh,J. (2005): Behavioral Study of the Cement Manufacturing Plant-A Numerical Approach, Journal of Mathematics and System Sciences, Vol. 1, pp 50-70.

7. Goel, P. & Singh, J. (1997): Availability Analysis of A Thermal Power Plant Having Two Imperfect Switches, Proc. (Reviewed) of 2nd Annual Conference of ISITA.
8. Gupta, P., Singh, J. & Singh , I. P. (2004): Availability Analysis of Soap Cakes production System – A Case Study, Proc. National Conference on Emerging Trends in Manufacturing System, SLIET, Longowal (Punjab) Pb 283-295.
9. Kumar, S. and Goel, P. (2014): Availability Analysis of Two Different Units System with a Standby Having Imperfect Switch Over Device in Banking Industry, Aryabhatta Journal of Mathematics & Informatics, ISSN: 0975-7139, Vol. 6, No. 2, p 299-304.
10. Gupta V. K. (2011): Analysis of a single unit system using a base State Aryabhatta J. of Maths & Info. Vol. 3 (1) pp 59-66