Pumping Lemma: A Leeway Classification of a few kinds of semi groups with space valued Fuzzy Mustiest Languages weakly inside principles

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Abstract: Before tackling the question we should perhaps begin by saying what a semigroup is. A non-empty set S endowed with a single binary operation ° is known as a semigroup if for evey x,y,z [2][3]in S, $(xy)z = x(yz)$. If in addition there exists 1 in S such that, for every x in S, $1x = x1 = z$ we say that S is a semigroup with identity or (more usually) a monoid. In this article, we give a definition of an space valued fuzzy weakly inside ideal. We study some interesting properties [4] [5] [6] of space valued fuzzy weakly inside ideals and the relationship between space valued fuzzy weakly interior ideals and space valued fuzzy principles. We characterize some semigroups by using interval valued fuzzy weakly interior principles. Moreover, we found theorems of the homorphic image and the preimage of an space valued fuzzy weakly inside principle in semigroups [7].

Keywords: Period valued fuzzy weakly inside principles, normal semigroup, intra-normal semigroup, semisimple semigroup, and weakly normal semigroup, Pumping Lemma.

1. INTRODUCTION

Suppose S is a semigroup with identity monoid, we shall be confining ourself today to semigroups that have no additional structure [1]. Thus, though semigroups feature quite prominently in parts of functional analysis, the algebraic structure of those semigroups is usually very straightforward and so they scarely rate a mention in any algebraic theory. Equally, although they are often of greater algebraic interest, We shall say nothing about topological semigroups. Assusme that begin by answering a slightly different question: Who studies semigroups? Sectopm 20 in Mathematical Reviews is entitled “ Groups and Generalizatrions” and has two **leper colonies** at the end, is known as 20M semigroups and 20N other generalizations. In the introduction to their algebraic theory of semigroups in 1961 Clifford and Preston [2] remarked that about thirty papers on semigroups per year were currently appearing. Incidentally, comparable figures for other generalizations are about on third of these. So it is clear that of all generalizations of the group concept the semigroup is the one that has attracted the most interest by far. We shall in due course hazard a guess as to why this is so. Mathematics are rightly a bit suppicious of theories whose only motive seems to be to generalize existing theories-and if the only motivation for semigroup theory were to examine group theoretical results with a view to generalization, then we would have no very convincing answer to the question of in this title.

Definition: For clearly every ring (R,+, .) [2] is a semigroup if we simply neglect the operation + . The converse is certainly not true: that is, there are semigroups (S, .) [3] with zero on which it is not possible to define an operation + so as to create a ring (S, +, .) . The easiest way to see this is to recall the known result that a ring (R, +, .) with the property that $x^2 = x \; for \; all \; x \; in \; R$ is necessarily commutative satisfies $xy = yx \; for \; every \; x \; in \; R$. Assume that

$S = (A \times B) \cup \{0\},\ where\ A, B\ are\ non-empty\ sets\ and\ 0$ does not belongs to $A \times B, and\ make\ S$ into a semigroup with zero by defining $(a,b)0 = 0(a,b) = 00 = 0, (a_1,b_1)(a_2,b_2) = (a_1,b_2)$.

Definition: Suppose that a semigroup (S, .) is normal iff $(for\ each\ a \in S)(\exists\ a'belongs\ S)\ aa'a = a, a'aa' = a' so, simply\ we\ take\ a' = xax\ we\ have\ aa'a = a, a'aa' = a'$.

The element $a' is\ usually\ is\ known\ as\ an\ inverse\ of\ a$, but it should be noted that this is a weaker concept of inverse than the one used in group theory: Observe that four element semigroup with Cayley table as hold [4] [5]:

	w	x	y	z
w	w	x	y	z
x	w	x	w	x
y	y	z	y	z
z	y	z	y	z

It is easy to check that every element is an inverse of every other element.

Theorem 1: The following conditions on a regular semigroup S are equivalent:

(i) Idempotents commute; (ii) Inverse are unique.

Proof: Suppose that idempotents commute. Assume that $a', a^*\ be\ an\ inverse\ of\ a, Then\ a' = a'aa' = a'aa^*aa' = a'aa^*aa^*\ aa' = a^*aa'aa^*aa' = a^*aa'aa'aa^* = a^*aa^* = a^*$ Assume that $e, f\ be\ idempotents\ and\ let\ x\ be\ the\ unique\ invese\ of\ ef: efxef = ef,$ $xefx = x. Then\ fxe\ is\ idempotenet, since\ (fxe)^2 = f(xefx)e = fxe; and\ ef\ is\ an$ inverse of $fxe: (fxe)(ef)(fxe) = f(xefx)e = fxe, (ef)(fxe)(ef) = efxef - ef$ But an idempotent $i\ is\ its\ own\ unique\ inverse\ (iii = i, iii = i) and\ so\ ef = fxe, and\ idempotent$. Similarly $fe\ is$ idempotent.

The unique inverse of $ef\ is\ thus\ ef\ itself$. On the other hand $fe\ is\ an\ inverse\ of\ ef, since$

$(ef)(fe)(ef) = (ef)^2 = ef, (fe)(ef)(fe) - (fe)^2 = fe$. It follows that $ef = fe, as\ required$.

That argument goes back to the early 1950s, to some basic work by Vagner and [16] [17] Preston [12] [13] [14]. Regular semigroups satisfying either one of the conditions are satisfied. This is known as inverse semigroup. Let me give a not very well known example due to Schein and McAlister. Let G be a group and let $K(G)$ be the set of all right semigroups of G. This includes G itself and also the semigroups of the subgroup1, which are effectively the elements of G. By the definition of an operation * on $K(G)$ by $Ha * kb = (H \vee aka^{-1})ab$. This is a natural definition: [11] it is not hard to check that $Ha * kb$ is the smallest coset containing the product $Hakb$. clearly $Hakb = (H \vee aka^{-1})ab \subseteq (H \vee aka^{-1})ab$. Conversely, suppose that $Hakb \subseteq Pc(\in K(G))$. then in particular $ab \in Pc$ and so $Pc = Pab$. Now $Had \subseteq Hakb \subseteq Pab$ and so H $\subseteq$ P; also $(aka^{-1})ab = akb \subseteq Hakb \subseteq Pab$ and so $aka^{-1} \subseteq P$. Thus $H \vee aka^{-1} \subseteq P$ and so $(H \vee aka^{-1})ab \subset Pab = Pc$. It is a routine matter to check that * is an associative [15] operation and that $(a^{-1}Ha)a^{-1}$ is an inverse of Ha in the semigroup$(K(G), *)$. Now suppose that Ha is idempotent: [10]

$Ha = Ha * Ha = (H \vee aHa^{-1})a^2$. In fact the idempotent of $(K(G), *)$ are precisely the subgroups of G. for any two subgroups $H, K\ of\ G$ then $H * K = H \vee K = K * H$. Thus idepotents commute and so $(K(G), *)$ is an inverse semigroup. The normal subgroups $N\ of\ G$ are such that, for all $Ha\ in\ K(G),\ N * Ha = (N \vee H)a = (NH)a.\ Ha * N = (H \vee aNa^{-1})a = (H \vee N)a = (NH)a,$ and so are central idempotents in $(K(G), *)$. Conversely, if N is a central idempotent then for all $a\ in\ G. Na = N * 1a = 1a * N = (1 \vee aNa^{-1})a = aN\ and\ so\ N\ is\ normal$.

The main reason that semigroups turn up in mathematics is that one is very often interested in self-maps of a set of one kind or another, and whenever f,g,h are such maps it is automatically the case that $(f\circ g)\circ h = f\circ (g \circ h)$.

II. Green's Equivalences

The equivalences of $\mathfrak{D}, \mathfrak{J}, \mathfrak{H}, \mathfrak{R}$ **and** $\mathfrak{L}$

If a is an element of a semigroup S, the smallest left ideal containing a is $Sa \cup \{a\}$, which we may conventiently write as S^1a, and which we shall call the principal left ideal generated by a. An equivalence relation $\mathfrak{L}$ on S is then defined by the rule that $a\ \mathfrak{L}$ b if and only if $a\ and\ b$ generate the same principal left ideal, i.e. if and only if $S^1a = S^1b$. Similarly, [18] we define $\mathfrak{R}$ **by** the rule that $a\ \mathfrak{R}$ b if and only if $a\ and\ b$ generate the same principal right ideal, i.e. if and only if $aS^1 = bS^1$.

Lemma 2.1 : Assume that a, b be the elements of a semigroup S. Then $a\ \mathfrak{L}$ b if and [18] only if there exist $x, y\ in\ S^1$ such that $xa = b, yb = a$ Also, $a\ \mathfrak{R}$ b if and only if there exist $u, v\ in\ S^1$ such that $au = b, bv = a$.

Lemma 2.2: Suppose $\mathfrak{L}$ is a right congruence and $\mathfrak{R}$ is a left congruence. We have the intersection of two equivalences is again an equivalence. Since the intersection of $\mathfrak{L}$ **and** $\mathfrak{R}$ is of great importance in the development of the theory, [18] we reserve for it the letter $\mathfrak{H}$ **.**

Proposition 2.3 : The relations $\mathfrak{L}$ **and** $\mathfrak{R}$ Commute.

Proof: If $(a, b) \in \mathfrak{L}$ **and** $\mathfrak{R}$ then there exists $c \in S\ such\ that\ a\ \mathfrak{L}$ **c, c** $\mathfrak{L}$ **b.** That is, there exists$x, y, u, v \in S^1$ such that $xa = c, cu = b; yc = a, bv = c$. If we write d for the element $ycu\ of\ S, then\ au = ycu = d\ and\ dv = ycuv = ybv = yc = a$, [18] from which it follows that $a\ \mathcal{R}$ **d** also, $yb = ycu = d\ and\ xd = xycu = xau = cu = b$. We deduce that $(a, b) \in \mathfrak{R} \circ \mathfrak{L}$. Thus $\mathcal{L}\circ \mathcal{R} \subseteq \mathcal{R}\circ\mathcal{L}$. The principal [18] two-sided ideal generated by an element a of S is $S^1aS^1 = SaS \cup aS \cup Sa \cup \{a\}, and\ we\ write\ a\ \mathcal{J}$ b if $S^1aS^1 = S^1bS^1$, i.e. if there exist $x, y, u, v\ in\ S^1$ for which $xay = b, ubv = a$. It is immediate that $\mathcal{L} \subseteq \mathcal{J}, \mathcal{R} \subseteq \mathcal{J}$; hence, since $\mathcal{D}$ is the smallest equivalence containing $\mathfrak{L}$ **and** $\mathfrak{R}$ it follows that by[1] Green certain classes of semigroups we do have equality. Certainly in commutative semigroups we have $\mathcal{D} = \mathcal{J} = \mathcal{H} = \mathcal{R} = \mathcal{L}$.

Preposition 2.4 : If $S\ is\ a\ periodic\ semigroup, then\ \mathcal{D} = \mathcal{J}$.

Proof: Supp ose that $a, b\ in\ S\ are\ such\ that$ a $\mathcal{J}$ b. Then there exist $x, y, u, v\ in\ S^1\ such\ that$

$xay = b, ubv = a. To$ prove the required result we require to show that there exists $c\ in\ S$ for which $a\ \mathcal{L}\ \boldsymbol{c}\ ,\ \boldsymbol{c}\ \mathcal{R}\ \boldsymbol{b}$. Now , it follows easily from the equations are hold: [18]

$$a = (ux)a(yv) = (ux)^2a(yv)^2 = (ux)^3a(yv)^3 = \cdots,$$

$$b = (xu)b(uy) = (xu)^2b(uy)^2 = (xu)^3a(vy)^3 = \cdots$$

Since S is periodic we can by find an $m\ for\ which\ (ux)^m$ is idempotent. Then if we assume that $c = xa, we\ have\ a = (ux)^m a(yv)^m = (ux)^m(ux)^m a(yv)^m = (ux)^m a = (ux)^{m-1}uc$, and so

$a\ \mathcal{L}\ c$ .Also ,$cy = xay = b, and\ if\ we\ choose\ n\ so\ that\ (uy)^m$ is idempotent. We have

$$c = xa = x\ (ux)^{n+1}a(yv)^{n+1} = (xu)^{n+1}xay(uy)^n v$$

$$= (xu)^{n+1}b(uy)^{2n}v = (xu)^{n+1}b(uy)^{n+1}(uy)^{n-1}v = b(uy)^{n-1}v.$$

Proposition 2.5: If S is a semigroup satisfying $min_L and min_R$ $then$ $\mathcal{D} = \mathcal{J}$.

Proof: If S is a semigroup satisfying $min_L and min_R$ $then$ so does $S^1, for\ S^1$ has exactly the same set of principal left and right ideals as S except for S^1 itself. We may thus assume that S has an identity element. If the, we have a $\mathcal{J}$ b , we may assert that there exist $p, q, r, s\ in\ S\ such$

That $paq = b, rbs = a$. It follows that the set $X = \{x \in S: (\exists y \in S)xay = b\}$ is non-empty, and hence so also is the subgroup $\wedge = \{\boldsymbol{L_x}: \boldsymbol{x} \in \boldsymbol{X}\}$ of $S/\ \mathcal{L}$. The condition min_L allows us to select a minimal element $\boldsymbol{L_u}$ **in** $\wedge$**;** thus $uav = b\ for\ some\ element\ v\ of\ S$.

Now $uruavsv = b$ and so $\boldsymbol{L_{uru}}\ \boldsymbol{in}\ \wedge$. we have $\boldsymbol{L_{uru}} \leq \boldsymbol{L_u}$**,** it follows by the minimality of $\boldsymbol{L_u}\ \boldsymbol{in} \wedge$ **that** $\boldsymbol{L_{uru}} = \boldsymbol{L_u}$**.** Hence we have $\boldsymbol{L_u} = \boldsymbol{L_{uru}} \leq \boldsymbol{L_{ru}} \leq \boldsymbol{L_u}$ and so $\boldsymbol{ru}\ \mathcal{L}\ \boldsymbol{u}$.

Lemma 2.5: Suppose that a, b be $\mathcal{R}$ – equivalent elements in a semigroup $S\ and\ let\ s, s'in\ S'\ be$ such that $as = b, bs' = a$. Then the right translations $\rho_s|L_a, \rho_s|L_b$ are mutually inverse [18] $\mathcal{R}$ – class preserving bijections from $L_a\ onto\ L_b$ and from $L_b\ onto\ L_a$ respectively. The left-right dual is proved in an analogous way:

Lemma 2.6: Suppose that a, b be $\mathcal{D}$ – equivalent elements in a semigroup S, then $|H_a| = |H_b|$.

Proof: If $c\ is\ such\ that\ \boldsymbol{a}\ \mathcal{R}$ **c, c** $\mathcal{L}$ b, and if $s, t, s', t'in\ S'are\ such\ that\ as = c, cs' = a$

$tc = b, t'b = c$, then by the preceding lemmas $(\rho_s|H_a)$ is a bijection onto $H_c$and $(\lambda_t|H_c)$is a bijection on to H_b. Thus $\rho_s\lambda_t: x \to txs\ is\ a\ bijection\ from\ H_a\ onto\ H_b$, it follows that $|H_a| = |H_b|$.

Lemma 2.7: If $x, y \in S\ are\ such\ that\ xy \in H_x\ then\ (\rho_y|H_x)$ is a bijection of H_x onto itself. If $xy \in H_y$ then $(\lambda_x|H_y)$ is a bijection of H_y onto itself.

Theorem 2.8: (Green's Theorem) If H is an $\mathfrak{H}$ **– class** [18] in a semigroup S then either $H^2 \cap H = \emptyset\ or$

$H^2 = H\ is\ a\ subgroup\ of\ S$.

Proof: Suppose that $H^2 \cap H \neq \emptyset$, [15] so that there exist a, b such that $ab \in H$. By the lemma, $\rho_b and\ \lambda_a$ are bijections of H onto itself. Hence $hb \in H\ and\ ah \in H$ for every $h\ in\ H$. Again by the lemma it follows that $\lambda_h and\ \rho_h$ are bijections of H onto itself. $Hh = hH = H\ for\ every$

$h\ in\ H. Hence\ certainly\ H^2 = H$.

Definition 2.9: A multi-set finite automaton $A = (Q, \Sigma, \delta, q_0, F)$ [8] is said to be a deterministic multiset finite automaton if the following tow conditions are satisfied as follows:

(i) For all $q \in Q, a \in \Sigma, if\ (q, a, q') \in \delta$ and $(q, a, q'') \in \delta$, then $q' = q''$.

(ii) For each state $q \in Q, if\ (q, a, q') \in \delta$, then for all b$\neq a$ and for all $q'' \in Q, (q. b, q'') \notin \delta$. [8] [9]

Obviously, a deterministic multi-set finite automaton represents a special case of a multi-set finite automaton. It is the well-known fact that in classical automata theory, the family of all languages accepted by non-deterministic finite automata equals the family of all languages by deterministic finite automata.

Theorem 2.10: The family of all languages accepted by deterministic multi-set finite automata is the proper subfamily of the family of all languages accepted [6] by multi-set finite automata.

A. Fuzzy multi-set finite automata: consider fuzzy sets with truth values in the unit interval [0,1] i.e., a fuzzy set in a universe set X is any mapping [12] $A: X \to [0,1], A(x)$ being interpreted as the truth degree of the fact that "

x belongs to A" and being called membership value. A fuzzy relation R between sets X and Y is defined as a mapping [13] $R: X \times Y \to [0,1]$. Analogously, a fuzzy ternary relation R is defined as a mapping $R: X \times Y \times Z \to [0,1]$ for any fuzzy set A, the set $supp(A) = \{a \in X \,|A(a) > 0\}$ is called support of A.

Definition 2.11: A fuzzy multi-set finite automaton (FMFA) is an ordered quintuple $A = (Q, \Sigma, \delta, q_0, F)$ where Q is a nonempty finite set of states, Σ is the input alphabet, q_0 is the initial state, $F: Q \to [0,1]$ [14]is a fuzzy set in Q , and $\delta: Q \times \Sigma \times Q \to [0,1]$ is the fuzzy transition relation.

A state $q \in Q$ is called a final state of A if F(q) > 0. We extend the fuzzy relation δ to fuzzy relation $\delta^*: Q \times \Sigma^{\oplus} \times Q \to [0,1]$ in the following way.

$\delta^*(q, 0_\Sigma, r) = 0 \; for \; r \neq q \; and \delta^*(q, 0_\Sigma, q) = 1,$

$\delta^*(q, \alpha, s) = \max_{a \in \Sigma, r \in Q} \{\delta(q, \alpha, r) \wedge \delta^*(r, \alpha \ominus \langle a \rangle, s)\}$. The fuzzy multiset language $L(A)$ accepted by the FMFA A is defined by $L(A) = \max_{q \in Q} \delta^*(q, 0_\Sigma, r) \wedge F(q)\}$ for all $\alpha \in \Sigma^{\oplus}$ and is called a FMFA-language [15] [16]. We will use examples of graphical representation which is suitable for fuzzy finite automata a labeled directed graph in which its nodes represent states of the automaton, the initial which its nodes represent states of the automaton, the initial state is indicated by the arrow pointing at it from nowhere, each final state q is depicted by double circle including the value of F(q), and each arc in the graph coincides with a non-null transition.

Example 2.12: Consider FMFA $A = (Q, \Sigma, \delta, q_0, F)$ with graphical representation from Figure 1.

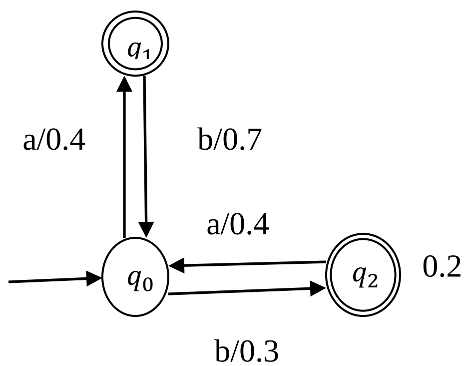


Fig.1.

It means that $Q = \{q_0, q_1, q_2\}$, Σ = {a, b},$\delta = \{q_0, a, q_1\} = \delta(q_2, a, q_0) = 0.4,$
$\delta(q_0, b, q_2) = 0.3, \delta(q_i, x, q_j) = 0$ Otherwise, $(q_0) = 0, F(q_1) = 1, F(q_2) = 0.2$. Then, for example $\delta^*(q_0, \langle a \rangle \oplus \langle b \rangle, q_0) = \max\{\delta(q_0, a, q_1) \wedge \delta(q_1, b, q_0), \delta(q_0, b, q_2) \wedge \delta(q_2, a, q_0)$

= max {0.4∧0.7, 0.3∧0.4}= 0.4

Obviously $\delta^*(q_0, \langle a \rangle \oplus \langle b \rangle^2, q_0) = 0, \delta^*(q_0, \langle a \rangle \oplus \langle b \rangle^2, q_1) = 0, \delta^*(q_0, \langle a \rangle \oplus \langle b \rangle^2, q_2) = 0.3,$
And $L(A)(\langle a \rangle \oplus \langle b \rangle^2) = \max\{\delta^*(q_0, \langle a \rangle \oplus \langle b \rangle^2, q_2) \wedge F(q_2) = 0.3 \wedge 0.2 = 0.2.$

It is easy to see that $L(A)(\alpha) = \begin{cases} 0.4 \; if |\alpha|_a = |\alpha|_b + 1, \\ 0.2 \; if \; |\alpha|_b = |\alpha|_a + 1, \\ o \qquad otherwise. \end{cases}$

The family of all FMFA languages equals the family of all fuzzy multi-set regular languages which are generated by fuzzy multi-set regular grammars.

Lemma 2.13: Let $A = (Q, \Sigma, \delta, q_0, F)$ be a FMFA. Then $\delta^*(q, \alpha \oplus \beta, s) \geq \delta^*(q, \alpha, r) \wedge \delta^*(r, \beta, s)$ for all $q, r, s \in Q, \alpha, \beta \in \Sigma^{\oplus}$.

Definition 2.14: A FMFA $A = (Q, \Sigma, \delta, q_0, F)$ is said to be deterministic (DFMFA) if the following two conditions are satisfied:

(i) For all $q \in Q, a \in \Sigma, if\ \delta(q, a, q') > 0\ and\ \delta(q, a, q'') > 0, then\ q' = q''$,

(ii) For each state $q \in Q, if\ \delta(q, a, q') > 0, then\ for\ all\ b \neq a$ and for all $q'' \in Q, \delta(q, a, q'')$ =0.

If a language is accepted by a DFMFA, we will call it DFMFA- language.
The following theorem is a straightforward consequence of Theorem 1.

Theorem 2.15 : The family of all DFMFA-languages is the proper subfamily of the family of all FMFA-languages.

III. PUMPING LEMMATA FOR FUZZY MULTISET LANGUAGES

Pumping lemma for regular languages represents well-known necessary condition for a language to be regular.
Pumping lemma for regular languages: If L is a regular language, then there is a number p (the pumping length) where, if w is any string in L of length at least $p, then\ w$ can be divided into three pieces, $w = xyz$, satisfying the following conditions:

(i) For each $i \geq 0, xy^i z \in L$,

(ii) $|y| > 0$, (iii) $|xy| \leq p$.

Theorem 3.1 : Let Σ be an alphabet and $L: \Sigma \to [0,1]$. If L is a FMFA-language, then there is a number p (the pumping length) where, if ω is any multiset of cardinality at least p such that $L(\omega) > 0, then\ \omega$ can be divided in three submultisets. $\omega = \alpha \oplus \beta \oplus \gamma$, satisfying the following condition:

(i) For each $i \geq 1, L(\omega) \leq L(\alpha \oplus \beta^i \oplus \gamma)$,

(ii) $card(\beta) > 0$,

(iii) $card(\alpha \oplus \beta) \leq p$.

Proof: Since L is a FMFA-language, there is a FMFA $A = (Q, \Sigma, \delta, q_0, F)$ which accepts L. Let us denote p = card (Q). Let $\omega = \langle a_1 \rangle^{k_1} \oplus\ \dots \oplus \langle a_n \rangle^{k_n}$ be a multiset with $a_1, \dots\ , a_n \in \Sigma,\ k_1, \dots\ , k_n \in N$, such that $k_1 +\ \dots +\ k_n \geq P\ and\ L(\omega) > 0$.

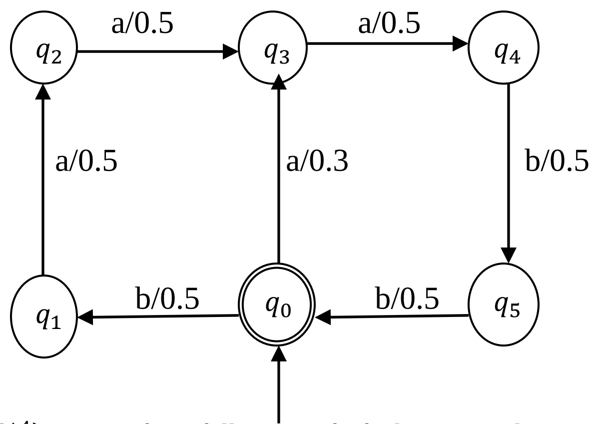
(i) By Definition 3, there are $b_1, \dots\ , b_j \in \{a_1, \dots\ , a_n\}, q_1, \dots\ , q_j \in Q\ with\ j = k_1 +\ \dots +\ k_n$ such that $L(\omega) = \delta(q_0, b_1, q_1) \wedge \delta(q_1, b_2, q_2) \wedge\ \dots \wedge \delta(q_{j-1}, b_j, q_j) \wedge F(q_j)$. Since $j \geq p$, by the Pigeonhole principle, a state q must be repeated in the sequence $q_0\ \dots\ q_j, i.e., \tilde{q} = q_k = q_m$ for some $0 \leq k \leq m \leq j$. Suppose that $k\ and\ m$ are the first such indexes, i.e. $\tilde{q} \neq q_l\ for\ all\ l \in \{0, \dots, k-1, k+1, \dots, m-1\}$. We put $\alpha = \langle b_1 \rangle \oplus \dots \oplus \langle b_k \rangle, \beta = \langle b_{k+1} \rangle \oplus \dots \oplus \langle b_m \rangle, \gamma = \langle b_{m+1} \rangle \oplus \dots \oplus \langle b_j \rangle$. Obviously $card(\beta) > 0\ and\ card(\alpha \oplus \beta) \leq p$. So, conditions (ii) and (iii) are satisfied.

By the Definition 3, Lemma 1, and commutatively of $\oplus$, we get for all $i \geq 1$:

$$L(\alpha \oplus \beta^i \oplus \gamma) = \max_{q \in Q}\{ \delta^*(q_0, \alpha \oplus \beta^i \oplus \gamma, q) \wedge F(q)\} \geq \delta^*(q_0, \alpha \oplus \beta^i \oplus \gamma, q_j) \wedge F(q) \geq \delta^*(q_m, \beta^{i-1}, q_m) \wedge \delta^*(\ q_0, \alpha \oplus \beta \oplus \gamma, q_j) \wedge F(q_j) = \delta^*(\ q_0, \alpha \oplus \beta \oplus \gamma, q_j) \wedge F(q_j) = L(\omega)$$

Hence condition (i) holds true.

Example 3.2: Consider FMFA $A = (Q, \Sigma, \delta, q_0, F)$ with graphical representation from Figure 2. As follows:



Clearly $L(A)(\langle a\rangle^4 \oplus \langle b\rangle^4) = 0.3$. If we follow proof of Theorem 3 then we have p = 6 $card\ (\langle a\rangle^4 \oplus \langle b\rangle^4) = 8$, and $\langle a\rangle^4 \oplus \langle b\rangle^4 = \alpha \oplus \beta \oplus \gamma$.Where $\alpha = 0_\Sigma, \beta = \langle a\rangle^2 \oplus \langle b\rangle^2 = \gamma$

Consequently $L(A)(\langle a\rangle^6 \oplus \langle b\rangle^6) = 0.5$, $L(A)(\langle a\rangle^8 \oplus \langle b\rangle^8) = L(A)(\langle a\rangle^{10} \oplus \langle b\rangle^{10}) = 0.3$,

$L(A)(\langle a\rangle^{12} \oplus \langle b\rangle^{12}) = 0.5\ etc.$

In case of DFMFA-languages, Theorem 3 gains more precise shape:

Theorem 3.3 : Let Σ be an alphabet and $L\colon \Sigma^{\oplus} \to [0,1]$. If L is a DFMFA-language, then there is a number p (the pumping length) where, if ω is any multiset of cardinality at least p such that $L(\omega) > 0$, then ω can be divided into three submultisets, $\omega = \alpha \oplus \beta \oplus \gamma$, satisfying the following conditions:

(i) For each $i \geq 1, L(\omega) = L(\alpha \oplus \beta^i \oplus \gamma)$,
(ii) $card(\beta) > 0$,
(iii) $card(\alpha \oplus \beta) \leq p$.

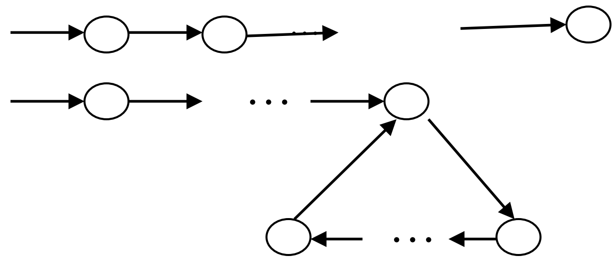
Proof: Condition (i) follows directly from the deterministic way of computation. The rest is identical with the earlier theorem.

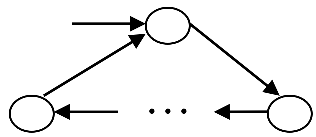
IV. SOME REMARKS TO DFMFA-LANGUAGES

Similarly some closure properties of FMA-languages were studied; we will have a look to these properties of DFMFA-languages.

Restrictions put on DFMFA lead to rather simple automata see their simplified graphical representations

(without depicting final states) on Figure 3. So fuzzy languages accepted by these automata are not too complex. In the next statement, we present some negative results concerning DFMFA-languages.





Definition 4.1: Let Σ be an alphabet and L, L_1, L_2: $\Sigma^{\oplus} \to [0,1]$. We define the addition $L_1 \oplus L_2$, the union $L_1 \cup L_2$ and the closure $L^{\oplus}$ by

$$(L_1 \oplus L_2)(\gamma) = \max_{\substack{\alpha,\beta \in \Sigma^{\oplus} \\ \alpha \oplus \beta = \gamma}} \{L_1(\alpha) \wedge L_2(\beta)\} \; for\ all\ \gamma \in \Sigma^{\oplus},$$

$(L_1 \cup L_2 = \max\{(L_1(\gamma), L_2(\gamma)\}\ for\ all\ \gamma \in \Sigma^{\oplus}$, $L^{\oplus} = L^0 \cup L^1 \cup L^2 \cup \ldots$

Where $L^0(\alpha) = \begin{cases} 1 \text{ if } \alpha = 0_\Sigma \\ 0 \text{ otherwise} \end{cases}$ and $L^i = L \oplus L^{i-1}$ for all $i \geq 2$.

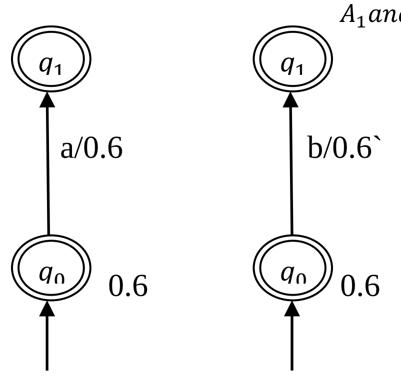
Assertion 4.2: The family of DFMFA-languages is not closed under the operations of union, addition, and closure.

Proof: Let $\Sigma = \{a, b\}$ and L_1, L_2 be two DFMFA-languages defined by

$L_1(\alpha) = \begin{cases} 0.6\ iF\ \alpha \in \{0_\Sigma, \langle a \rangle\}, \\ 0 \quad\quad otherwise \end{cases}$

$L_2(\alpha) = \begin{cases} 0.6\ iF\ \alpha \in \{0_\Sigma, \langle b \rangle\}, \\ 0 \quad\quad otherwise \end{cases}$ Clearly, these languages are accepted by DFMFA

$A_1 and\ A_2$ from Figure 4



DVMFA A_1 DFMFA A_2

Languages $(L_1 \cup L_2)$ and $L_1 \oplus L_2$ are defined by

$(L_1 \cup L_2)(\alpha) = \begin{cases} 0.6 \ iF \ \alpha \in \{0_\Sigma, \langle a \rangle, \langle b \rangle\}, \\ 0 \quad otherwise \end{cases}$ $(L_1 \oplus L_2)(\alpha) = \begin{matrix} 0.6 \ iF \ \alpha \in \{0_\Sigma, \langle a \rangle, \langle b \rangle \ \langle a \rangle \oplus \langle b \rangle\}, \\ 0 \quad otherwise \end{matrix}$

None of the languages $(L_1 \cup L_2)$ and $(L_1 \oplus L_2)$ is a DFMFA-language because the corresponding DFMFA should accept both multiset $\langle a \rangle$ and $\langle b \rangle$, however in the initial state each DFMFA can read only one input, either a or b , not both. Now consider DFMFA-language L_3 defined by $L_3(\alpha) = \begin{matrix} \{ \ 1 \ iF \ \alpha \in \{0_\Sigma, \langle a \rangle, \langle b \rangle \ \langle a \rangle \oplus \langle b \rangle, \langle a \rangle \oplus \langle b \rangle^3 \ \}, \\ 0 \quad otherwise. \end{matrix}$

Since $(L_3 \oplus L_3)(\ \langle a \rangle^2 \oplus \langle b \rangle^2 = 1 = (L_3 \oplus L_3)(\ \langle a \rangle \oplus \langle b \rangle^3)$, each DFMFA accepting $(L_3 \oplus L_3)$ should accept two different multisets of the same cardinality (with membership value equal to 1) which is not possible. Regarding the fact that $(L_3 \oplus L_3)$ is included in $L_3^{\oplus}$ it implies that $L_3^{\oplus}$ is not a DFMFA-language.

V.CONCLUSION

In this paper, the notion of deterministic fuzzy multi-set finite automata was introduced and some properties of the equivalent languages were shown. Since the relations every languages accepted by deterministic fuzzy multi-set finite automata is the appropriate subfamily of all languages accepted by non-deterministic fuzzy multi-set finite automata, Description for languages of together families were presented. The future research can be directed to enlarging the theory of fuzzy multi-set languages to the area of both deterministic and non-deterministic pushdown automata.

VI. Acknowledgment

Premier, I might want to offer my genuine thanks to my counsel Prof. E. Keshava Reddy for the nonstop help of my PhD study and research, for his understanding, inspiration, energy, just as enormous learning. His direction helped me in all the season of research just as composing of this Research Paper. I couldn't have envisioned having a superior counsel just as coach for my Ph.D. consider

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