Intuitionistic Fuzzy Bi-Magic Labeling Graphs

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Abstract

In this paper, we introduce the concepts of bi-magic labeling of intuitionistic fuzzy graphs. We examine some properties of bi-magic labeling on intuitionistic fuzzy path, cycle and star graphs.

Keywords: Bi-magic labeling of intuitionistic fuzzy graphs; path; cycle; star graphs.
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1 Introduction

Graph theory concepts was introduced by Euler in 1736 [5]. Fuzzy sets were introducted by L.Zadeh [13] in 1965, as an expansion of the old style of sets and it has showed useful applications in many areas. Rosenfeld [12] introduced the concept of fuzzy graph in 1975. The extension of fuzzy set was introduced by Atanassov [2] in 1986 as intuitionistic fuzzy sets, which incorporated as degree of membership and degree of non-membership grades. In 1994, Sovan and Atanassov [3] introduced the concept of intuitionistic fuzzy graph. A graph labeling is an assignment of integers to the edges or vertices or both subject to certain condition. The idea of graph labeling was introduced by Rosa [11]. Kotzig and Rosa [7] defined a magic labeling of finite graphs. A. Nagoor Gani et al, [8] introduced the concept of fuzzy magic labeling graphs.Ameenal Bibi and Devi [1], defined the fuzzy bi-magic labeling for cycle and star graphs. In this paper, we introduce the concepts of bi-magic labeling of intuitionistic fuzzy graphs. We examine some properties of bi-magic labeling on intuitionistic fuzzy path, cycle and star graphs.

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2 Preliminaries

Definition 2.1. [9,10] An Intuitionistic Fuzzy Graph(IFG) with underlying set V is defined to be a pair G = (A, B) where

1. The function $\mu_A : V \to [0, 1]$, $\gamma_A : V \to [0, 1]$, denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_A(v_i) + \gamma_A(v_i) \leq 1$ for all $v_i \in V$.
2. The functions $\mu_B : E \subseteq V \times V \to [0, 1]$, $\gamma_B : E \subseteq V \times V \to [0, 1]$ are defined by $\mu_B(v_i, v_j) \leq min[\mu_A(v_i), \mu_A(v_j)]$ *and* $\gamma_B(v_i, v_j) \leq max[\gamma_A(v_i), \gamma_A(v_j)]$, where $0 \leq \mu_B(v_i, v_j)+ \gamma_B(v_i, v_j) \leq 1$ for all $(v_i, v_j) \in E$.

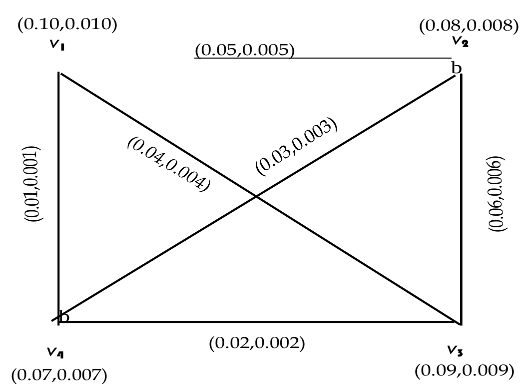
Definition 2.2. [9, 10] A Path P_n in an IFG is a sequence of distinct vertices $v_1, v_2, ..., v_n$ such that $0 < \mu_B(v_i, v_{i+1}), \gamma_B(v_i, v_{i+1}) \leq 1$; $1 \leq i \leq n - 1$; $n - 1$ is called the length of the path P_n. A path P_n is called Cycle if $v_1 = v_n$ for $n \geq 3$.

Definition 2.3. [9, 10] A Star in an IFG graph consists of two vertex sets U and V with $|U| = 1$ and $|V| = n$ such that $0 < \mu_B(u, v_i), \gamma_B(u, v_i) \leq 1$; $1 \leq i \leq n$.

3 Intuitionistic Fuzzy Bi-Magic Labeling Graphs

Definition 3.1. An intuitionistic fuzzy graph $G = (A, B)$ is said to be an intuitionistic fuzzy bi-magic[briefly $\mathsf{IFB} - \mathsf{M}$] graph if $\mathsf{Bm}_\mu(G) = \mu_A(u) + \mu_B(u, v) + \mu_A(v)$ and $\mathsf{Bm}_\gamma(G) = \gamma_A(u) + \gamma_B(u, v) + \gamma_A(v)$ has two different magic values $\mathsf{Bm}_1(G)$, $\mathsf{Bm}_2(G)$ for all $u, v \in V$. Where $\mathsf{Bm}_1(G) = (\mathsf{Bm}_{\mu_1}(G), \mathsf{Bm}_{\gamma_1}(G))$ and $\mathsf{Bm}_2(G) = (\mathsf{Bm}_{\mu_2}(G), \mathsf{Bm}_{\gamma_2}(G))$. Bi-magic labeling of an intuitionistic fuzzy graph G is denoted as $\mathsf{Bm}(G) = (\mathsf{Bm}_1(G), \mathsf{Bm}_2(G))$.

Example 3.1. *Consider an intuitionistic fuzzy graph G=(A,B) such that* $V = \{v_1, v_2, v_3, v_4\}$ *and* $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3), (v_2, v_4)\}$.



$\mathsf{IFB} - \mathsf{M}$ labeling Graph G

$\mathsf{Bm}_1(G) = (0.23, 0.023)$ and $\mathsf{Bm}_2(G) = (0.18, 0.018)$. Hence G is an $\mathsf{IFB} - \mathsf{M}$ labeling graph.

Theorem 3.1. *For all $n \geq 3$, the path P_n is an $\mathrm{IFB}-\mathrm{M}$ labeling graph.*

Proof. Let P_n be any path with $n \geq 3$. Then $v_1, v_2, v_3, \ldots, v_n$ and $v_1v_2, v_2v_3, \ldots, v_{n-1}v_n$ are vertices and edges of P_n.
Let $l = min\{x : n < 3(10)^x, x = 0, 1, 2, \ldots\}$ and $s_1 = 10^{-(l+1)}$, $s_2 = 10^{-(l+2)}$ where s_1 and s_2 are the set of membership and non-membership degree in an intuitionistic fuzzy labeling.
The intuitionistic fuzzy vertex and edge labeling is defined as follows:
Case - (1) When n is odd

$$\mu_A(v_{2k}) = (2n-k)s_1;\ 1 \leq k \leq \frac{n-1}{2},$$
$$\gamma_A(v_{2k}) = (2n-k)s_2;\ 1 \leq k \leq \frac{n-1}{2}.$$

$$\mu_A(v_{2k-1}) = min\{\mu_A(v_{2i}) | 1 \leq i \leq \frac{n-1}{2}\} - ks_1;\ 1 \leq k \leq \frac{n+1}{2},$$
$$\gamma_A(v_{2k-1}) = min\{\gamma_A(v_{2i}) | 1 \leq i \leq \frac{n-1}{2}\} - ks_2;\ 1 \leq k \leq \frac{n+1}{2}.$$

$$\mu_B(v_kv_{k+1}) = (\frac{n-1}{2} + k)s_1;\ 1 \leq k \leq \frac{n-1}{2},$$
$$\gamma_B(v_kv_{k+1}) = (\frac{n-1}{2} + k)s_2;\ 1 \leq k \leq \frac{n-1}{2}.$$

$$\mu_B(v_kv_{k+1}) = (k - \frac{n-1}{2})s_1;\ \frac{n+1}{2} \leq k \leq n-1,$$
$$\gamma_B(v_kv_{k+1}) = (k - \frac{n-1}{2})s_2;\ \frac{n+1}{2} \leq k \leq n-1.$$

Sub case - (i) When k is even
Let $k = 2a$ where $a \in Z^+$.
For each edge (v_k, v_{k+1}) the $\mathrm{IFB}-\mathrm{M}$ labeling are

$$\mathrm{Bm}_{\mu_1}(P_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ 1 \leq k \leq \frac{n-1}{2}$$
$$= \mu_A(v_{2a}) + \mu_B(v_{2a}, v_{2a+1}) + \mu_A(v_{2a+1}),$$
$$= (2n-a)s_1 + (\frac{n-1}{2} + 2a)s_1 + min\{\mu_A(v_{2i}) | 1 \leq i \leq \frac{n-1}{2}\} - (a+1)s_1,$$
$$\mathrm{Bm}_{\mu_1}(P_n) = (\frac{5n-3}{2})s_1 + min\{\mu_A(v_{2i}) | 1 \leq i \leq \frac{n-1}{2}\}.$$

$$\mathrm{Bm}_{\mu_2}(P_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ \frac{n+1}{2} \leq k \leq n-1$$
$$= \mu_A(v_{2a}) + \mu_B(v_{2a}, v_{2a+1}) + \mu_A(v_{2a+1}),$$

$$= (2n-a)s_1 + (2a - \frac{n-1}{2})s_1 + min\{\mu_A(v_{2i})|1 \le i \le \frac{n-1}{2}\} - (a+1)s_1,$$

$$Bm_{\mu_2}(P_n) = (\frac{3n-1}{2})s_1 + min\{\mu_A(v_{2i})|1 \le i \le \frac{n-1}{2}\}.$$

Sub case - (ii) When k is odd

Let $k = 2a + 1$ where $a \in Z^+$.

For each edge (v_k, v_{k+1}) the IFB − M labeling are

$$Bm_{\mu_1}(P_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ 1 \le k \le \frac{n-1}{2}$$

$$= \mu_A(v_{2a+1}) + \mu_B(v_{2a+1}, v_{2a+2}) + \mu_A(v_{2a+2}),$$

$$= min\{\mu_A(v_{2i})|1 \le i \le \frac{n-1}{2}\} - (a+1)s_1 + (\frac{n-1}{2} + 2a + 1)s_1 + (2n-(a+1))s_1,$$

$$Bm_{\mu_1}(P_n) = (\frac{5n-3}{2})s_1 + min\{\mu_A(v_{2i})|1 \le i \le \frac{n-1}{2}\}.$$

$$Bm_{\mu_2}(P_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ \frac{n+1}{2} \le k \le n-1$$

$$= \mu_A(v_{2a+1}) + \mu_B(v_{2a+1}, v_{2a+2}) + \mu_A(v_{2a+2}),$$

$$= min\{\mu_A(v_{2i})|1 \le i \le \frac{n-1}{2}\} - (a+1)s_1 + (2a+1 - \frac{n-1}{2})s_1 + (2n-(a+1))s_1,$$

$$Bm_{\mu_2}(P_n) = (\frac{3n-1}{2})s_1 + min\{\mu_A(v_{2i})|1 \le i \le \frac{n-1}{2}\}.$$

Similarly we can find,

$$Bm_{\gamma_1}(P_n) = (\frac{5n-3}{2})s_2 + min\{\gamma_A(v_{2i})|1 \le i \le \frac{n-1}{2}\},$$

$$Bm_{\gamma_2}(P_n) = (\frac{3n-1}{2})s_2 + min\{\gamma_A(v_{2i})|1 \le i \le \frac{n-1}{2}\}.$$

Hence IFB − M labeling of a even length path P_n are

$Bm_1(P_n) = (Bm_{\mu_1}(P_n), Bm_{\gamma_1}(P_n))$ and $Bm_2(P_n) = (Bm_{\mu_2}(P_n), Bm_{\gamma_2}(P_n))$.

Case - (2) When n is even

$$\mu_A(v_{2k}) = (2n-k)s_1;\ 1 \le k \le \frac{n}{2},$$

$$\gamma_A(v_{2k}) = (2n-k)s_2;\ 1 \le k \le \frac{n}{2}.$$

$$\mu_A(v_{2k-1}) = min\{\mu_A(v_{2i})|1 \le i \le \frac{n}{2}\} - ks_1;\ 1 \le k \le \frac{n}{2},$$

$$\gamma_A(v_{2k-1}) = min\{\gamma_A(v_{2i})|1 \le i \le \frac{n}{2}\} - ks_2;\ 1 \le k \le \frac{n}{2}.$$

$$\mu_B(v_k, v_{k+1}) = (\frac{n-2}{2} + k)s_1;\ 1 \le k \le \frac{n}{2},$$
$$\gamma_B(v_k, v_{k+1}) = (\frac{n-2}{2} + k)s_2;\ 1 \le k \le \frac{n}{2}.$$

$$\mu_B(v_k, v_{k+1}) = (k - \frac{n}{2})s_1;\ \frac{n}{2} + 1 \le k \le n-1,$$
$$\gamma_B(v_k, v_{k+1}) = (k - \frac{n}{2})s_2;\ \frac{n}{2} + 1 \le k \le n-1.$$

Sub case - (i) When k is even

Let $k = 2a$ where $a \in Z^+$.

For each edge (v_k, v_{k+1}) the IFB − M labeling are

$$\mathrm{Bm}_{\mu_1}(P_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ 1 \le k \le \frac{n}{2}$$
$$= \mu_A(v_{2a}) + \mu_B(v_{2a}, v_{2a+1}) + \mu_A(v_{2a+1}),$$
$$= (2n-a)s_1 + (\frac{n-2}{2} + 2a)s_1 + min\{\mu_A(v_{2i}) | 1 \le i \le \frac{n}{2}\} - (a+1)s_1,$$
$$\mathrm{Bm}_{\mu_1}(P_n) = (\frac{5n-4}{2})s_1 + min\{\mu_A(v_{2i}) | 1 \le i \le \frac{n}{2}\}.$$

$$\mathrm{Bm}_{\mu_2}(P_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ \frac{n}{2} + 1 \le k \le n-1$$
$$= \mu_A(v_{2a}) + \mu_B(v_{2a}, v_{2a+1}) + \mu_A(v_{2a+1}),$$
$$= (2n-a)s_1 + (2a - \frac{n}{2})s_1 + min\{\mu_A(v_{2i}) | 1 \le i \le \frac{n}{2}\} - (a+1)s_1,$$
$$\mathrm{Bm}_{\mu_2}(P_n) = (\frac{3n-2}{2})s_1 + min\{\mu_A(v_{2i}) | 1 \le i \le \frac{n}{2}\}$$

Sub case - (ii) When k is odd

Let $k = 2a + 1$ where $a \in Z^+$.

For each edge (v_k, v_{k+1}) the IFB − M labeling are

$$\mathrm{Bm}_{\mu_1}(P_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ 1 \le k \le \frac{n}{2}$$
$$= \mu_A(v_{2a+1}) + \mu_B(v_{2a+1}, v_{2a+2}) + \mu_A(v_{2a+2}),$$
$$= min\{\mu_A(v_{2i}) | 1 \le i \le \frac{n}{2}\} - (a+1)s_1 + (\frac{n-2}{2} + 2a + 1)s_1 + (2n - (a+1))s_1,$$
$$\mathrm{Bm}_{\mu_1}(P_n) = (\frac{5n-4}{2})s_1 + min\{\mu_A(v_{2i}) | 1 \le i \le \frac{n}{2}\}.$$
$$\mathrm{Bm}_{\mu_2}(P_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ \frac{n}{2} + 1 \le k \le n-1$$
$$= \mu_A(v_{2a+1}) + \mu_B(v_{2a+1}, v_{2a+2}) + \mu_A(v_{2a+2}),$$
$$= min\{\mu_A(v_{2i}) | 1 \le i \le \frac{n}{2}\} - (a+1)s_1 + (2a + 1 - \frac{n}{2})s_1 + (2n - (a+1))s_1,$$
$$\mathrm{Bm}_{\mu_2}(P_n) = (\frac{3n-2}{2})s_1 + min\{\mu_A(v_{2i}) | 1 \le i \le \frac{n}{2}\}.$$

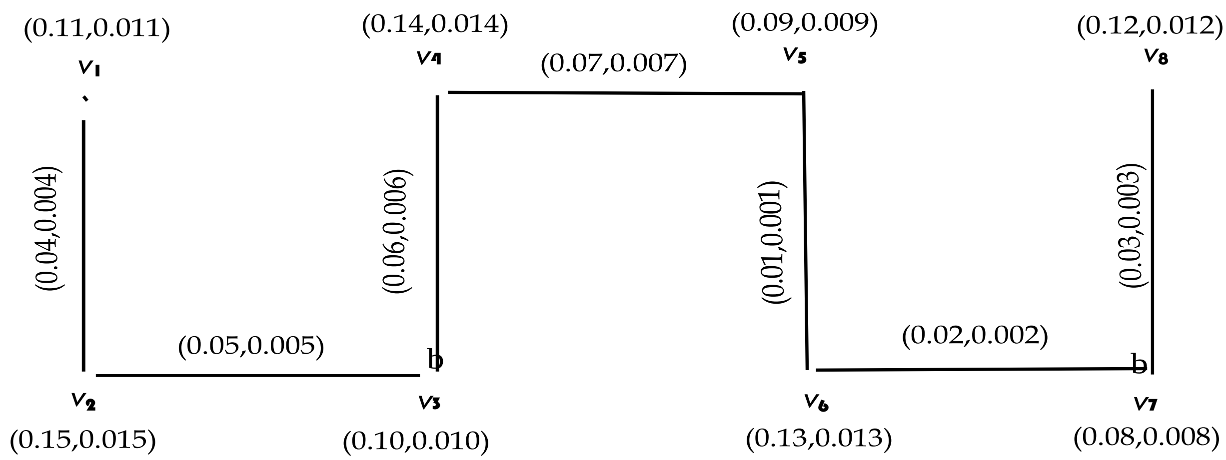
Similarly we can find,

$$\mathrm{Bm}_{\gamma_1}(P_n) = \left(\frac{5n-4}{2}\right)s_2 + min\{\gamma_A(v_{2i}) | 1 \le i \le \frac{n}{2}\},$$

$$\mathrm{Bm}_{\gamma_2}(P_n) = \left(\frac{3n-2}{2}\right)s_2 + min\{\gamma_A(v_{2i}) | 1 \le i \le \frac{n}{2}\}.$$

Hence IFB − M labeling of an odd length path P_n are
$\mathrm{Bm}_1(P_n) = (\mathrm{Bm}_{\mu_1}(P_n), \mathrm{Bm}_{\gamma_1}(P_n))$ and $\mathrm{Bm}_2(P_n) = (\mathrm{Bm}_{\mu_2}(P_n), \mathrm{Bm}_{\gamma_2}(P_n))$. □

Example 3.2. *Consider an intuitionistic fuzzy path graph P_8 such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_6), (v_6, v_7), (v_7, v_8)\}$.*



IFB − M labeling graph P_8

$\mathrm{Bm}_1(P_8) = (0.30, 0.030)$ and $\mathrm{Bm}_2(P_8) = (0.23, 0.023)$. Hence P_8 is an IFB − M labeling graph.

Theorem 3.2. *For all $n \ge 3$, the cycle C_n is an IFB − M labeling graph.*

Proof. Let C_n be any path with $n \ge 3$. Then $v_1, v_2, v_3, ..., v_n$ and $v_1v_2, v_2v_3, ..., v_nv_1$ are vertices and edges of C_n.
Let s_1 and s_2 such that $s_1 = \{10^{-(l+1)} | n \le 5(10)^l, l \in W\}$ and $s_2 = \{10^{-(l+2)} | n \le 5(10)^l, l \in W\}$ where s_1 *and* s_2 choose for set of membership and non-membership degree in intuitionistic fuzzy labeling.
The intuitionistic fuzzy labeling is defined as follows:
Case - (1) When n is odd

$$\mu_A(v_{2k}) = (2n - k + 1)s_1;\ 1 \le k \le \frac{n-1}{2},$$

$$\gamma_A(v_{2k}) = (2n - k + 1)s_2;\ 1 \le k \le \frac{n-1}{2}.$$

$$\mu_A(v_{2k-1}) = min\{v_{2i} | 1 \leq i \leq \frac{n-1}{2}\} - ks_1,$$
$$\gamma_A(v_{2k-1}) = min\{v_{2i} | 1 \leq i \leq \frac{n-1}{2}\} - ks_2.$$

$$\mu_B(v_k, v_{k+1}) = (\frac{n-1}{2} + k)s_1; 1 \leq k \leq \frac{n+1}{2},$$
$$\gamma_B(v_k, v_{k+1}) = (\frac{n-1}{2} + k)s_2; 1 \leq k \leq \frac{n+1}{2}.$$

$$\mu_B(v_k, v_{k+1}) = (k - \frac{n+1}{2})s_1; \frac{n+3}{2} \leq k \leq n-1,$$
$$\gamma_B(v_k, v_{k+1}) = (k - \frac{n+1}{2})s_2; \frac{n+3}{2} \leq k \leq n-1.$$

Sub case - (i) When k is even

Let $k = 2a$ where $a \in Z^+$.

For each edge (v_k, v_{k+1}) the IFB − M labeling are

$$Bm_{\mu_1}(C_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}), 1 \leq k \leq \frac{n+1}{2}$$
$$= \mu_A(v_{2a}) + \mu_B(v_{2a}, v_{2a+1}) + \mu_A(v_{2a+1}),$$
$$= (2n + 1 - a)s_1 + (\frac{n-1}{2} + 2a)s_1 + min\{v_{2i} | 1 \leq i \leq \frac{n-1}{2}\} - (a+1)s_1,$$
$$Bm_{\mu_1}(C_n) = (\frac{5n-1}{2})s_1 + min\{v_{2i} | 1 \leq i \leq \frac{n-1}{2}\}.$$

$$Bm_{\mu_2}(C_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}), \frac{n+3}{2} \leq k \leq n-1$$
$$= \mu_A(v_{2a}) + \mu_B(v_{2a}, v_{2a+1}) + \mu_A(v_{2a+1}),$$
$$= (2n + 1 - a)s_1 + (2a - \frac{n+1}{2})s_1 + min\{v_{2i} | 1 \leq i \leq \frac{n-1}{2}\} - (a+1)s_1,$$
$$Bm_{\mu_2}(C_n) = (\frac{3n-1}{2})s_1 + min\{v_{2i} | 1 \leq i \leq \frac{n-1}{2}\}.$$

Sub case - (ii) When k is odd

Let $k = 2a + 1$ where $a \in Z^+$.

For each edge (v_k, v_{k+1}) the IFB − M labeling are

$$Bm_{\mu_1}(C_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}), 1 \leq k \leq \frac{n+1}{2}$$
$$= \mu_A(v_{2a+1}) + \mu_B(v_{2a+1}, v_{2a+2}) + \mu_A(v_{2a+2}),$$
$$= min\{v_{2i} | 1 \leq i \leq \frac{n-1}{2}\} - (a+1)s_1 + (\frac{n-1}{2} + (2a+1))s_1 + (2n + 1 - (a+1))s_1,$$
$$Bm_{\mu_1}(C_n) = (\frac{5n-1}{2})s_1 + min\{v_{2i} | 1 \leq i \leq \frac{n-1}{2}\}.$$

$$\mathrm{Bm}_{\mu_2}(C_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}), \frac{n+3}{2} \le k \le n-1$$
$$= \mu_A(v_{2a+1}) + \mu_B(v_{2a+1}, v_{2a+2}) + \mu_A(v_{2a+2}),$$
$$= min\{v_{2i} | 1 \le i \le \frac{n-1}{2}\} - (a+1)s_1 + ((2a+1) - \frac{n+1}{2})s_1 + (2n+1-(a+1))s_1,$$
$$\mathrm{Bm}_{\mu_2}(C_n) = (\frac{3n-1}{2})s_1 + min\{v_{2i} | 1 \le i \le \frac{n-1}{2}\}.$$

Similarly we can find

$$\mathrm{Bm}_{\gamma_1}(C_n) = (\frac{5n-1}{2})s_2 + min\{v_{2i} | 1 \le i \le \frac{n-1}{2}\}.$$
$$\mathrm{Bm}_{\gamma_2}(C_n) = (\frac{3n-1}{2})s_2 + min\{v_{2i} | 1 \le i \le \frac{n-1}{2}\}.$$

Hence IFB − M labeling of a odd cycle C_n are
$\mathrm{Bm}_1(C_n) = (\mathrm{Bm}_{\mu_1}(C_n), \mathrm{Bm}_{I\gamma_1}(C_n))$ and $\mathrm{Bm}_2(C_n) = (\mathrm{Bm}_{\mu_2}(C_n), \mathrm{Bm}_{\gamma_2}(C_n))$.

Case - (2) When n is even

$$\mu_A(v_{2k-1}) = (2n-k+1)s_1; 1 \le k \le \frac{n}{2},$$
$$\gamma_A(v_{2k-1}) = (2n-k+1)s_2; 1 \le k \le \frac{n}{2}.$$

$$\mu_A(v_{2k}) = min\{v_{2i-1} | 1 \le i \le \frac{n}{2}\} - ks_1,$$
$$\gamma_A(v_{2k}) = min\{v_{2i-1} | 1 \le i \le \frac{n}{2}\} - ks_2.$$

$$\mu_B(v_1, v_n) = s_1,$$
$$\gamma_B(v_1, v_n) = s_2.$$

$$\mu_B(v_k, v_{k+1}) = (\frac{n}{2} + k)s_1; 1 \le k \le \frac{n}{2},$$
$$\gamma_B(v_k, v_{k+1}) = (\frac{n}{2} + k)s_2; 1 \le k \le \frac{n}{2}.$$

$$\mu_B(v_k, v_{k+1}) = (k+1-\frac{n}{2})s_1; \frac{n+2}{2} \le k \le n-1,$$
$$\gamma_B(v_k, v_{k+1}) = (k+1-\frac{n}{2})s_2; \frac{n+2}{2} \le k \le n-1.$$

Sub case - (i) When k is even
Let $k = 2a$ where $a \in Z^+$.

For each edge (v_k, v_{k+1}) the $\mathrm{IFB}-\mathrm{M}$ labeling are

$$\mathrm{Bm}_{\mu_1}(C_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ 1 \leq k \leq \frac{n}{2}$$
$$= \mu_A(v_{2a}) + \mu_B(v_{2a}, v_{2a+1}) + \mu_A(v_{2a+1}),$$
$$= min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\} - as_1 + (\frac{n}{2} + 2a)s_1 + (2n + 1 - (a + 1))s_1,$$
$$\mathrm{Bm}_{\mu_1}(C_n) = (\frac{5n}{2})s_1 + min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\}.$$

$$\mathrm{Bm}_{\mu_2}(C_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ \frac{n+2}{2} \leq k \leq n - 1$$
$$= \mu_A(v_{2a}) + \mu_B(v_{2a}, v_{2a+1}) + \mu_A(v_{2a+1}),$$
$$= min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\} - as_1 + (2a + 1 - \frac{n}{2})s_1 + (2n + 1 - (a + 1))s_1,$$
$$\mathrm{Bm}_{\mu_2}(C_n) = (\frac{3n+2}{2})s_1 + min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\}.$$

Sub case - (ii) When k is odd
Let $k = 2a + 1$ where $a \in Z^+$.
For each edge (v_k, v_{k+1}) the $\mathrm{IFB}-\mathrm{M}$ labeling are

$$\mathrm{Bm}_{\mu_1}(C_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ 1 \leq k \leq \frac{n}{2}$$
$$= \mu_A(v_{2a+1}) + \mu_B(v_{2a+1}, v_{2a+2}) + \mu_A(v_{2a+2}),$$
$$= (2n + 1 - (a + 1))s_1 + (\frac{n}{2} + 2a + 1)s_1 + min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\} - (a + 1)s_1$$
$$\mathrm{Bm}_{\mu_1}(C_n) = (\frac{5n}{2})s_1 + min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\}.$$

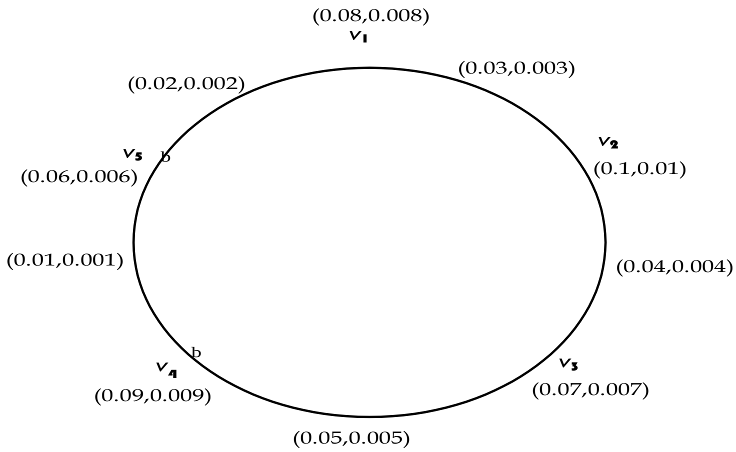
$$\mathrm{Bm}_{\mu_2}(C_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1}),\ \frac{n+2}{2} \leq k \leq n - 1$$
$$= \mu_A(v_{2a+1}) + \mu_B(v_{2a+1}, v_{2a+2}) + \mu_A(v_{2a+2}),$$
$$= (2n + 1 - (a + 1))s_1 + (2a + 2 - \frac{n}{2})s_1 + min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\} - (a + 1)s_1,$$
$$\mathrm{Bm}_{\mu_2}(C_n) = (\frac{3n+2}{2})s_1 + min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\}.$$

similarly we can find,

$$\mathrm{Bm}_{\gamma_1}(C_n) = (\frac{5n}{2})s_2 + min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\}.$$
$$\mathrm{Bm}_{\gamma_2}(C_n) = (\frac{3n+2}{2})s_2 + min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\}.$$

Hence $\mathrm{IFB}-\mathrm{M}$ labeling of a even cycle C_n are
$\mathrm{Bm}_1(C_n) = (\mathrm{Bm}_{\mu_1}(C_n), \mathrm{Bm}_{\gamma_1}(C_n))$ and $\mathrm{Bm}_2(C_n) = (\mathrm{Bm}_{\mu_2}(C_n), \mathrm{Bm}_{\gamma_2}(C_n))$. □

Example 3.3. *Consider an intuitionistic fuzzy cycle* C_5 *such that* $V = \{v_1, v_2, v_3, v_4, v_5\}$ *and* $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_1)\}$.



IFB − M labeling of C_5

$Bm_1(C_5) = (0.21, 0.021)$ and $Bm_2(C_5) = (0.16, 0.016)$. Hence C_5 is an IFB − M labeling graph.

Theorem 3.3. *For any* $n \geq 2$, *star graph* $S_{1,n}$ *is an* IFB − M *graph.*

Proof. Let $S_{1,n}$ be a star graph with $u, v_1, v_2, v_3, ..., v_n$ as vertices and $uv_1, uv_2, uv_3, ..., uv_n$ as edges.

Let s_1 and s_2 such that $s_1 = \{10^{-(l+1)} | n \leq 5(10)^l, l \in W\}$ and $s_2 = \{10^{-(l+2)} | n \leq 5(10)^l, l \in W\}$ where s_1 and s_2 choose for set of membership and non-membership degree in intuitionistic fuzzy labeling.

The intuitionistic fuzzy labeling is defined as follows:

Case - (1) When n is odd

$$\mu_A(u) = (2n + 1)s_1,$$
$$\gamma_A(u) = (2n + 1)s_2.$$

$$\mu_A(v_k) = \mu_A(u) - ks_1;\ 1 \leq k \leq n,$$
$$\gamma_A(v_k) = I_A(u) - ks_2;\ 1 \leq k \leq n.$$

$$\mu_B(u, v_k) = \left(\frac{n+1}{2} + k - 1\right)s_1;\ 1 \leq k \leq \frac{n+1}{2},$$
$$\gamma_B(u, v_k) = \left(\frac{n+1}{2} + k - 1\right)s_2;\ 1 \leq k \leq \frac{n+1}{2}.$$

$$\mu_B(u, v_k) = \left(k - \frac{n+1}{2}\right)s_1;\ \frac{n+3}{2} \leq k \leq n,$$
$$\gamma_B(u, v_k) = \left(k - \frac{n+1}{2}\right)s_2;\ \frac{n+3}{2} \leq k \leq n.$$

Sub case - (i) k is even

Let $k = 2a$ where $a \in Z^{+}$.

For each edge (u, v_k) the IFB − M labeling are

$$\mathrm{Bm}_{\mu_1}(S_{1,n}) = \mu_A(u) + \mu_B(u.v_k) + \mu_A(v_k), 1 \le k \le \frac{n+1}{2}$$
$$= \mu_A(u) + \mu_B(u.v_{2a}) + \mu_A(v_{2a}),$$
$$= (2n+1)s_1 + (\frac{n+1}{2} + 2a - 1)s_1 + (2n+1)s_1 - 2as_1,$$
$$\mathrm{Bm}_{\mu_1}(S_{1,n}) = (\frac{9n+3}{2})s_1.$$

$$\mathrm{Bm}_{\mu_2}(S_{1,n}) = \mu_A(u) + \mu_B(u.v_k) + \mu_A(v_k), \frac{n+3}{2} \le k \le n$$
$$= \mu_A(u) + \mu_B(u.v_{2a}) + \mu_A(v_{2a}),$$
$$= (2n+1)s_1 + (2a - \frac{n+1}{2})s_1 + (2n+1)s_1 - 2as_1,$$
$$\mathrm{Bm}_{\mu_2}(S_{1,n}) = (\frac{7n+3}{2})s_1.$$

Sub case - (ii) k is odd

Let $k = 2a + 1$ where $a \in Z^{+}$.

For each edge (u, v_k) the IFB − M labeling are

$$\mathrm{Bm}_{\mu_1}(S_{1,n}) = \mu_A(u) + \mu_B(u.v_k) + \mu_A(v_k), 1 \le k \le \frac{n+1}{2}$$
$$= \mu_A(u) + \mu_B(u.v_{2a+1}) + \mu_A(v_{2a+1}),$$
$$= (2n+1)s_1 + (\frac{n+1}{2} + 2a)s_1 + (2n+1)s_1 - (2a+1)s_1,$$
$$\mathrm{Bm}_{\mu_1}(S_{1,n}) = (\frac{9n+3}{2})s_1.$$

$$\mathrm{Bm}_{\mu_2}(S_{1,n}) = \mu_A(u) + \mu_B(u.v_k) + \mu_A(v_k), \frac{n+3}{2} \le k \le n$$
$$= \mu_A(u) + \mu_B(u.v_{2a+1}) + \mu_A(v_{2a+1}),$$
$$= (2n+1)s_1 + ((2a+1) - \frac{n+1}{2})s_1 + (2n+1)s_1 - (2a+1)s_1,$$
$$\mathrm{Bm}_{\mu_2}(S_{1,n}) = (\frac{7n+3}{2})s_1.$$

Similarly we can find

$$\mathrm{Bm}_{\gamma_1}(S_{1,n}) = (\frac{9n+3}{2})s_2,$$
$$\mathrm{Bm}_{\gamma_2}(S_{1,n}) = (\frac{7n+3}{2})s_2.$$

Hence IFB − M labeling of a star graph $S_{1,n}$ are
$Bm_1(S_{1,n}) = (Bm_{\mu_1}(S_{1,n}), Bm_{\gamma_1}(S_{1,n}))$ and $Bm_2(S_{1,n}) = (Bm_{\mu_2}(S_{1,n}), Bm_{\gamma_2}(S_{1,n}))$
Case - (2) When n is even

$$\mu_A(u) = (2n+1)s_1,$$
$$\gamma_A(u) = (2n+1)s_2.$$

$$\mu_A(v_k) = \mu_A(u) - ks_1;\ 1 \le k \le n,$$
$$\gamma_A(v_k) = I_A(u) - ks_2;\ 1 \le k \le n.$$

$$\mu_B(u, v_k) = (\frac{n}{2} + k)s_1;\ 1 \le k \le \frac{n}{2},$$
$$\gamma_B(u, v_k) = (\frac{n}{2} + k)s_2;\ 1 \le k \le \frac{n}{2}.$$

$$\mu_B(u, v_k) = (k - \frac{n}{2})s_1;\ \frac{n+2}{2} \le k \le n,$$
$$\gamma_B(u, v_k) = (k - \frac{n}{2})s_2;\ \frac{n+2}{2} \le k \le n.$$

Sub case - (i) k is even
Let $k = 2a$ where $a \in Z^+$. For each edge (u, v_k) the IFB − M labeling are

$$Bm_{\mu_1}(S_{1,n}) = \mu_A(u) + \mu_B(u.v_k) + \mu_A(v_k),\ 1 \le k \le \frac{n}{2}$$
$$= \mu_A(u) + \mu_B(u.v_{2a}) + \mu_A(v_{2a}),$$
$$= (2n+1)s_1 + (\frac{n}{2} + 2a)s_1 + (2n+1)s_1 - 2as_1,$$
$$Bm_{\mu_1}(S_{1,n}) = (\frac{9n+4}{2})s_1.$$

$$Bm_{\mu_2}(S_{1,n}) = \mu_A(u) + \mu_B(u.v_k) + \mu_A(v_k),\ \frac{n+2}{2} \le k \le n$$
$$= \mu_A(u) + \mu_B(u.v_{2a}) + \mu_A(v_{2a}),$$
$$= (2n+1)s_1 + (2a - \frac{n}{2})s_1 + (2n+1)s_1 - 2as_1,$$
$$Bm_{\mu_2}(S_{1,n}) = (\frac{7n+4}{2})s_1.$$

Sub case - (ii) k is odd
Let $k = 2a + 1$ where $a \in Z^+$.
For each edge (u, v_k) the IFB − M labeling are

$$Bm_{\mu_1}(S_{1,n}) = \mu_A(u) + \mu_B(u.v_k) + \mu_A(v_k),\ 1 \le k \le \frac{n}{2}$$
$$= \mu_A(u) + \mu_B(u.v_{2a+1}) + \mu_A(v_{2a+1}),$$

$$= (2n+1)s_1 + (\frac{n}{2} + 2a + 1)s_1 + (2n+1)s_1 - (2a+1)s_1,$$

$$\mathrm{Bm}_{\mu_1}(S_{1,n}) = (\frac{9n+4}{2})s_1.$$

$$\mathrm{Bm}_{\mu_2}(S_{1,n}) = \mu_A(u) + \mu_B(u.v_k) + \mu_A(v_k),\ \frac{n+2}{2} \le k \le n$$

$$= \mu_A(u) + \mu_B(u.v_{2a+1}) + \mu_A(v_{2a+1}),$$

$$= (2n+1)s_1 + ((2a+1) - \frac{n}{2})s_1 + (2n+1)s_1 - (2a+1)s_1,$$

$$\mathrm{Bm}_{\mu_2}(S_{1,n}) = (\frac{7n+4}{2})s_1.$$

Similarly we can find

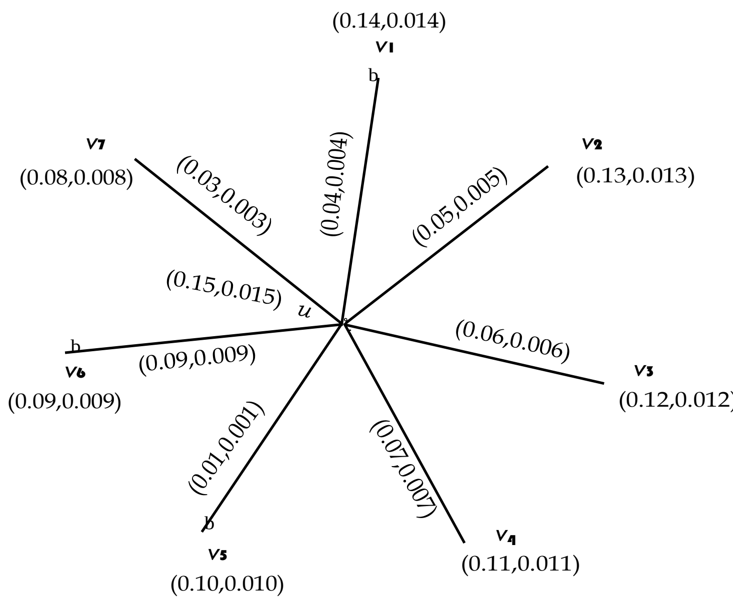
$$\mathrm{Bm}_{\gamma_1}(S_{1,n}) = (\frac{9n+4}{2})s_2,$$

$$\mathrm{Bm}_{\gamma_2}(S_{1,n}) = (\frac{7n+4}{2})s_2.$$

Hence IFB − M labeling of a star graph $S_{1,n}$ are

$\mathrm{Bm}_1(S_{1,n}) = (\mathrm{Bm}_{\mu_1}(S_{1,n}), \mathrm{Bm}_{\gamma_1}(S_{1,n}))$ and $\mathrm{Bm}_2(S_{1,n}) = (\mathrm{Bm}_{\mu_2}(S_{1,n}), \mathrm{Bm}_{\gamma_2}(S_{1,n}))$ □

Example 3.4 (H). *Consider an intuitionistic fuzzy star graph $K_{1,7}$ such that $V = \{u, v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $E = \{(u, v_1), (u, v_2), (u, v_3), (u, v_4), (u, v_5), (u, v_6), (u, v_7)\}$.*



IFB − M labeling of $K_{1,7}$

$\mathrm{Bm}_1(K_{1,7}) = (0.33, 0.033)$ and $\mathrm{Bm}_2(K_{1,7}) = (0.26, 0.026)$. Hence $K_{1,7}$ is an IFB – M labeling graph.

4 Conclusion

In this paper, the concepts of bi-magic labeling on intuitionistic fuzzy path, cycle and star graphs have been discussed. In future we can extend this bi-magic labeling on some intuitionistic fuzzy special graphs.

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